Wave Chaos and Coupling to EM Structures

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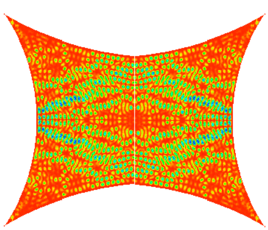
INSTITUTE FOR RESEARCH IN ELECTRONICS & APPLIED PHYSICS



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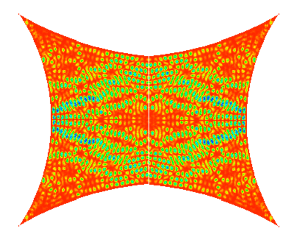
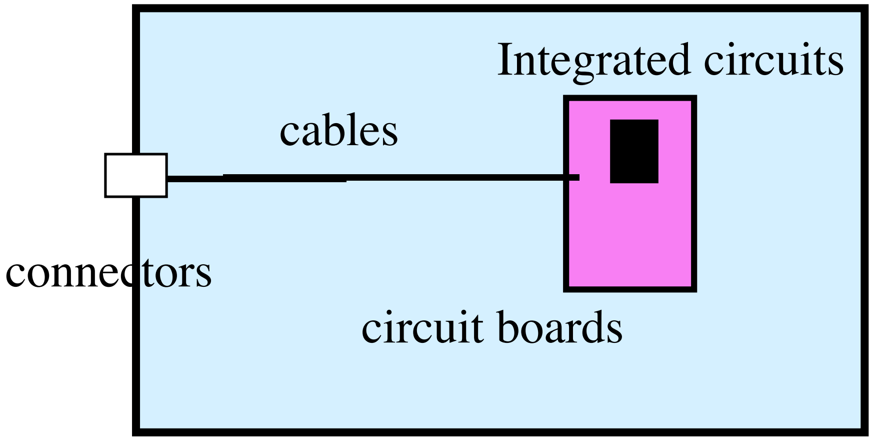


Electromagnetic Coupling in Computer Circuits

Schematic



• What can be said about coupling without solving in detail the complicated EM problem ?

• **Statistical Description !**

(Statistical Electromagnetics, Holland and St. John)

• Coupling of external radiation to computer circuits is a complex processes:

- apertures
- resonant cavities
- transmission lines
- circuit elements

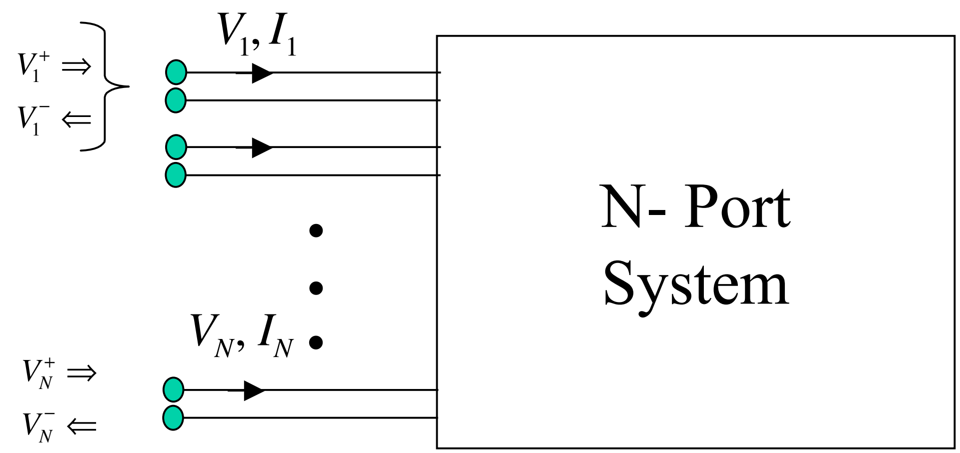
• Intermediate frequency range involves many interacting resonances

• System size >>Wavelength

• Chaotic Ray Trajectories

Z and S-Matrices
What is S_{ij} ?

N ports
- voltages and currents,
- incoming and outgoing waves



Z matrix

$$\begin{pmatrix} V_1 \\ V_2 \\ \vdots \\ V_N \end{pmatrix} = \boldsymbol{Z} \begin{pmatrix} I_1 \\ I_2 \\ \vdots \\ I_2 \end{pmatrix}$$

voltage current

S matrix

$$\begin{pmatrix} V_1^- \\ V_2^- \\ \vdots \\ V_{N1}^- \end{pmatrix} = \boldsymbol{S} \begin{pmatrix} V_1^+ \\ V_2^+ \\ \vdots \\ V_{N1}^+ \end{pmatrix}$$

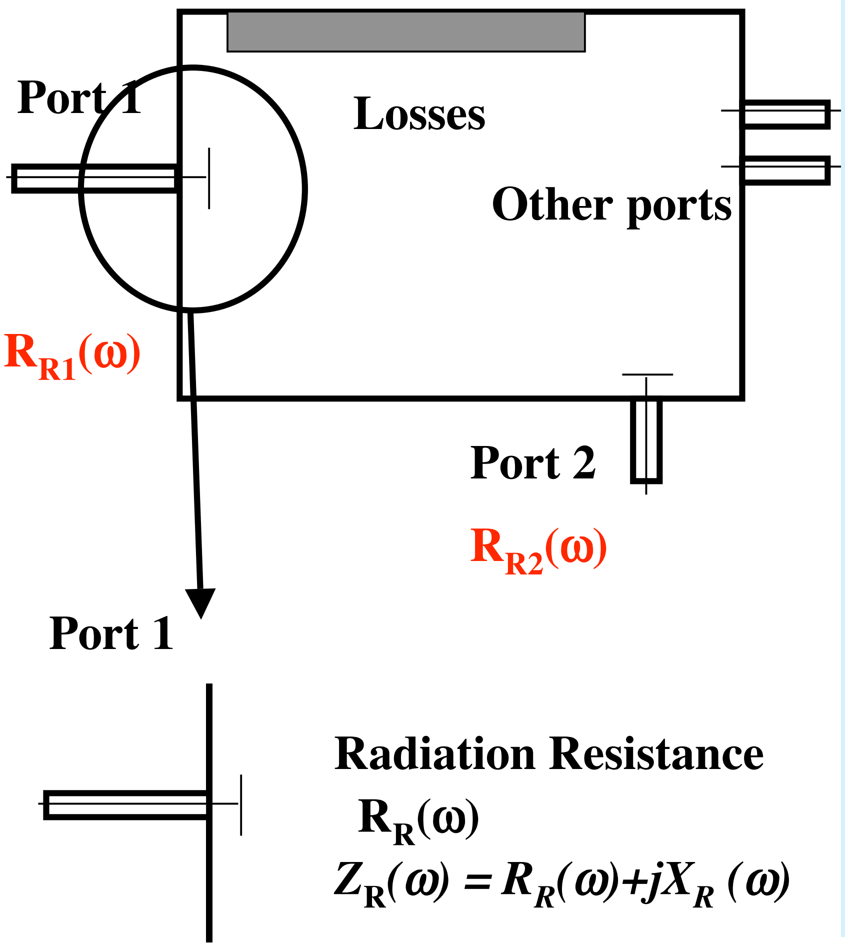
outgoing incoming

$$\boldsymbol{S} = (\boldsymbol{Z} + \boldsymbol{Z}_0)^{-1}(\boldsymbol{Z} - \boldsymbol{Z}_0)$$

$\boldsymbol{Z(\omega)}$, $\boldsymbol{S(\omega)}$

- Complicated function of frequency
- Details depend sensitively on unknown parameters

Statistical Model of Z Matrix



Statistical Model Impedance

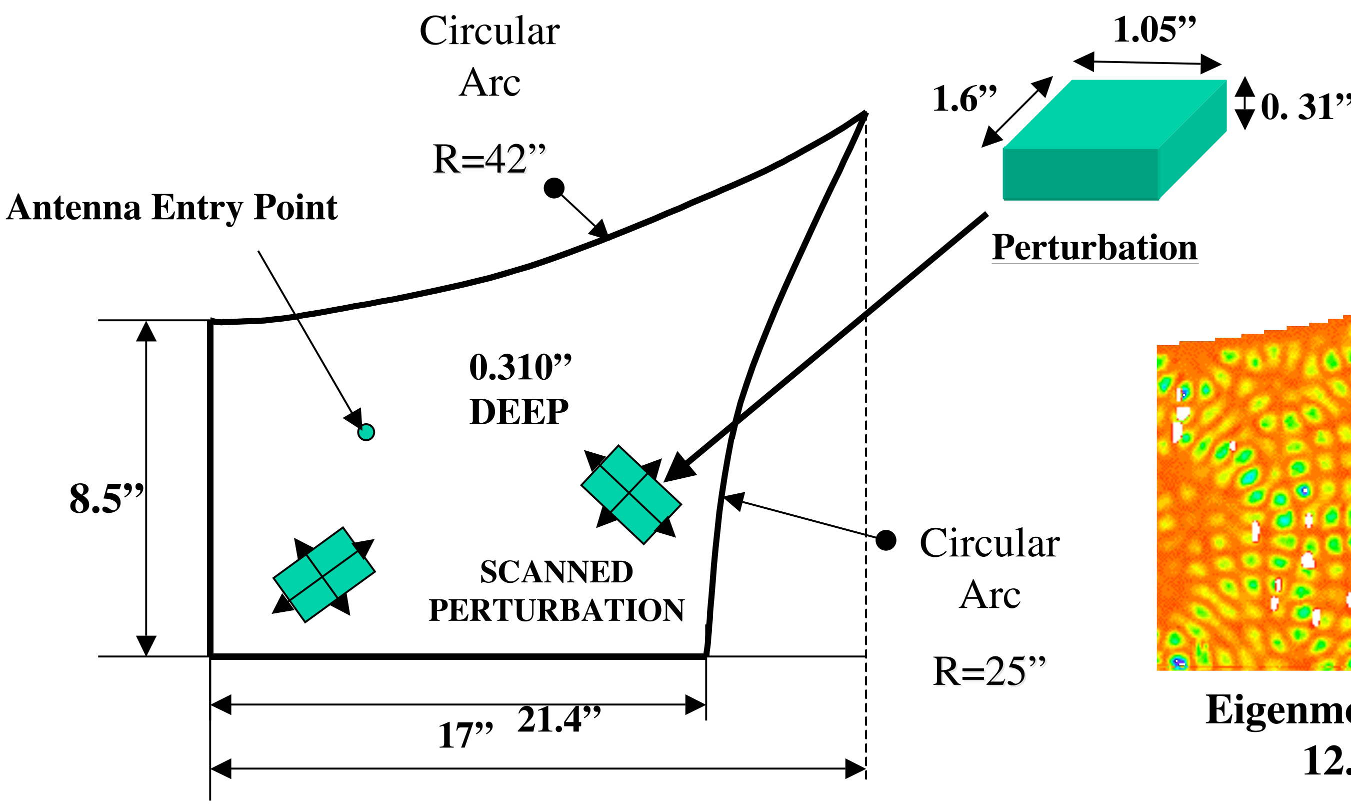
$$Z_{ij}(\omega) = -\frac{j}{\pi}\sum_n R_{Ri}^{1/2}(\omega_n) R_{Rj}^{1/2}(\omega_n) \frac{\Delta\omega_n^2 \, w_{in} w_{jn}}{\omega^2(1+jQ^{-1}) - \omega_n^2}$$

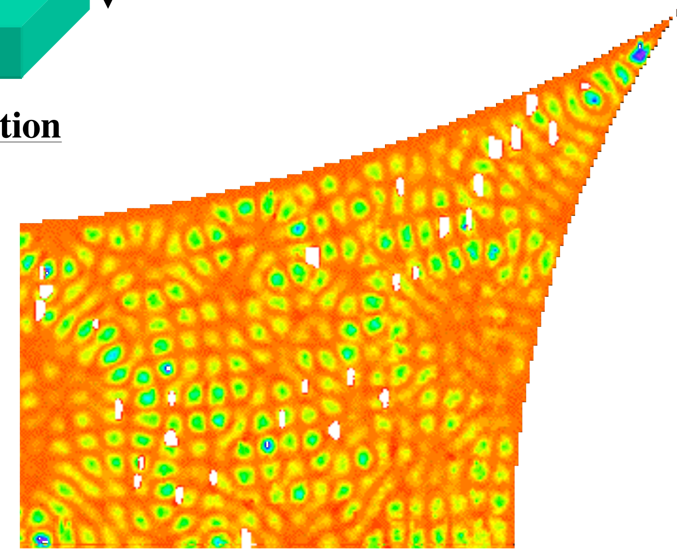
System parameters
- **Radiation Resistance** $R_{Ri}(\omega)$
- $\Delta\omega_n^2$ **- mean spectral spacing**
- Q **-quality factor**

Statistical parameters
- ω_n **- random spectrum**
- w_{in}**- Guassian Random variables**

EXPERIMENTAL Test

Sameer Hemmady, Steve Anlage

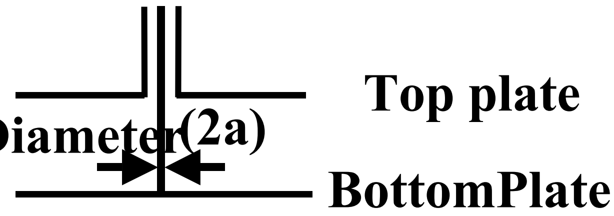




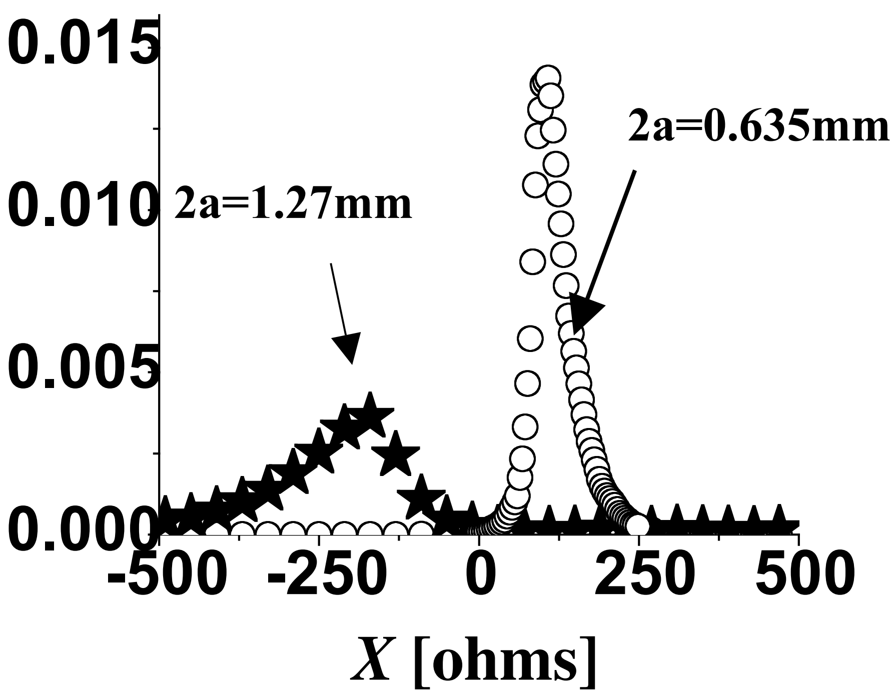
Eigenmode Image at 12.57GHz

- 2 Dimensional Quarter Bow Tie Wave Chaotic cavity
- Classical ray trajectories are chaotic - short wavelength - Quantum Chaos
- 1-port S and Z measurements in the 6 – 12 GHz range.
- Ensemble average through 100 locations of the perturbation

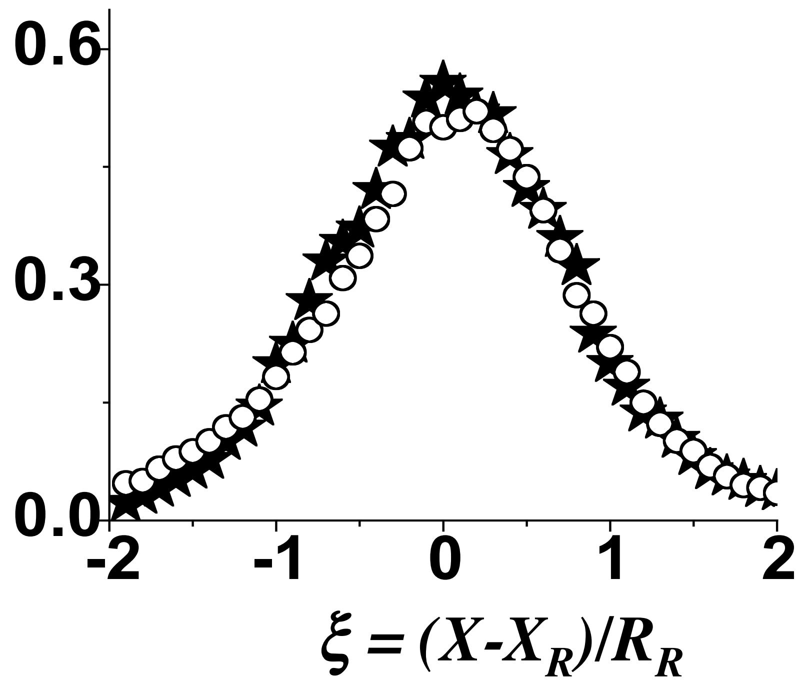
Importance of Normalization



Raw Data



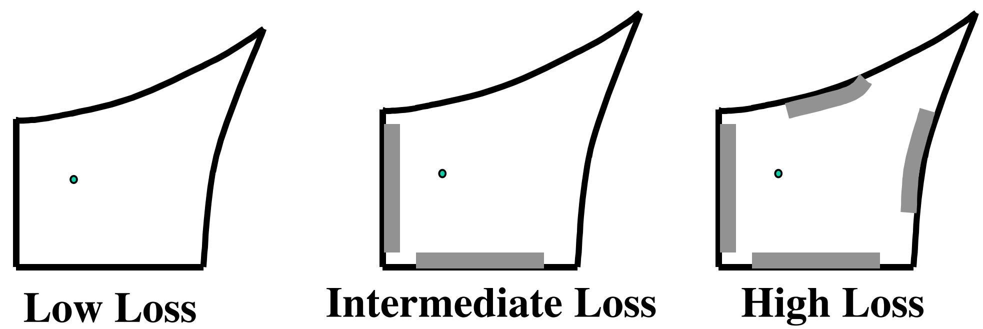
Normalized

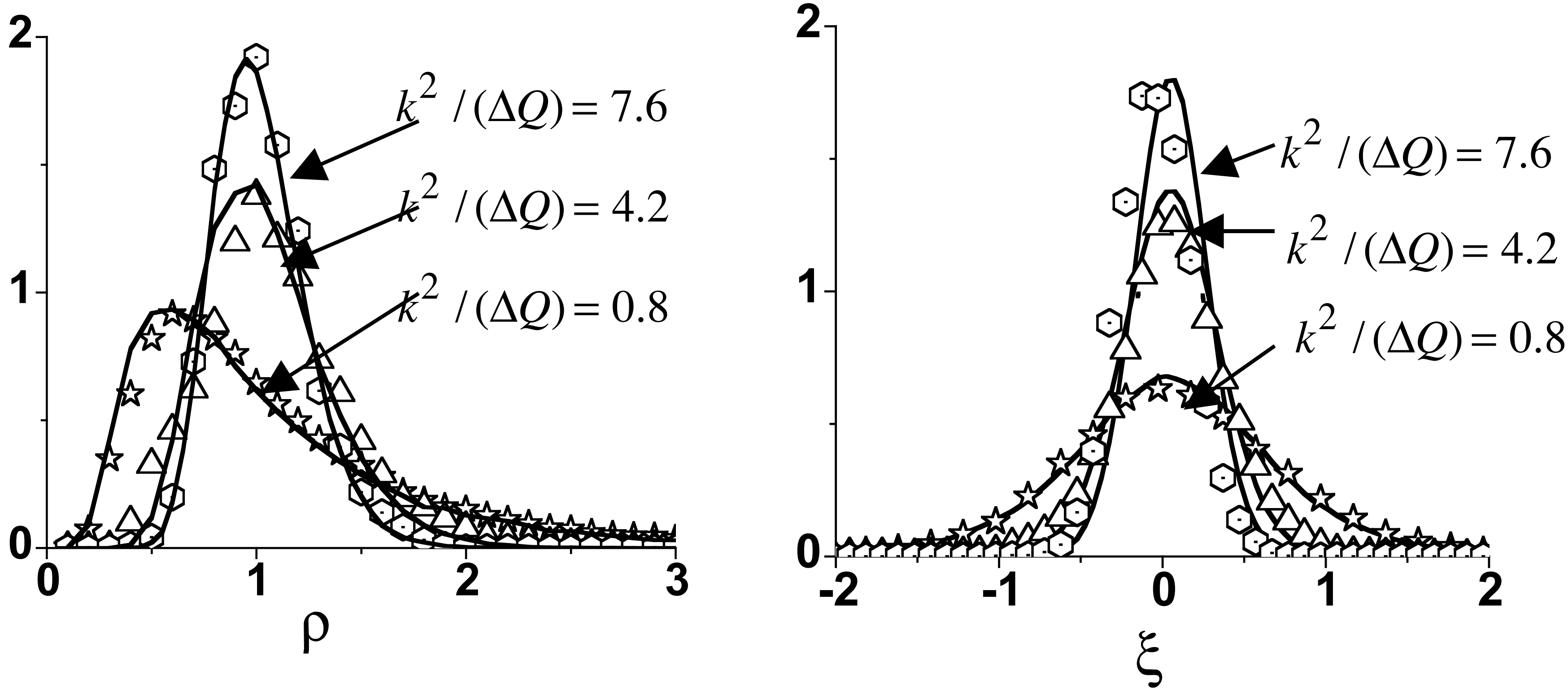


Testing the Effects of Increasing Loss

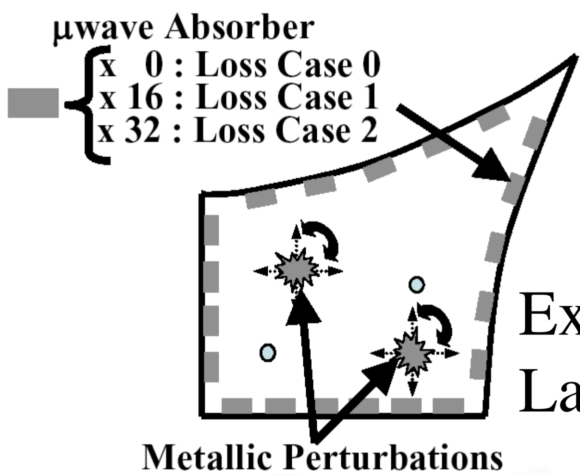
$$\rho = \mathrm{Re}(Z)\,/\,R_R$$

$$\xi = (\mathrm{Im}(Z) - X_R)\,/\,R_R$$



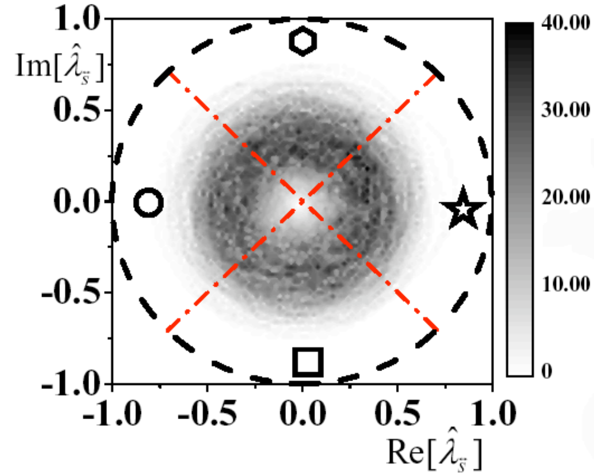


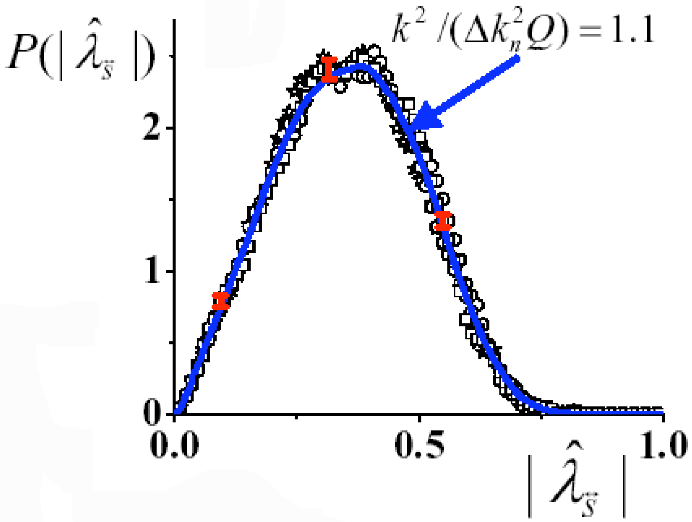
Multiple Ports

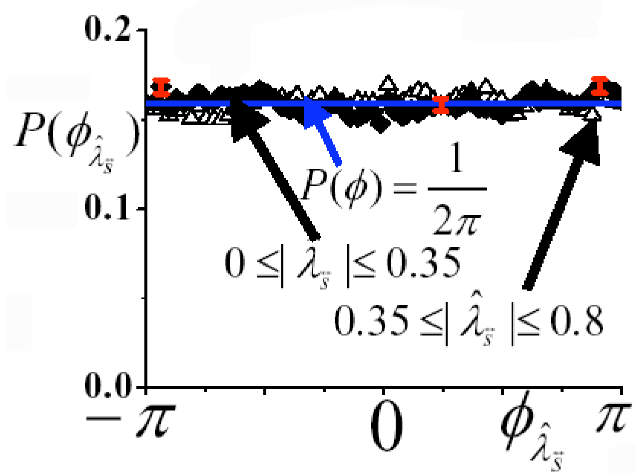


Experimental Layout

Complex eigenvalues
Normalizes S-matrix







Time Domain Model for Impedance Matrix

Frequency Domain

w_n- Guassian Random variables

$$Z(\omega) = -\frac{j\omega}{\pi}\sum_n \frac{R_R(\omega_n)}{\omega_n}\frac{\Delta\omega_n^2\, w_n^2}{\omega^2(1-jQ^{-1})-\omega_n^2}$$

Statistical Parameters

ω_n - random spectrum

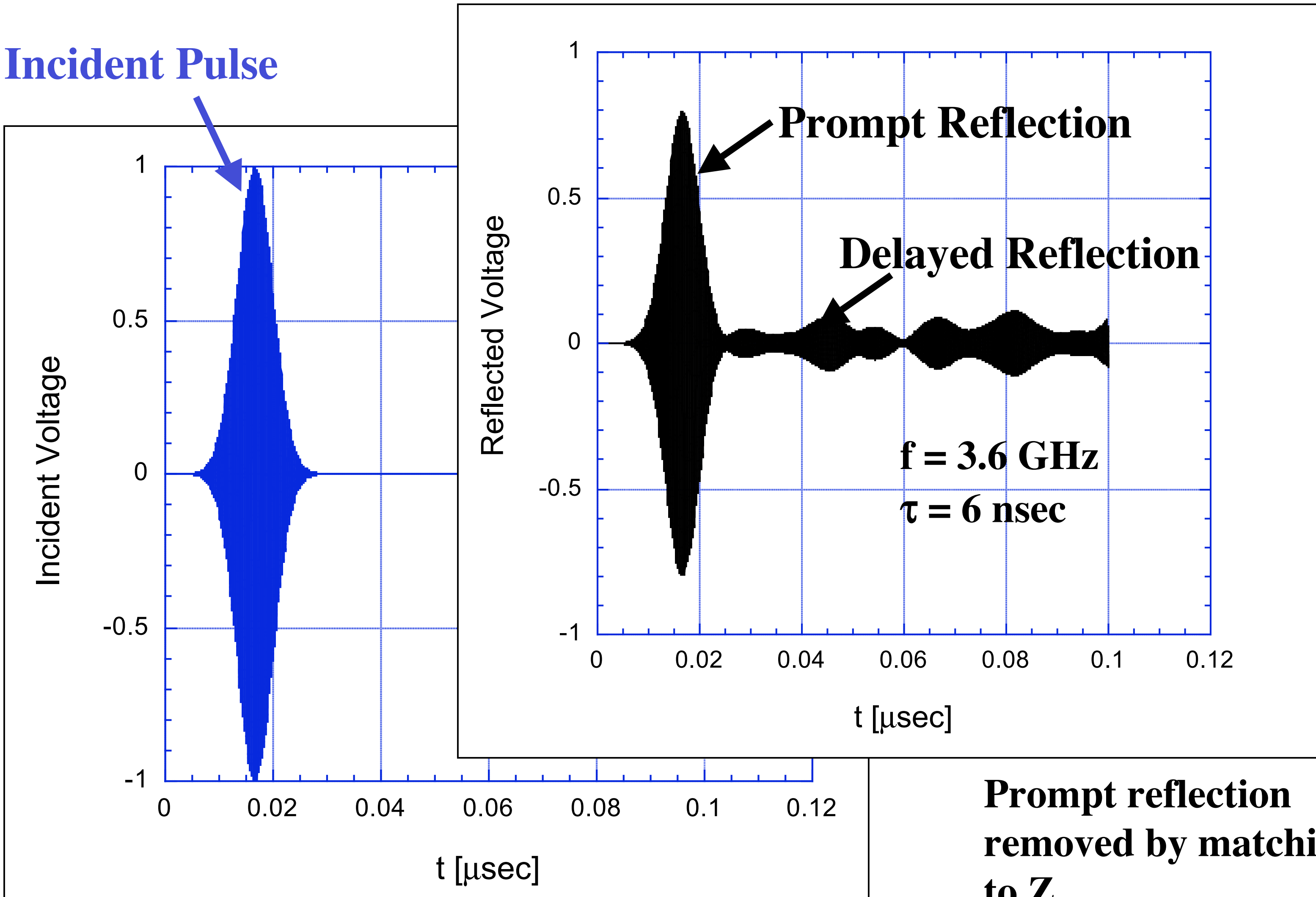
Time Domain

w_n- Guassian Random variables

$$\left(\frac{d^2}{dt^2}+2\nu_n\frac{d}{dt}+\omega_n^2\right)V_n(t) = -\frac{1}{\pi}\frac{R_R(\omega_n)\Delta\omega_n^2 w_n^2}{\omega_n}\frac{d}{dt}I(t)$$

$$V(t) = \sum_n V_n(t) \qquad \nu_n = \frac{\omega_n}{Q}$$

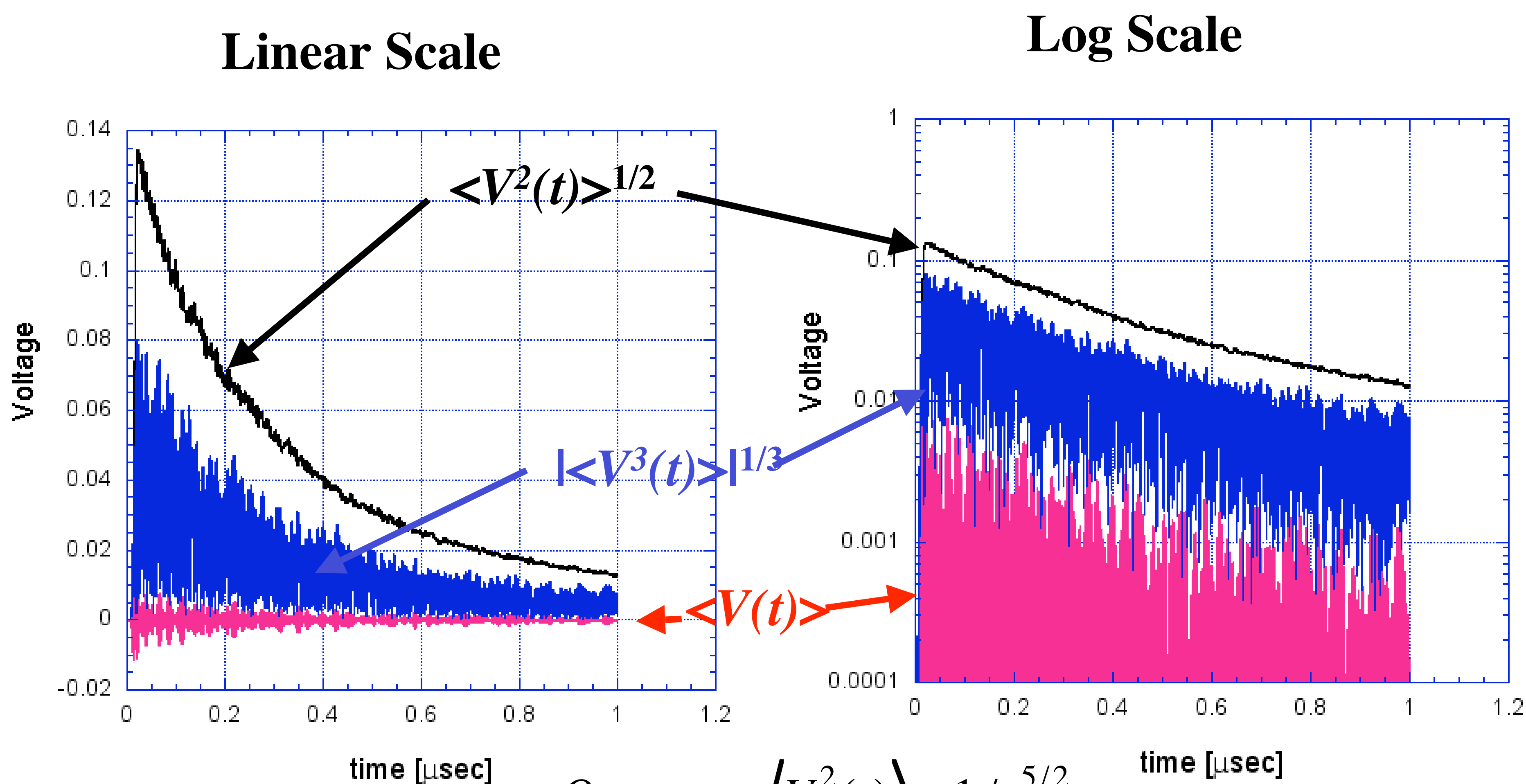
Incident and Reflected Pulses for One Realization



Prompt reflection removed by matching Z_0 to Z_R

Decay of Moments
Averaged Over 1000 Realizations

Prompt reflection eliminated



$$Q = \infty, \qquad \left\langle V^2(t) \right\rangle \approx 1/t^{5/2}$$

Decay of Port Voltage - Lossless Case

- One Port with an Incident Pulse: $\left\langle V^2(t)\right\rangle \approx 1/t^{5/2}$

- Two Ports Excited Through Port 1:

a) Ports 1 and 2 matched to Z_{rad} $\left\langle V_1^2(t)\right\rangle = 2\left\langle V_2^2(t)\right\rangle \approx 1/t^3$

$R_{rad\ 1,2} = Z_{0\ 1,2}$

$X_{rad\ 1,2} = 0$

b) Port 1 matched $\left\langle V_1^2(t)\right\rangle \approx 1/t^{5/2}$

Port 2 strongly mismatched $\left\langle V_2^2(t)\right\rangle \approx 1/t^{3/2}$

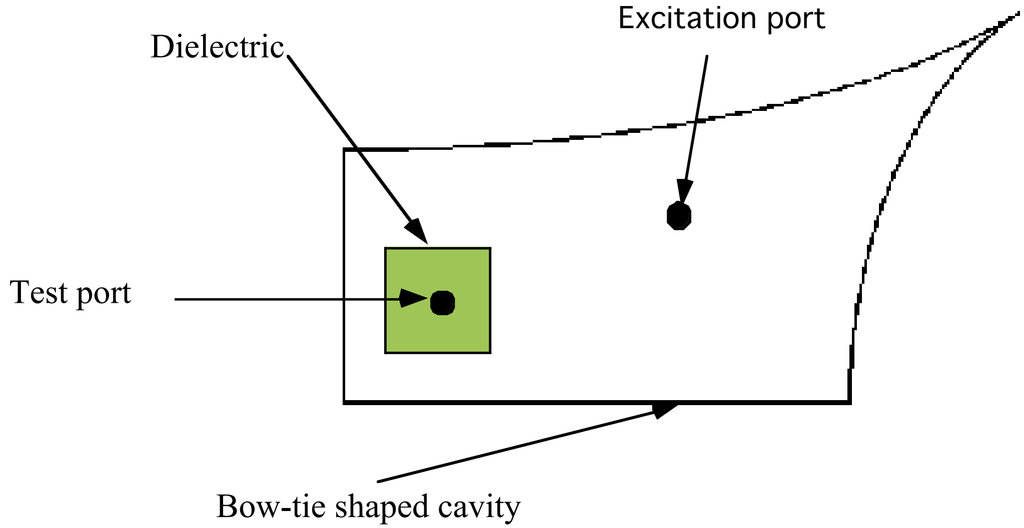
- N Ports Excited Through Port 1: $\left\langle V_1^2(t)\right\rangle = 2\left\langle V_{i\neq 1}^2(t)\right\rangle \approx 1/t^{(4+N)/2}$

All ports matched

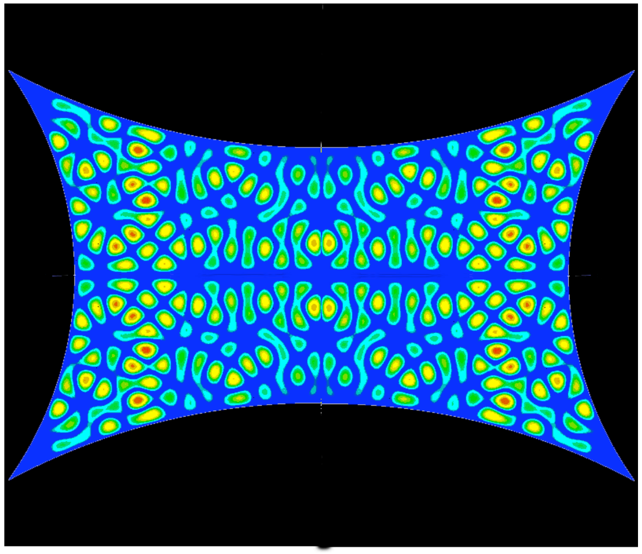
Future Work:

- Coupling in complex geometries involving enclosures, circuits and cables
- Scars - “Anomalous” hot spots
- Networks formed by transmission line links
- Statistical aspects of coupling of pulsed signals - fidelity

More Complexity in the Scatterer



- Can be addressed
  - -theoretically
  - -numerically
  - -experimentally



HFSS simulation courtesy J. Rodgers

Features:

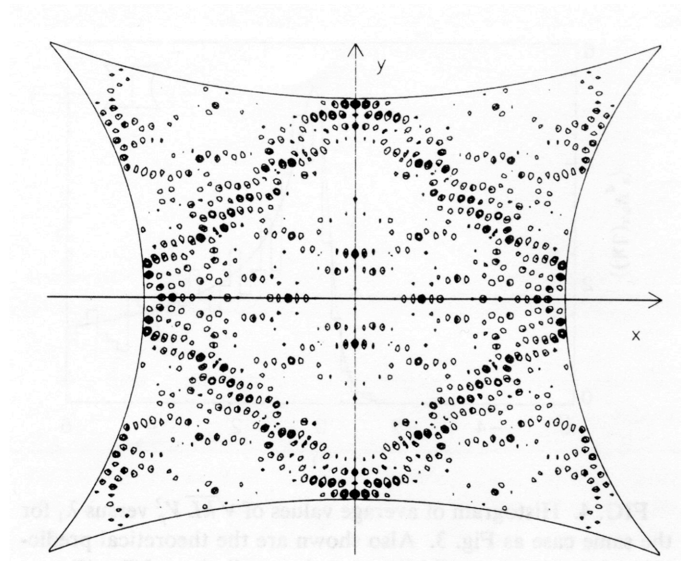
Ray splitting

Losses

Additional complications to be added later

Role of Scars?

- Eigenfunctions that do not satisfy random plane wave assumption



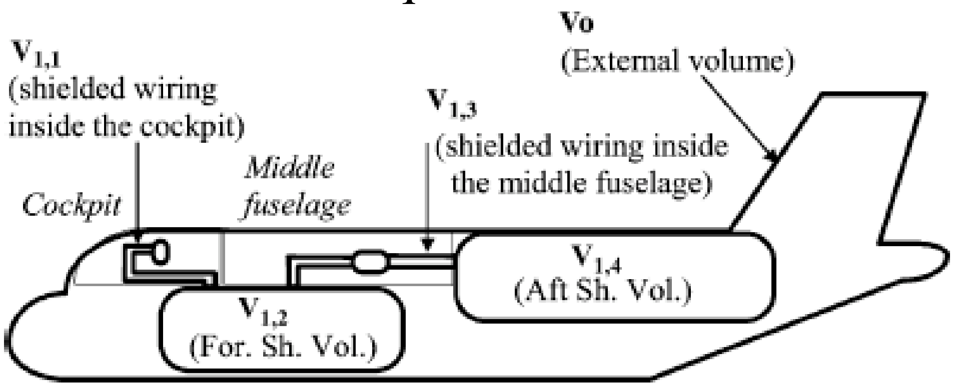
Bow-Tie with diamond scar

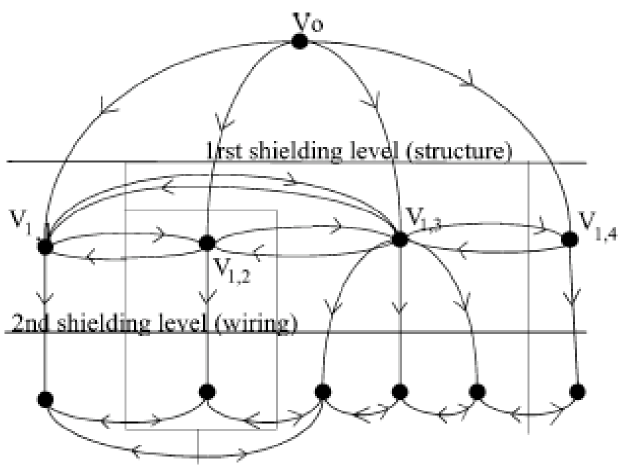
- Scars are not treated by either random matrix or chaotic eigenfunction theory
- Semi-classical methods

Ref: Antonsen et al., Phys. Rev E **51**, 111 (1995).

Electromagnetic Topology and Quantum Graphs

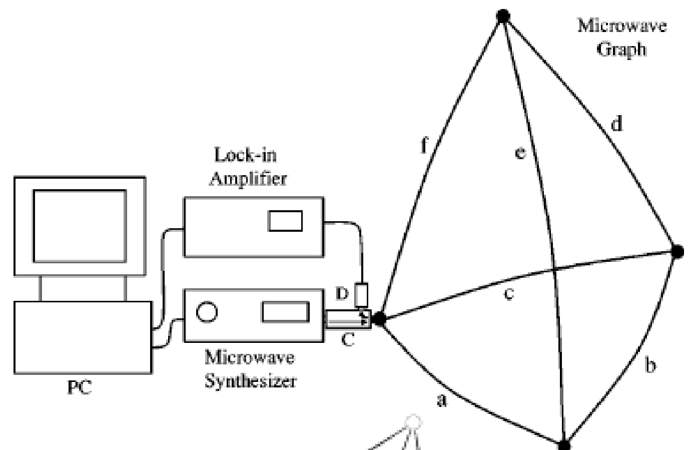
Electromagnetic Topology
BLT Equations

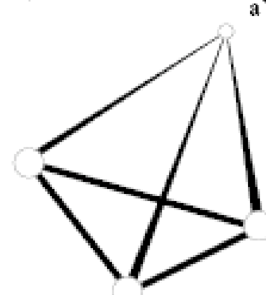




J.-P. Parmantier, IEEE Trans. Electromag. Compat. 46 (3) 359-367 (2004). "Numerical coupling models for complex systems and results"

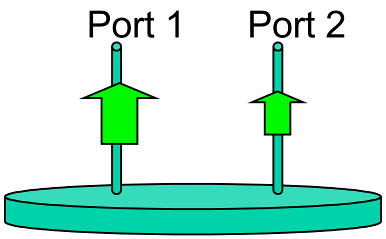
Quantum Graphs
Random Matrix Theory

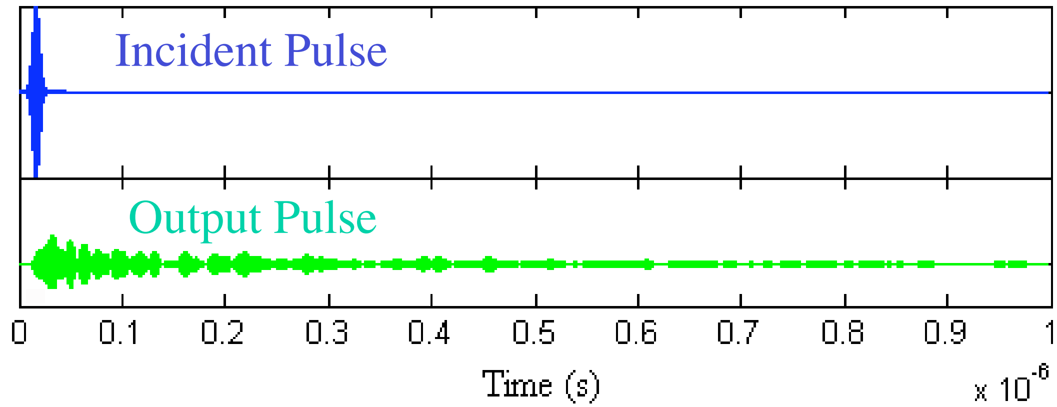


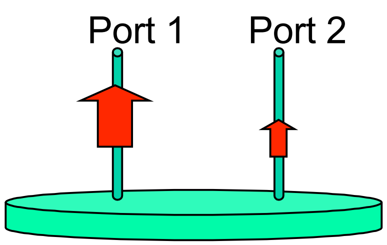


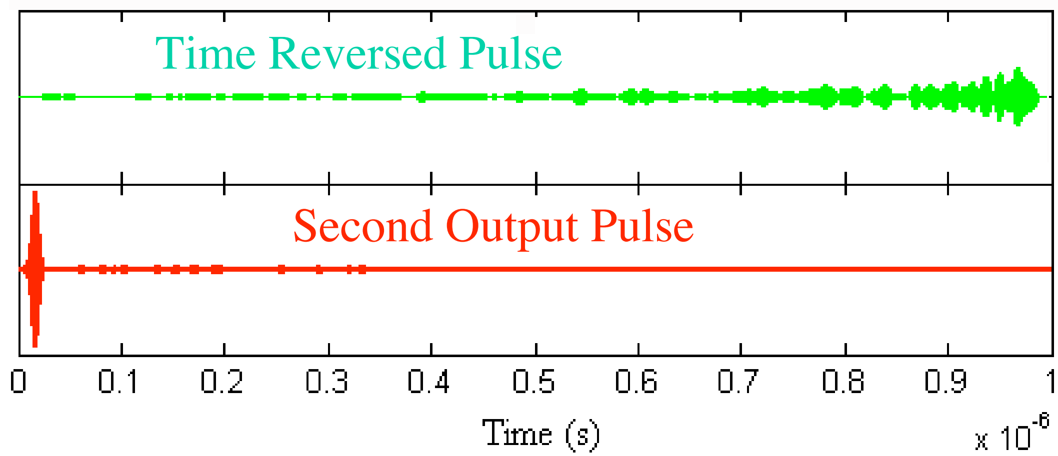
O. Hul, et al., Phys. Rev. E 69, 056205 (2004). "Experimental simulation of quantum graphs by microwave networks"

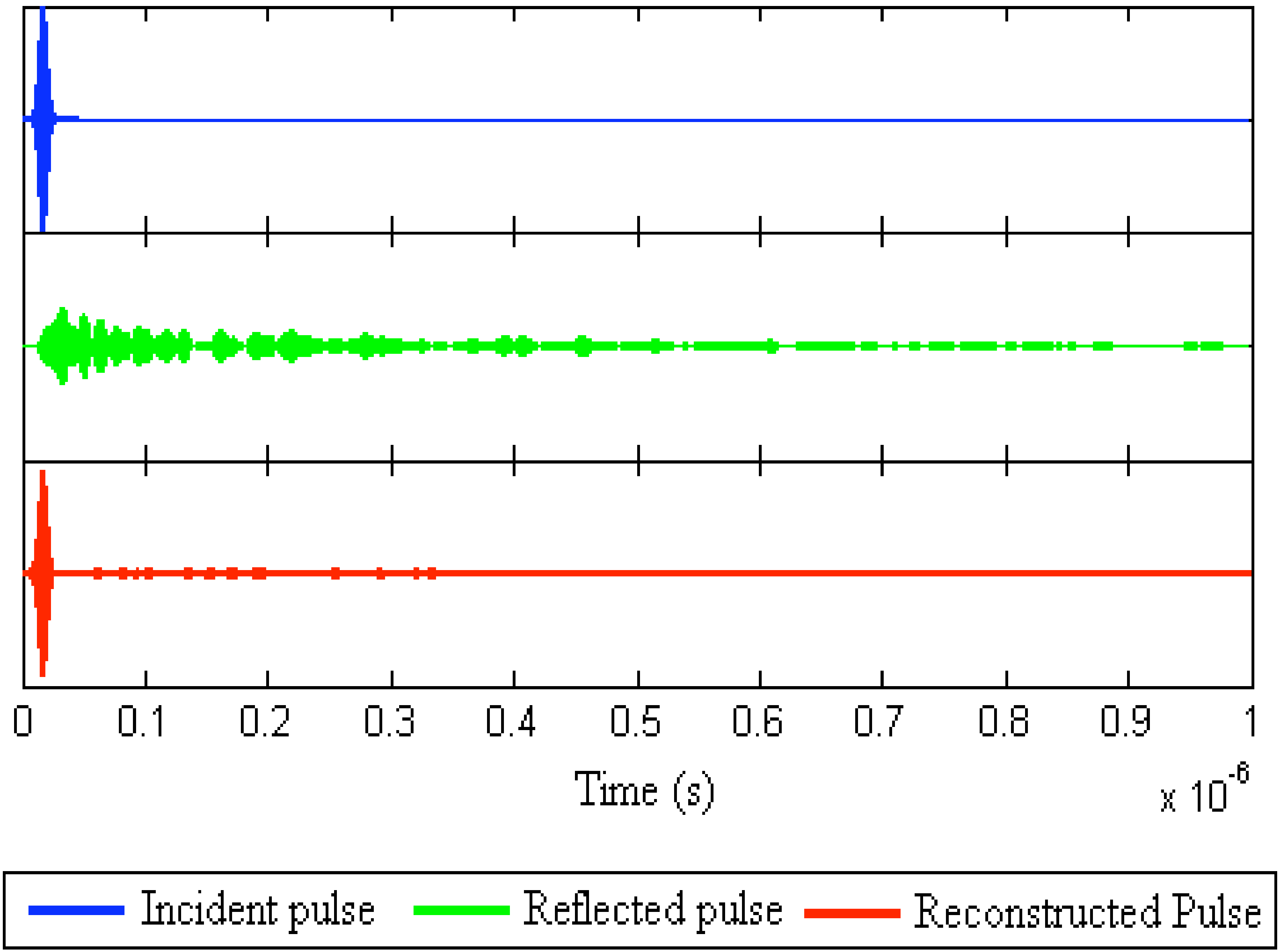
Fidelity of Pulses










0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
Time (s)
x 10-6
Incident pulse
Reflected pulse
Reconstructed Pulse

Publications

1. Zheng X, **Antonsen TM** , Ott E Statistics of impedance and scattering matrices in chaotic microwave cavities: Single channel case ELECTROMAGNETICS 26 (1): 3-35 JAN 2006
2. Zheng, X, **Antonsen TM** , Ott E Statistics of impedance and scattering matrices of chaotic microwave cavities with multiple ports ELECTROMAGNETICS 26 (1): 37-55 JAN 2006
3. Hemmady S, Zheng X, **Antonsen TM** , et al. Universal statistics of the scattering coefficient of chaotic microwave cavities PHYSICAL REVIEW E 71 (5): Art. No. 056215 Part 2 MAY 2005
4. Hemmady S, Zheng X, Ott E, et al. Universal impedance fluctuations in wave chaotic systems PHYSICAL REVIEW LETTERS 94 (1): Art. No. 014102 JAN 14 2005