Inducing Chaos in the p/n Junction

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Motivation

- Identify the origins of chaos in the driven resistor – inductor - varactor diode series circuit
- Establish a “universal” picture of chaos in circuits containing p/n junctions
- Identify new opportunities to induce chaos exploiting the p/n junction nonlinearity

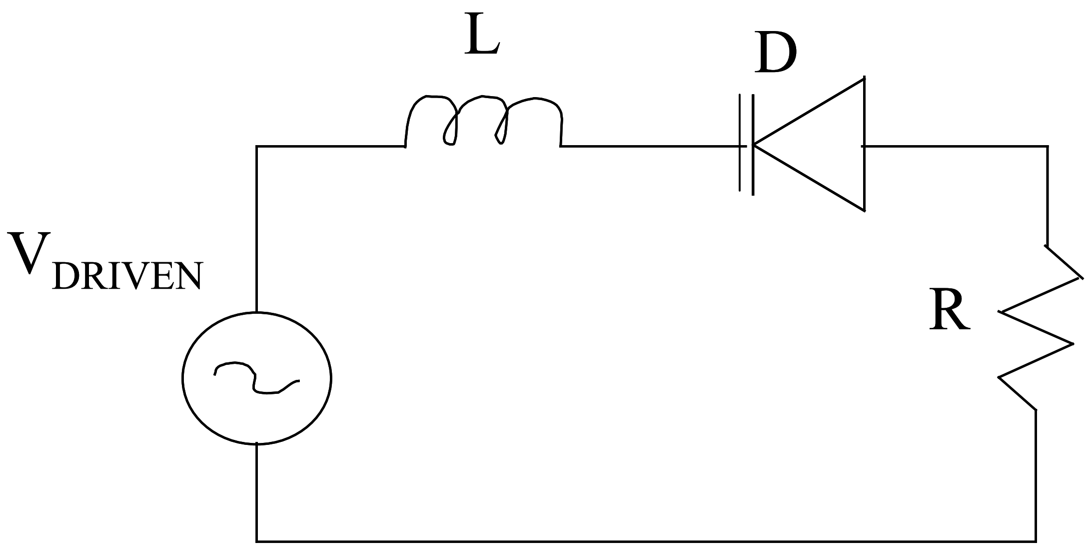
$\Rightarrow$ John Rodgers’ talk

Driven Resistor-Inductor-Diode Circuit

Studied since the 1980's

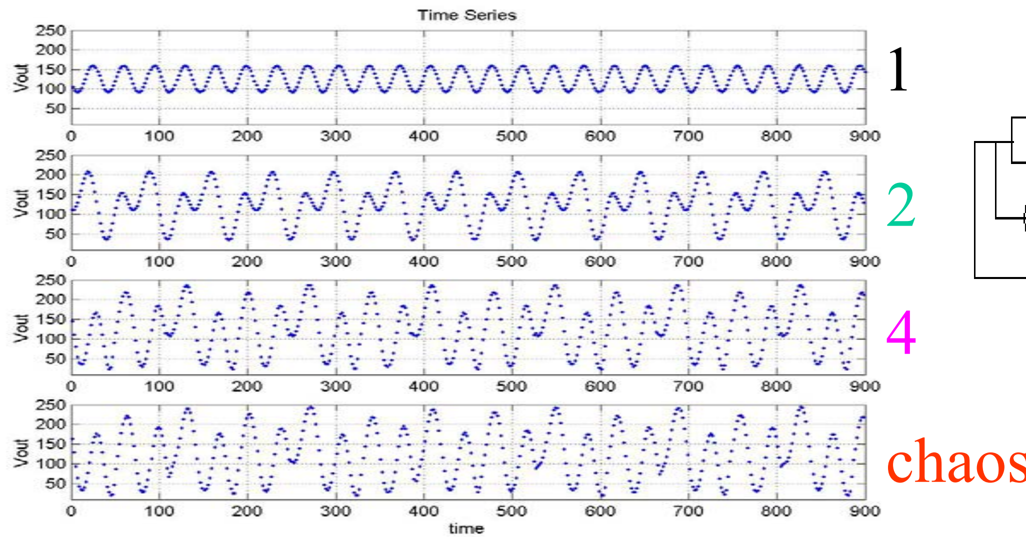
Why is the driven RLD circuit so important?

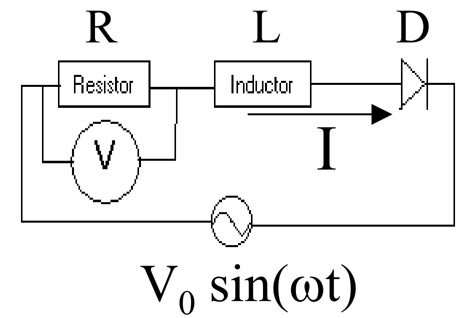
- Simplest passive circuit that displays period doubling and chaos
- It is a good model of the ubiquitous p-n junction and its nonlinearities



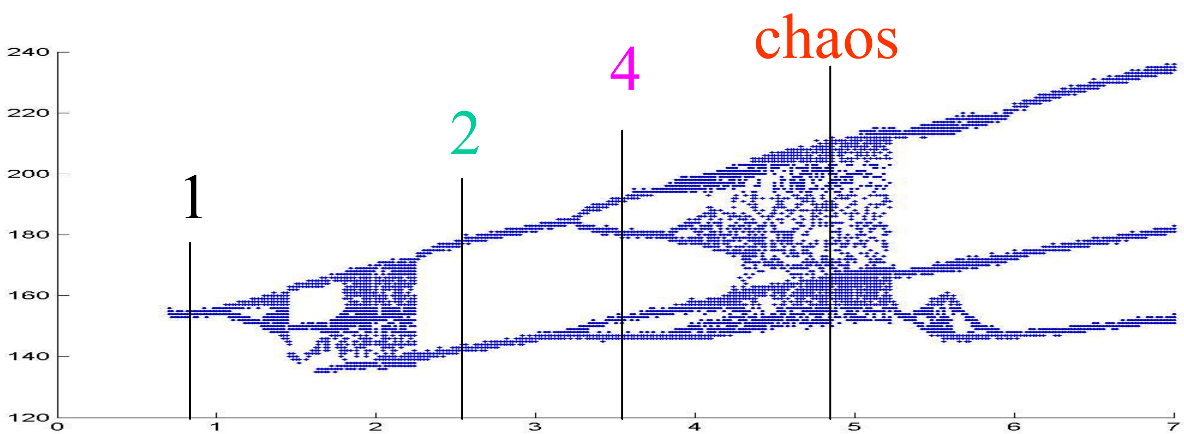
Chaos in the Driven RLD Circuit

Voltage across Resistor R ~ I





Maximum Voltage across Resistor R

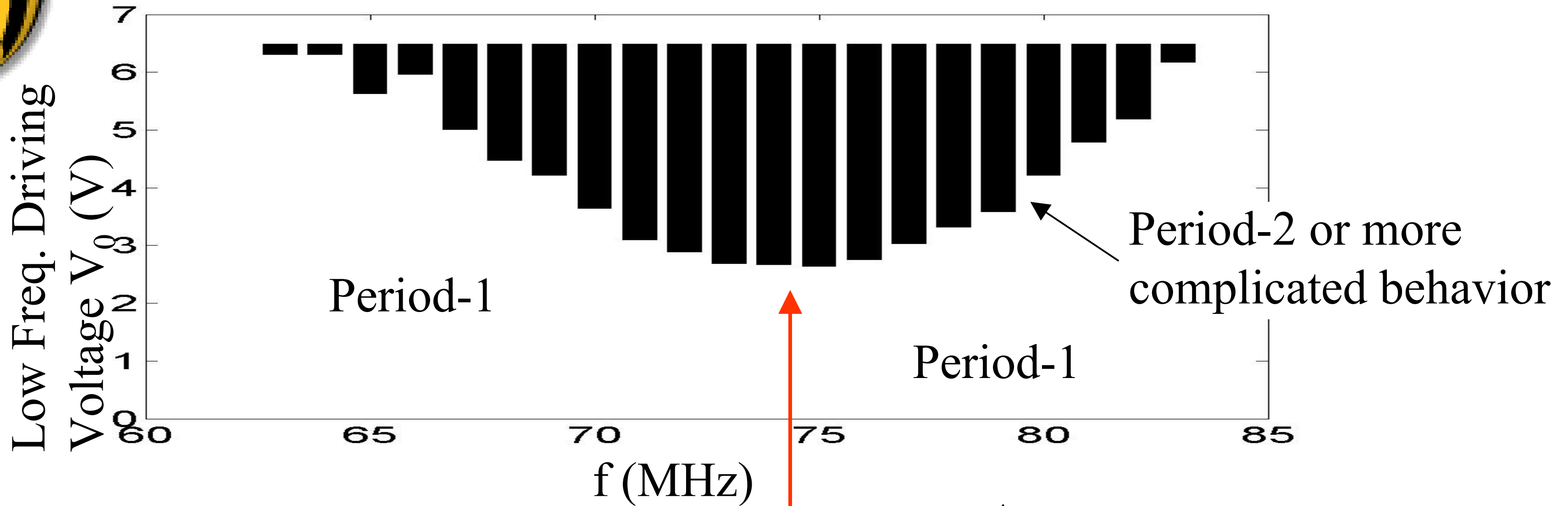


Driving Amplitude V_0 (V)

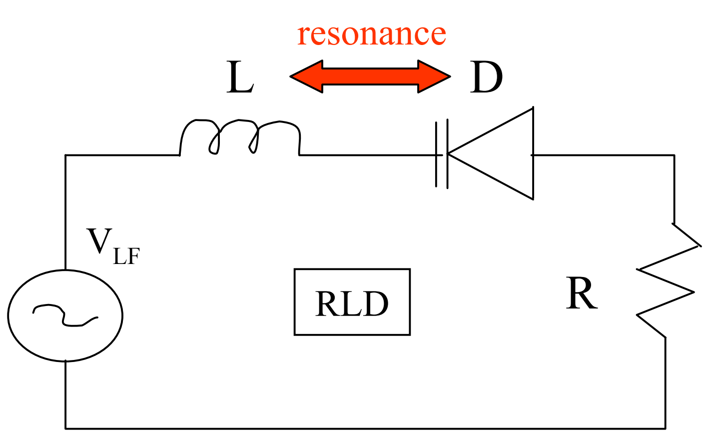
Bifurcation diagram

R = 25 Ω
L = 50 μH
D = NTE610
f = 2.5 MHz

Period-doubling phase diagrams



$$f_0 \approx 1 / \sqrt{L C_j(0)}$$



$V_{LF} = V_0 \cos(2\pi\, f\, t)$

Diode = NTE610

L = 390 nH

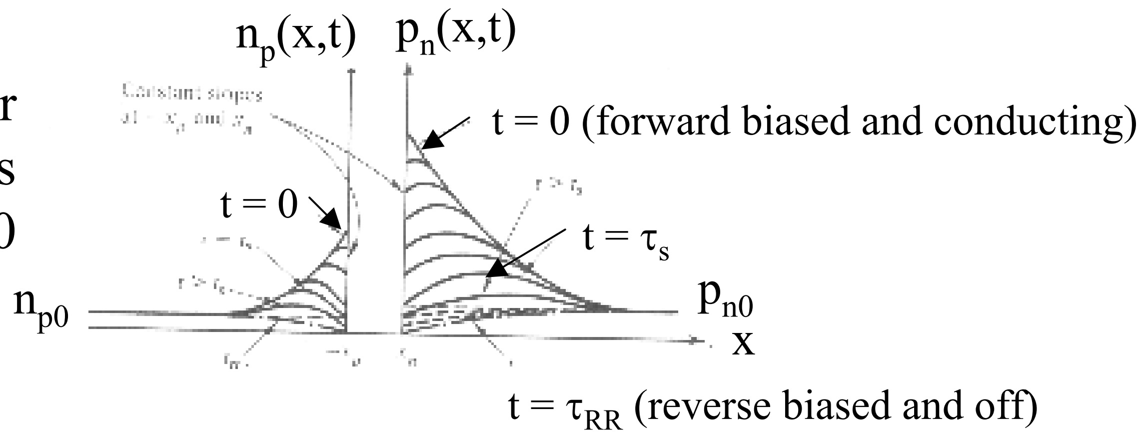
R = 25 Ω

$f_0 \approx 70$ MHz

Nonlinearity of the p-n Junction

The diffusive dynamics of majority and minority charge carriers in the p-n junction is complex and nonlinear

Charge distribution profiles for a forward-biased diode that is suddenly switched off at t = 0



All models of chaotic dynamics in p-n junctions approximate the charge dynamics using nonlinear lumped-elements

Resistor-Inductor-Diode Circuit

What is the cause of chaos?

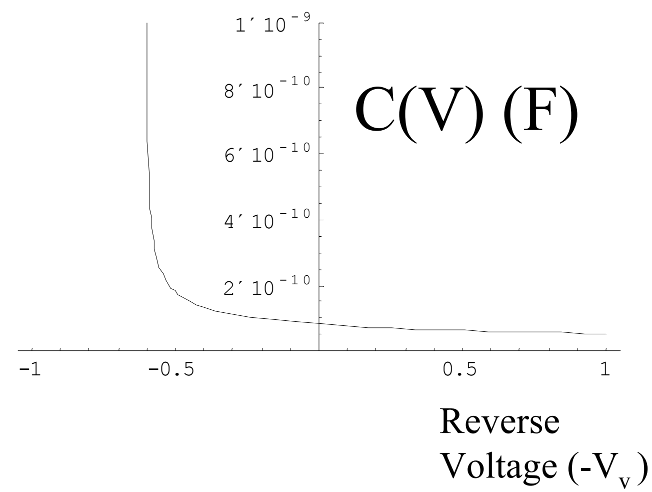
There are 3 competing forms of nonlinearity in this problem:

1. Nonlinear I-V curve $I_{rv}(Q)$. Traditional focus $\Rightarrow$ rectification

2. Nonlinear C(V) => $V_v(Q)$. C increases by x4 $\Rightarrow$ period doubling

   Van Buskirk + Jeffries, Chua, Crevier, etc.

$$f_0 = \frac{1}{2\pi\sqrt{LC(V)}}$$



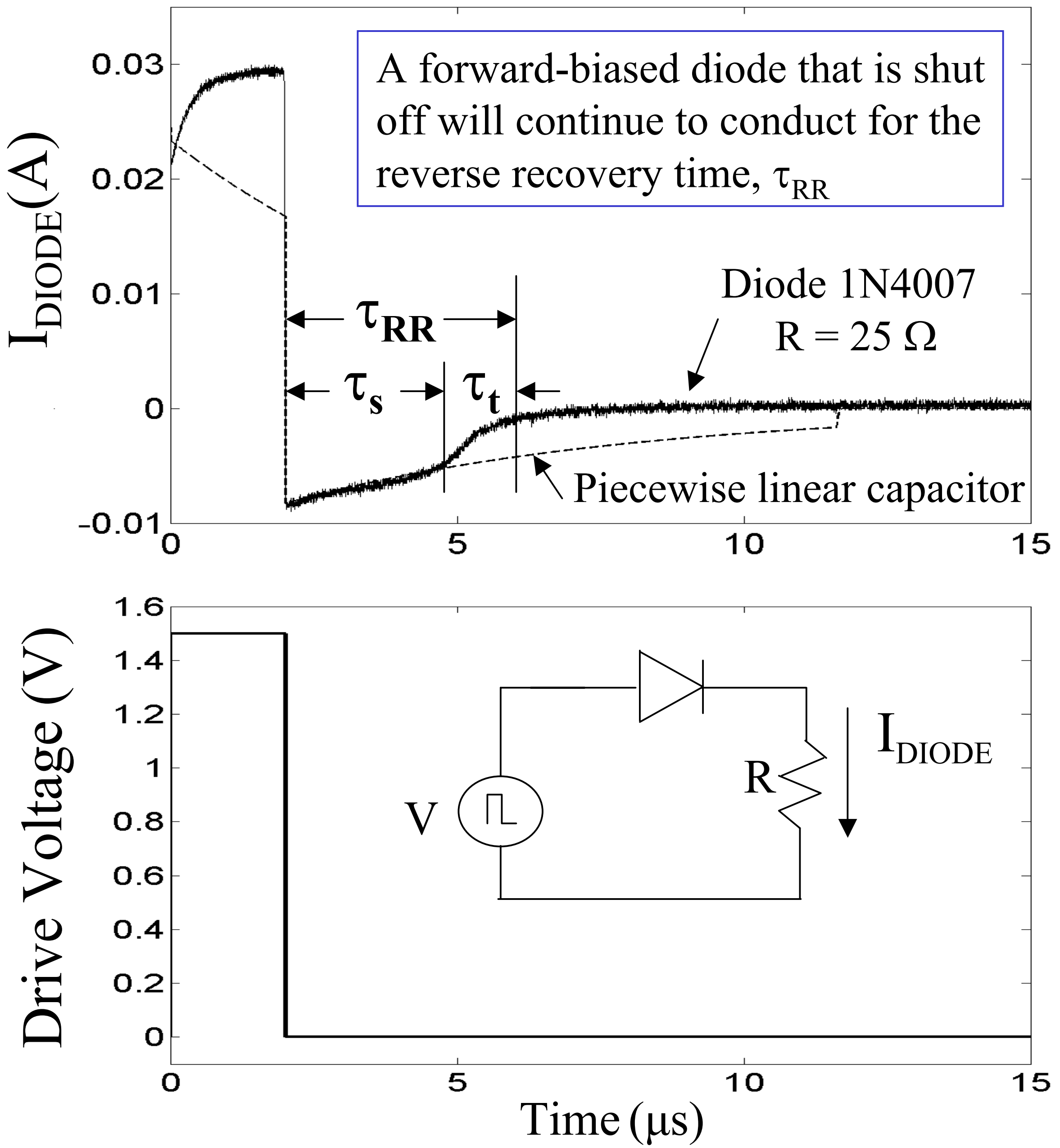
3. Finite minority carrier lifetime or reverse recovery time. Delayed feedback

   The p/n junction retains memory of previous fwd-bias current swings

   Rollins + Hunt

=> No consensus on the origin of chaos

Reverse Recovery Time τ_{RR}

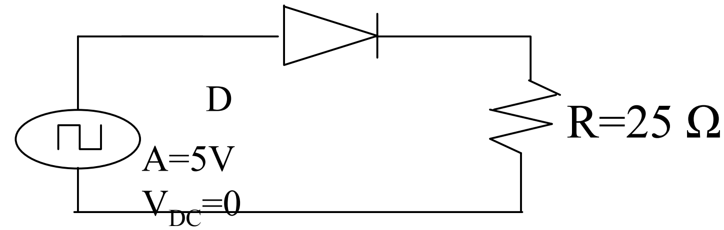


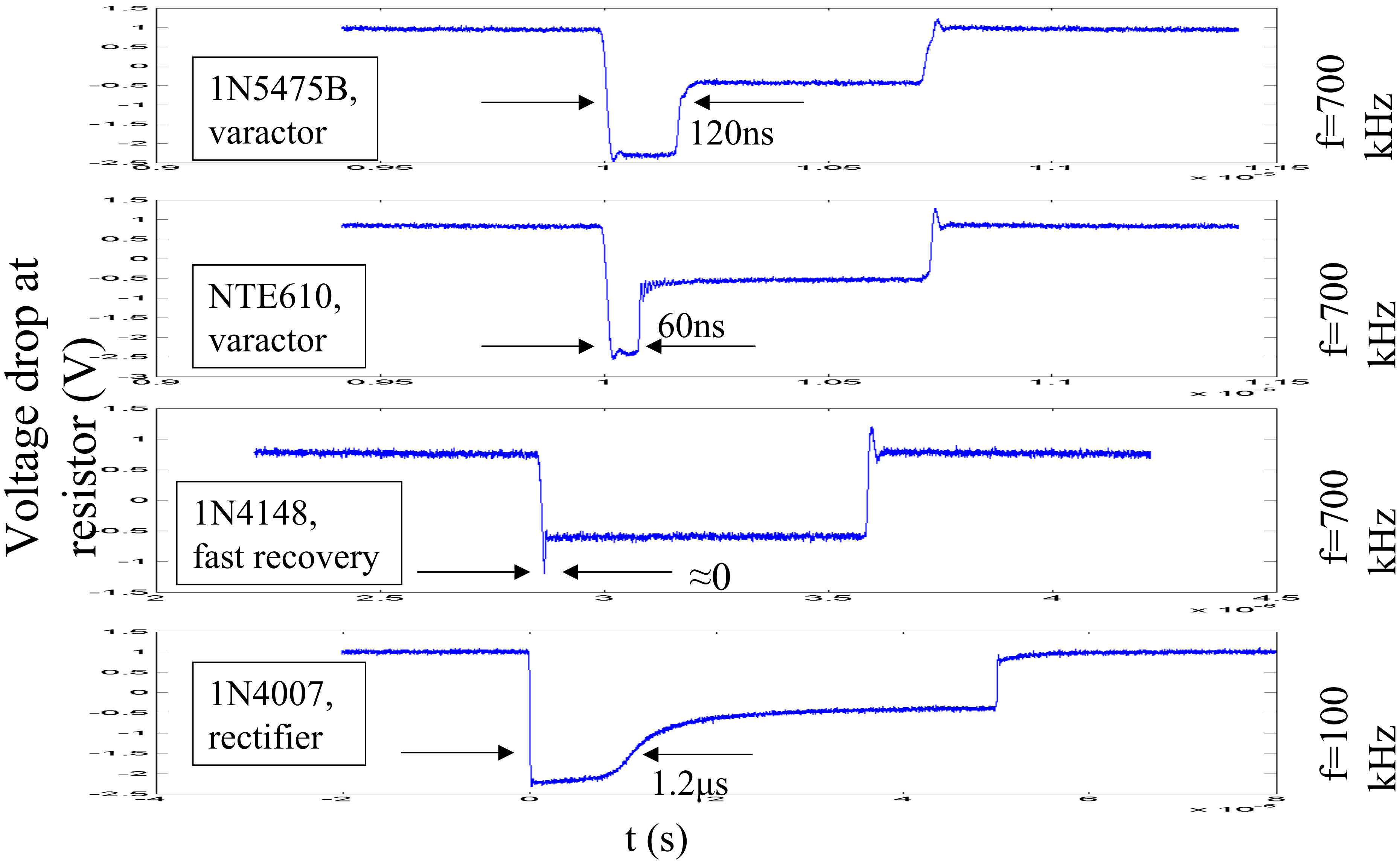
τ_s = **storage time**

τ_t = **transition time**

$\tau_{RR} = \tau_s + \tau_t$

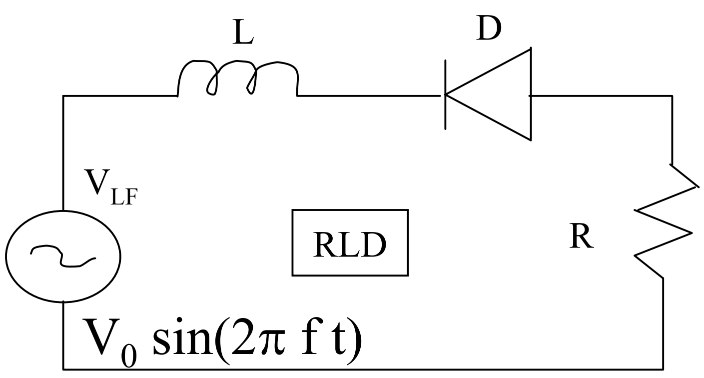
τ_{rr} vs. Diode





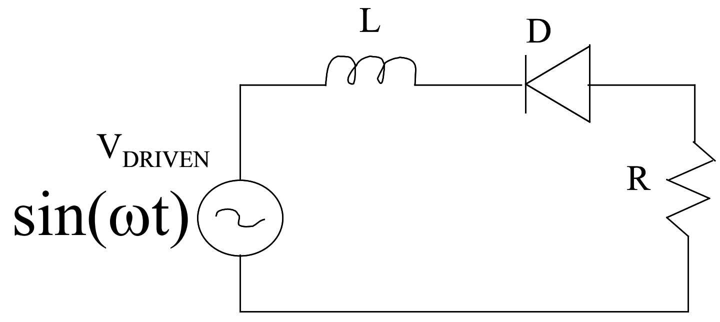
Search for Period Doubling and Chaos in Driven RLD Circuit

Diode	τ_{RR} (ns)	C_j (pF)	Results with $f_0 \sim 1/\tau_{RR}$	Results with $f_0 \sim 10/\tau_{RR}$	Results with $f_0 \sim 100/\tau_{RR}$
1N5400	7000	81	Period-doubling and chaos for $f/f_0 \sim 0.11 - 1.64$	Period-doubling and chaos $f/f_0 \sim 0.16 - 1.76$	No chaos, nor period-doubling
1N4007	700	19	Period-doubling and chaos for $f/f_0 \sim 0.13 - 2$	Period-doubling and chaos for $f/f_0 \sim 0.23 - 1.3$	No period doubling or chaos
1N5475B	160	82	Period-doubling and chaos for $f/f_0 \sim 0.66 - 2.2$	No chaos, nor period-doubling	No chaos, nor period-doubling
NTE610	45	16	Period-doubling and chaos for $f/f_0 \sim 0.14 - 3.84$	Period-doubling only for $f/f_0 \sim 1.17 - 3.25$	No chaos, nor period-doubling



$$f_0 = \frac{1}{2\pi\sqrt{LC_j}}$$

Circuit Chaos: Rule of Thumb
Driven Nonlinear Diode Resonator



For lumped element nonlinear diode resonator circuits, Period Doubling and Chaos are observed for sufficiently large driving amplitudes when;

$$\omega \sim \omega_0/10 \leftrightarrow 4\,\omega_0$$

<u>and</u>

$$\omega_0 < 10/\tau_{RR} \text{ to } 100/\tau_{RR}$$

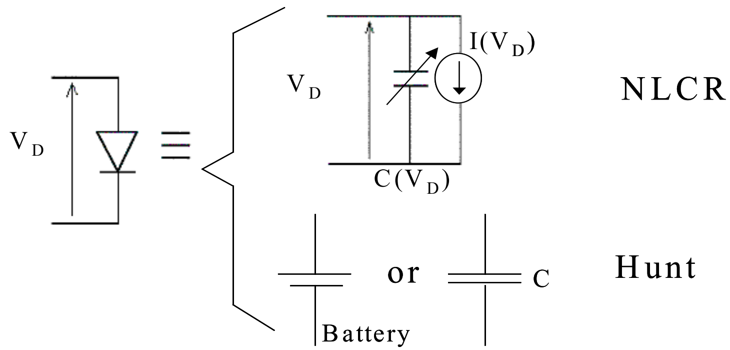
where $\omega_0 = \frac{1}{\sqrt{LC_j}}$ and C_j is the diode junction capacitance

τ_{RR} is the "reverse recovery" time of the diode

Circuit Chaos
Driven Nonlinear Diode Resonator

(technical detail)

Several kinds of models of the driven RLD circuit show behavior consistent with experiment

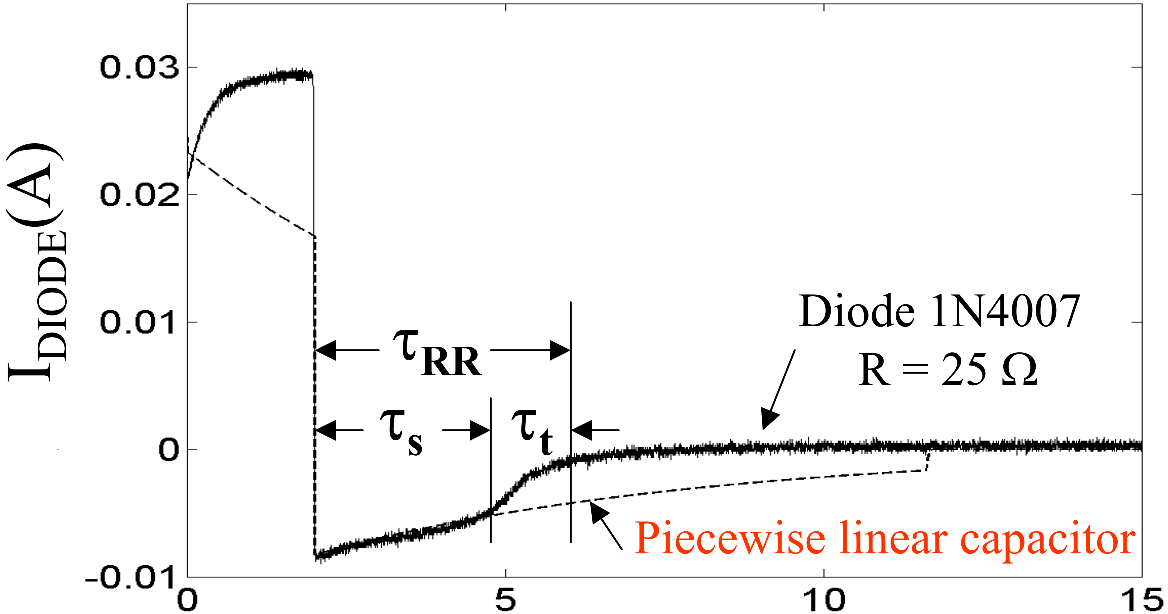


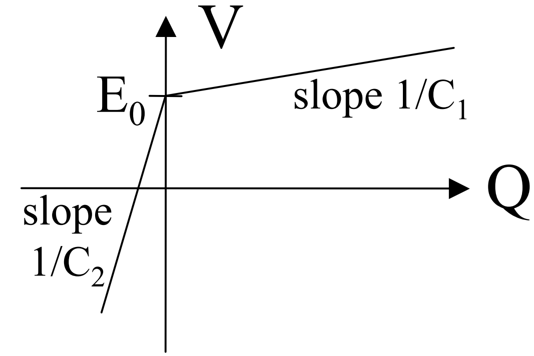
Universal Feature of All Models:

All models display a "reverse-recovery-like" phenomenon, associated with a charge storage mechanism.

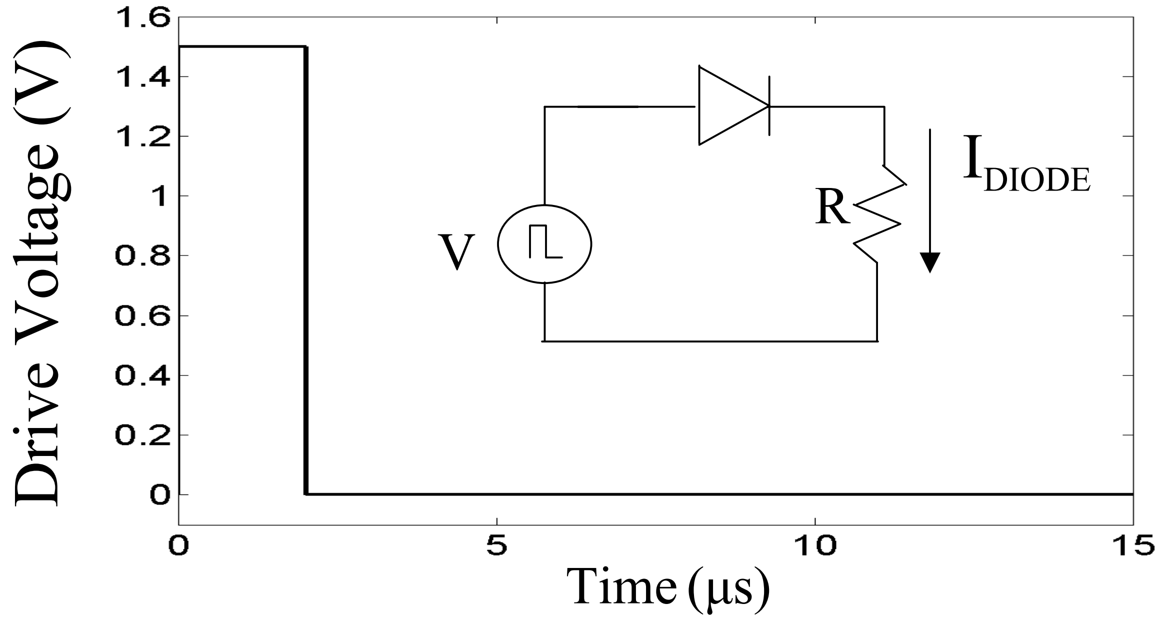
When $\omega >> 1/\tau_{RR}$ period doubling and chaos are strongly suppressed

Piecewise Linear Capacitor has a "Reverse Recovery Time τ_{RR}"





PLC Model
for the diode



Circuit Chaos
Driven Nonlinear Diode Resonator

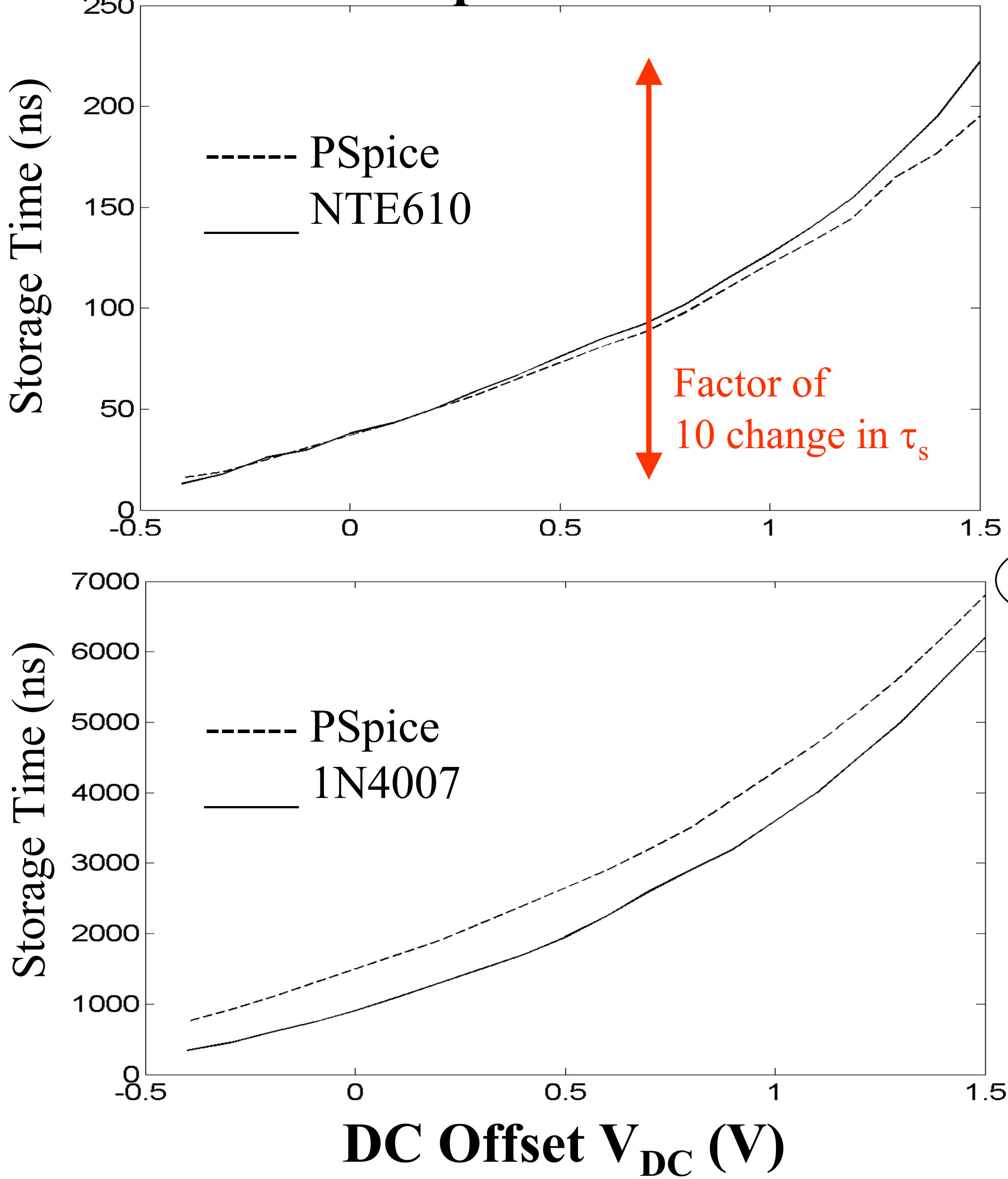
The reverse recovery time of the diode is itself a nonlinear function of many parameters, including;

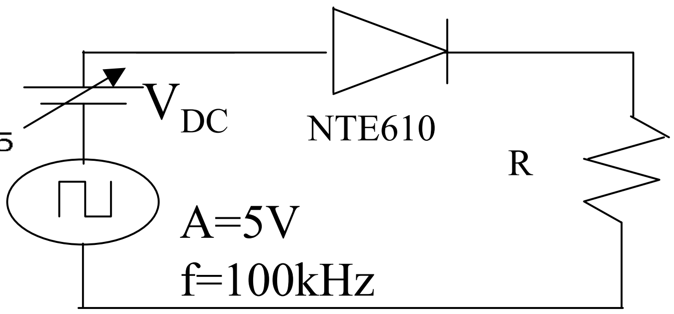
- History of current transients in the diode
- Pulse amplitude
- Pulse frequency
- Pulse duty cycle
- DC bias on the junction

} Influence τ_{RR}

These nonlinearities can expand the range of driving parameters over which period doubling and chaos are observed.

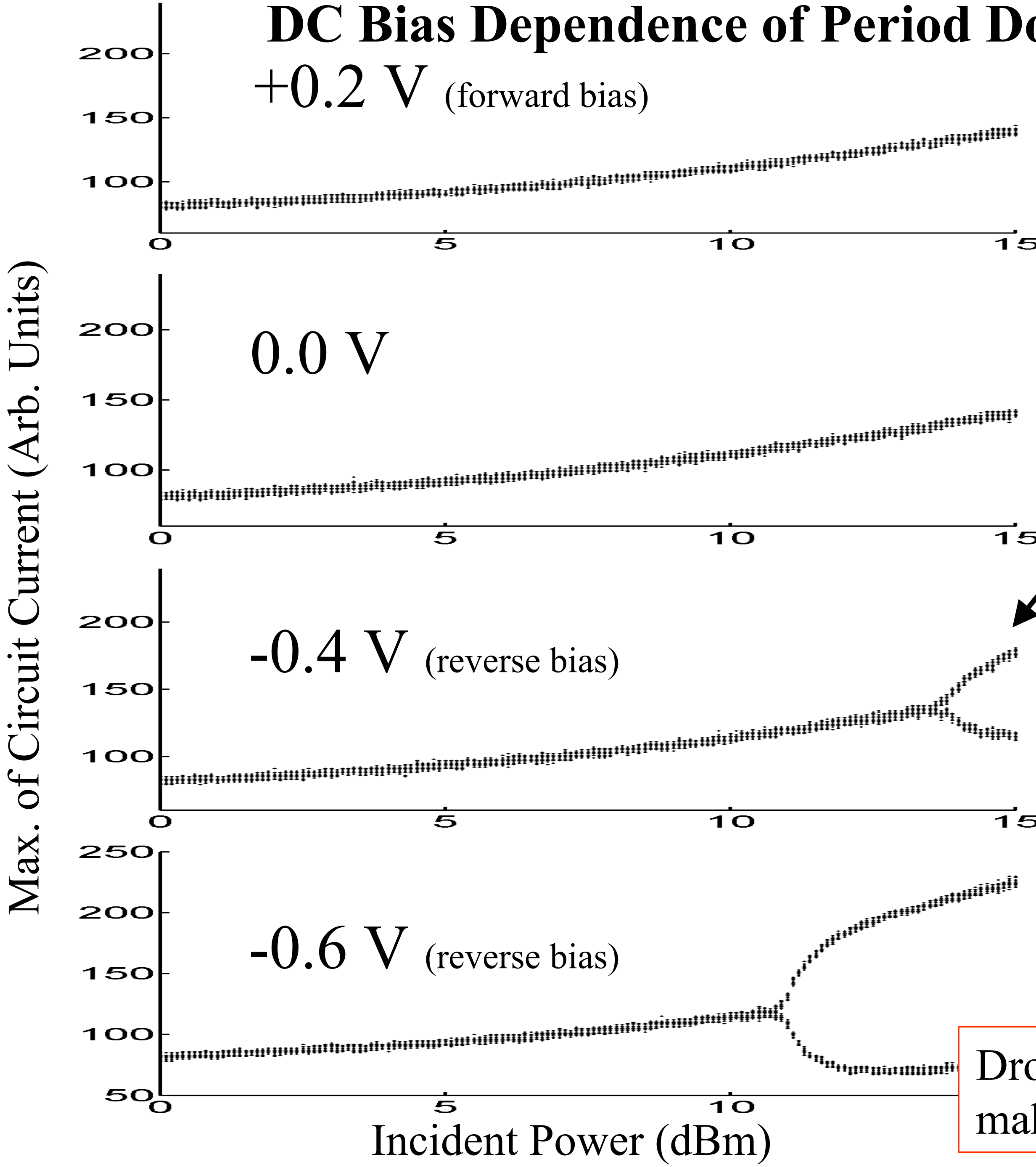
DC Bias Dependence of Reverse Recovery Time





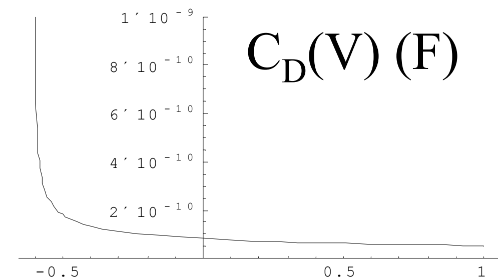
Forward bias: broader charge distribution in p/n junction, and a longer recovery time

DC Bias Dependence of Period Doubling



RLD
NTE 610 diode
R = 25 Ω, L = 10 μH
f =29 MHz >$1/\tau_{RR}$=22 MHz
> f_0 = 12.3 MHz

Reverse bias enhances period doubling in this case

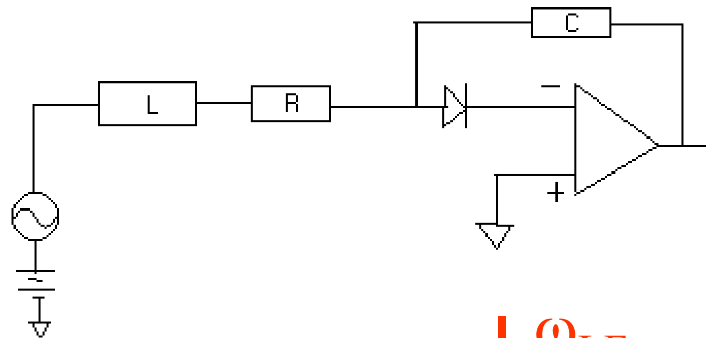


Reverse Voltage (-V_v

But reverse bias should reduce the C(V) nonlinearity!

Drop in τ_{RR} and C_D with V_{DC} makes $f \sim f_0 \sim 1/\tau_{RR}$

Circuit Chaos
More Complicated Circuits



RLD-TIA
Trans-Impedance
Amplifier

2-tone injection experiments:

(similar to Vavriv)

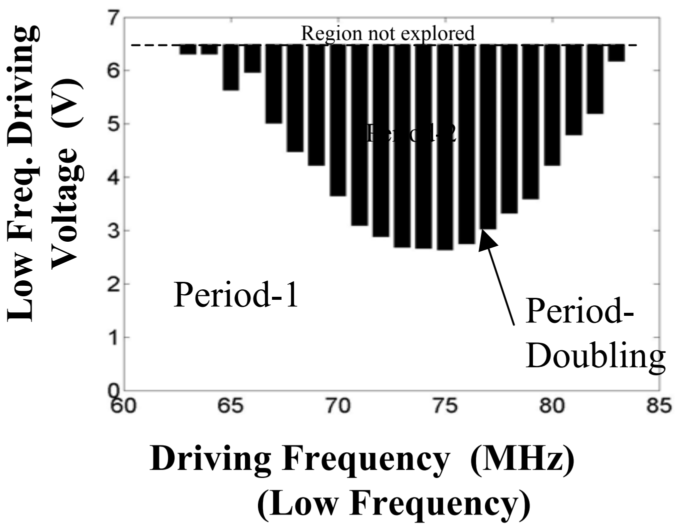
The ω_{HF} signal is rectified, introducing a DC bias on the p/n junction and increasing the circuit nonlinearity at ω_{LF}.

Our conclusion:

The combination of rectification and nonlinear dynamics in this circuit produces qualitatively new ways to influence circuit behavior by means of rf injection.

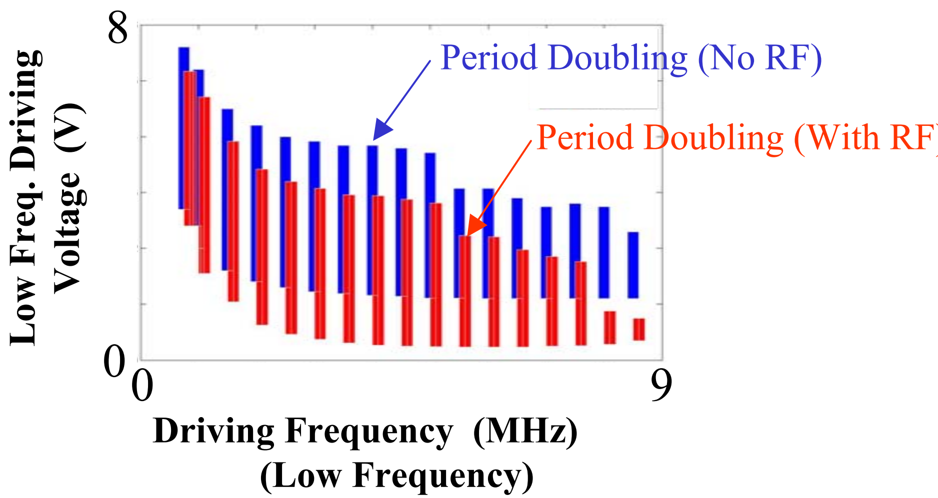
Two-Tone Injection of Nonlinear Circuits

Driven **RLD** Circuit



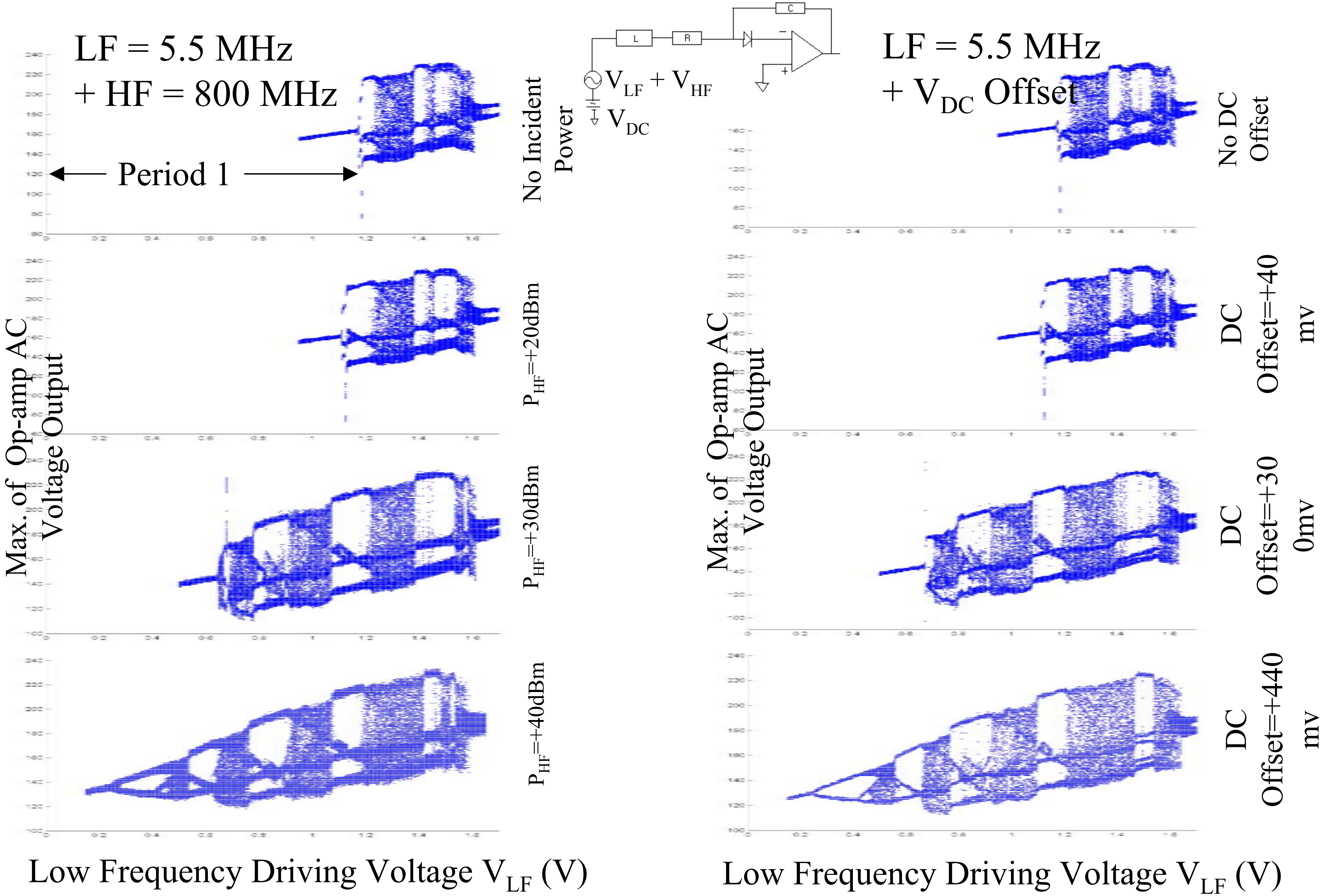
No change in period doubling behavior with or without RF

Driven **RLD/TIA** Circuit



RF injection causes significant drop in driving amplitude required to produce period-doubling!

RF Injection Lowers the Threshold for Chaos in Driven RLD/TIA



Two-Tone Injection of Nonlinear Circuits

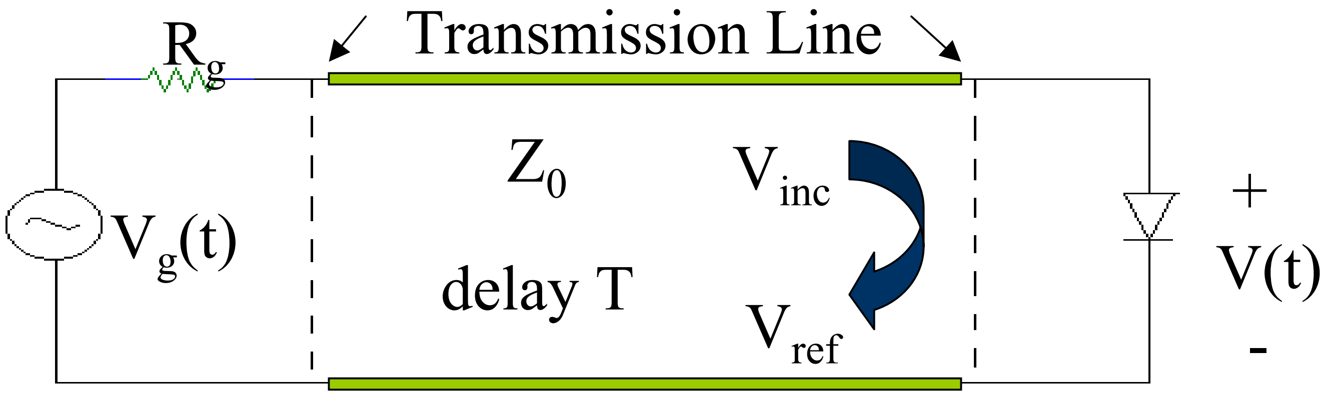
In this case …
The combination of rectification, nonlinear capacitance, and the DC-bias dependence of τ_{RR} produce complex dynamics

In general …
To understand the p/n junction embedded in more complicated circuits:

Nonlinear capacitance Rectification Nonlinearities of τ_{RR}	} All play a role!

$\Rightarrow$ More surprises are in store …

Chaos in the Driven Diode <u>Distributed</u> Circuit



A simple model of the ESD circuit on an IC

Delay differential equations for the diode voltage

$$1)\ 2V_{inc}(t) = V(t) + Z_o\left[gV + \frac{d}{dt}Q(V(t))\right]$$

$$2)\ V_{ref} = V(t) - V_{inc}(t)$$

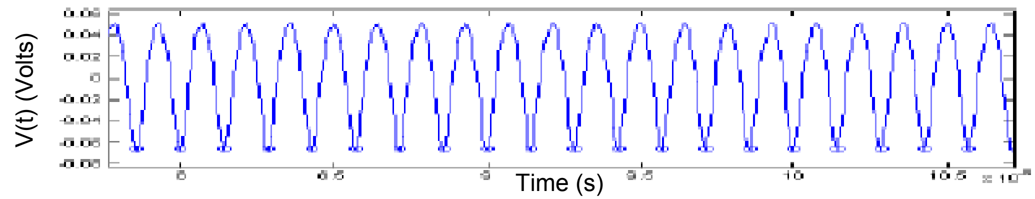
$$3)\ V_{inc}(t) = V_{ref}(t-2T) + V_g(t-T)$$

$$\frac{d}{dt}V(t) = \frac{-(1+Z_o g)}{Z_o C(V(t))}V(t) + \frac{\rho_g(1-Z_o g)}{Z_o C(V(t))}V(t-2T) + \frac{-\rho_g C(V(t))}{C(V(t-2T))}\frac{d}{dt}V(t-2T) + \frac{V_g \tau_g}{Z_o C(V(t))}\cos(w(t-T))$$

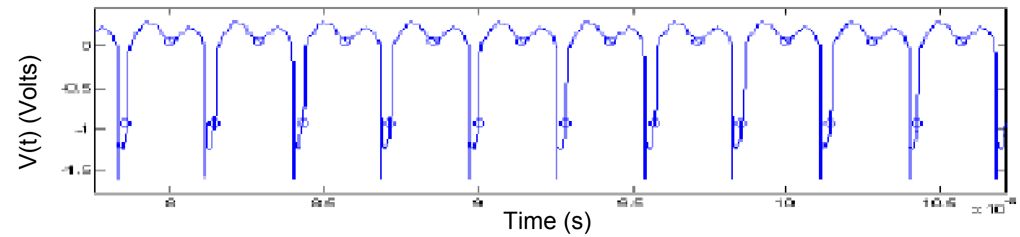
Chaos in the Driven Diode Distributed Circuit

Simulation results

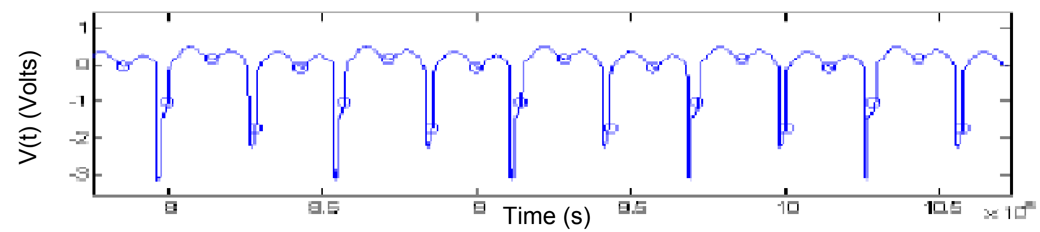
V_g = .5 V **Period 1**



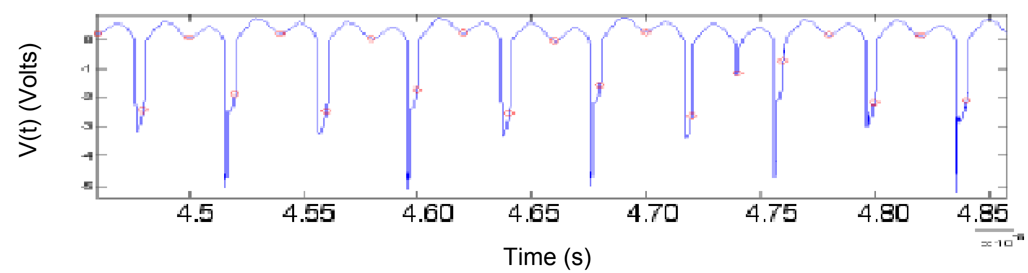
V_g = 2.25 V **Period 2**



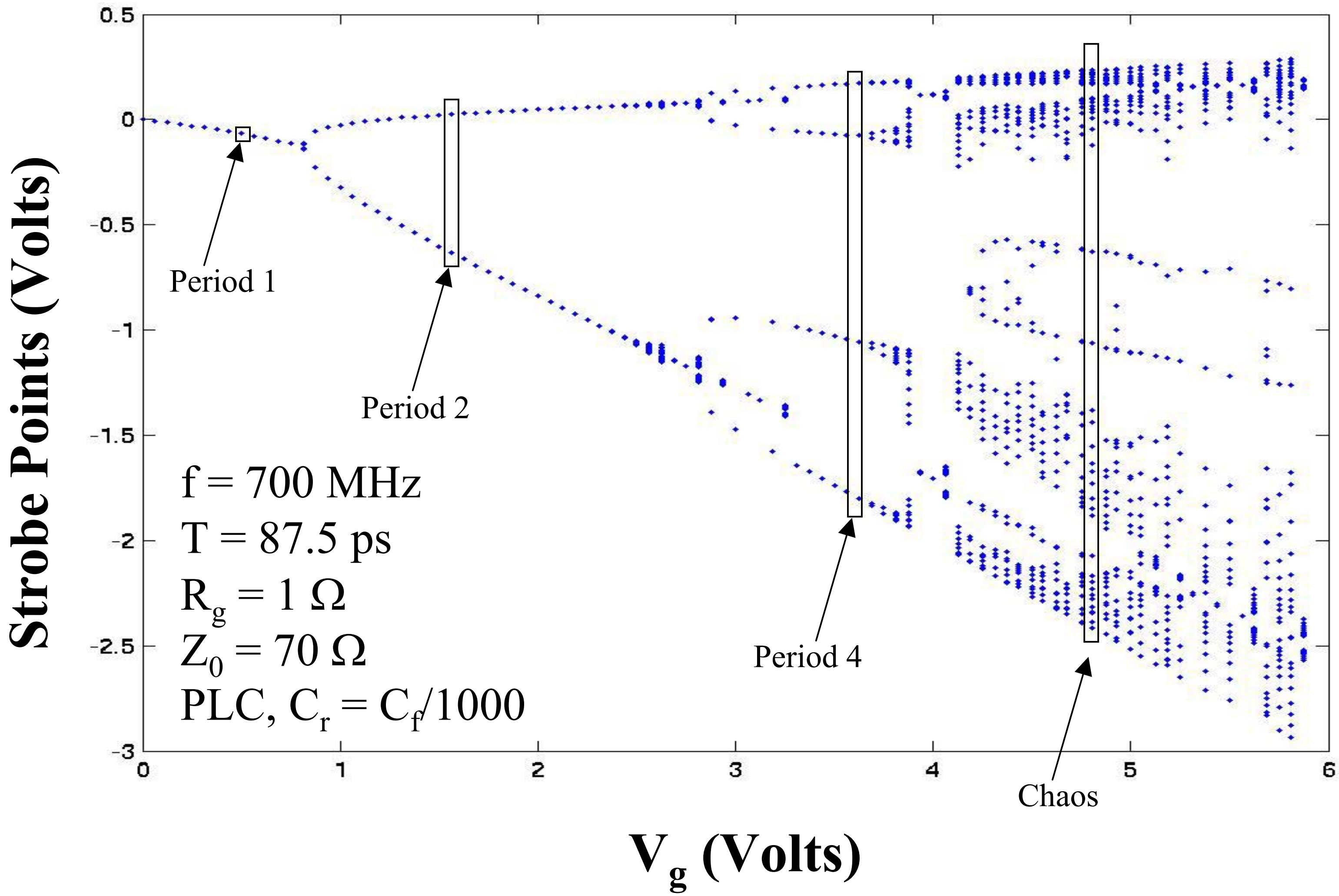
V_g = 3.5 V **Period 4**



V_g = 5.25 V **Chaos**



Chaos in the Driven Diode Distributed Circuit

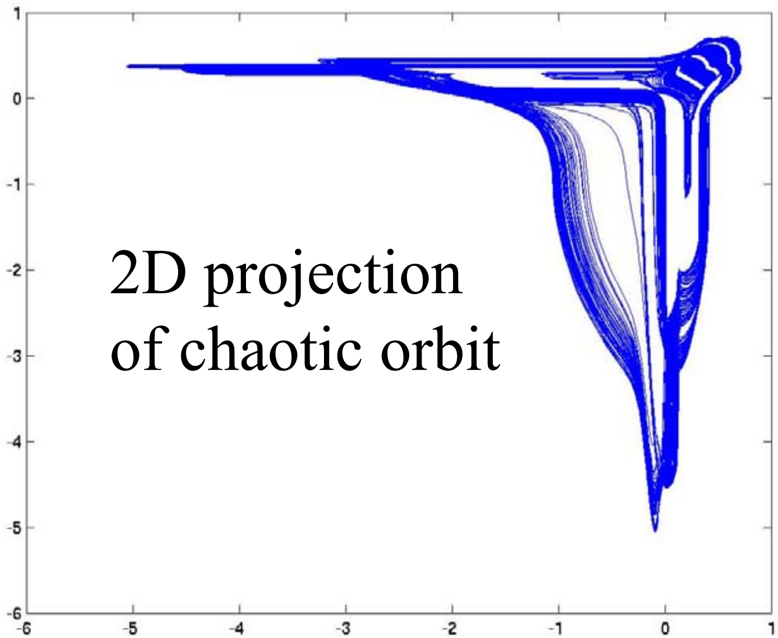


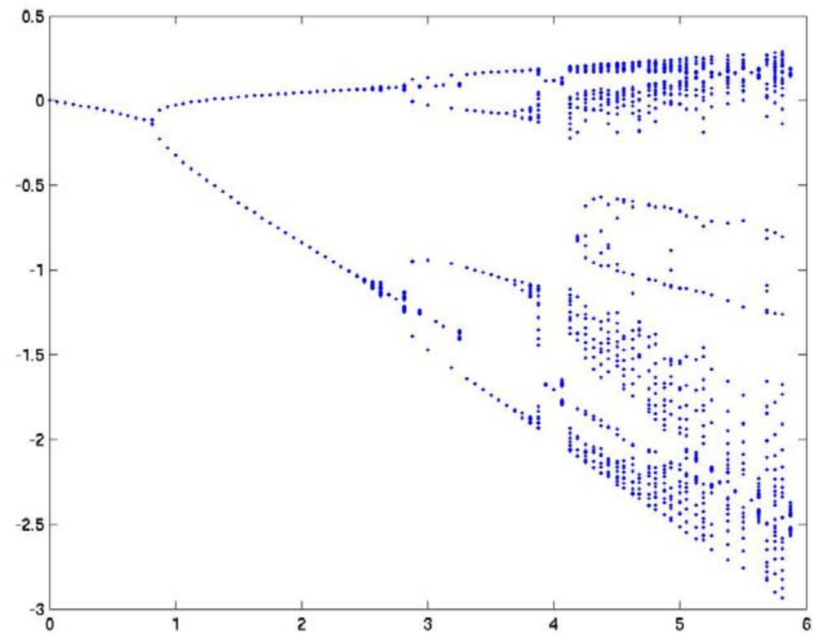
Challenges for the Future

- Ten parameters to explore:

$$C_f, C_r, g, Z_o, R_g, V_g, \omega, T, V_f, V_{gap}$$

- Experimental verification of numerical results





Conclusions about Chaos in the Driven p/n Junction

- A history-dependent recovery/discharge time scale is the key physics needed to understand chaos in the driven RLD circuit
- Nonlinear Capacitor (NLC) models have a τ_{RR}-like time scale
- Both the Hunt and NLC models have a history-dependent recovery time scale due to charge storage mechanisms
- Real diodes have strong nonlinearities of the reverse recovery time that are not captured in current models
- The addition of a TIA to the RLD circuit introduces a new way to influence nonlinear circuit behavior through rectification
- Embedding a diode in a distributed circuit offers new opportunities to induce chaos. See John Rodgers' talk

Recent Papers on the Nonlinear Diode Resonator and Related Circuits:

Renato Mariz de Moraes and Steven M. Anlage, **"Unified Model, and Novel Reverse Recovery Nonlinearities, of the Driven Diode Resonator,"** Phys. Rev. E **68**, 026201 (2003).

Renato Mariz de Moraes and Steven M. Anlage, **"Effects of RF Stimulus and Negative Feedback on Nonlinear Circuits,"** IEEE Trans. Circuits Systems I (in press).
http://arxiv.org/abs/nlin.CD/0208039

T. L. Carroll and L. M. Pecora, "**Parameter ranges for the onset of period doubling in the diode resonator**," Phys. Rev. E **66**, 046219 (2002)

DURIP 2004 proposal: "**Nonlinear and Chaotic Pulsed Microwave Effects on Electronics**"
Anlage, Granatstein and Rodgers