Inducing Chaos in the p/n Junction

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Motivation

• Identify the origins of chaos in the driven resistor – inductor - varactor diode series circuit
• Establish a “universal” picture of chaos in circuits containing p/n junctions
• Identify new opportunities to induce chaos exploiting the p/n junction nonlinearity
  ⇒ John Rodgers’ talk
Driven Resistor-Inductor-Diode Circuit
Studied since the 1980’s

Why is the driven RLD circuit so important?
- Simplest passive circuit that displays period doubling and chaos
- It is a good model of the ubiquitous p-n junction and its nonlinearities

![Driven RLD Circuit Diagram](image)
Chaos in the Driven RLD Circuit

Voltage across Resistor R \sim I

Maximum Voltage across Resistor R

Driving Amplitude $V_0$ (V)

Bifurcation diagram

- $R = 25 \ \Omega$
- $L = 50 \ \mu\text{H}$
- $D = \text{NTE610}$
- $f = 2.5 \ \text{MHz}$
Period-doubling phase diagrams

Low Freq. Driving Voltage $V_0$ (V)

Period-1

Period-2 or more complicated behavior

$f_0 \approx \frac{1}{\sqrt{LC_j(0)}}$

Diode = NTE610

L = 390 nH

R = 25 Ω

$f_0 \approx 70$ MHz

$V_{LF} = V_0 \cos(2\pi f t)$
Nonlinearity of the p-n Junction

The diffusive dynamics of majority and minority charge carriers in the p-n junction is complex and nonlinear.

Charge distribution profiles for a forward-biased diode that is suddenly switched off at $t = 0$

$t = 0$ (forward biased and conducting)

$t = \tau_s$

$t = \tau_{RR}$ (reverse biased and off)

All models of chaotic dynamics in p-n junctions approximate the charge dynamics using nonlinear lumped-elements.

Real Diode $\approx$ Approximate Nonlinear Lumped-element
Resistor-Inductor-Diode Circuit

What is the cause of chaos?

There are 3 competing forms of nonlinearity in this problem:

1. Nonlinear I-V curve $I_{rv}(Q)$. Traditional focus $\Rightarrow$ rectification

2. Nonlinear $C(V) \Rightarrow V_v(Q)$. $C$ increases by x4 $\Rightarrow$ period doubling
   Van Buskirk + Jeffries, Chua, Crevier, etc.

$$f_0 = \frac{1}{2\pi \sqrt{LC(V)}}$$

3. Finite minority carrier lifetime or reverse recovery time. Delayed feedback
   The p/n junction retains memory of previous fwd-bias current swings
   Rollins + Hunt

$\Rightarrow$ No consensus on the origin of chaos
Reverse Recovery Time $\tau_{RR}$

A forward-biased diode that is shut off will continue to conduct for the reverse recovery time, $\tau_{RR}$.

Diode 1N4007
$R = 25 \ \Omega$

$\tau_{RR}$
$\tau_s$ storage time
$\tau_t$ transition time
$\tau_{RR} = \tau_s + \tau_t$

Drive Voltage (V)

$V \parallel R$

$I_{DIODE}$

Diagram of a piecewise linear capacitor.
$\tau_{rr}$ vs. Diode

- **1N5475B, varactor**
  - $f=700$ kHz
  - Voltage drop at resistor ($V$) increases significantly.
  - Duration: 120 ns

- **NTE610, varactor**
  - $f=700$ kHz
  - Voltage drop at resistor ($V$) reduces to a small value.
  - Duration: 60 ns

- **1N4148, fast recovery**
  - $f=700$ kHz
  - Voltage drop at resistor ($V$) approaches zero.
  - Duration: approximately 0

- **1N4007, rectifier**
  - $f=100$ kHz
  - Voltage drop at resistor ($V$) increases gradually.
  - Duration: 1.2 $\mu$s
## Search for Period Doubling and Chaos in Driven RLD Circuit

<table>
<thead>
<tr>
<th>Diode</th>
<th>$\tau_{RR}$ (ns)</th>
<th>$C_j$ (pF)</th>
<th>Results with $f_0 \sim 1/\tau_{RR}$</th>
<th>Results with $f_0 \sim 10/\tau_{RR}$</th>
<th>Results with $f_0 \sim 100/\tau_{RR}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1N5400</td>
<td>7000</td>
<td>81</td>
<td>Period-doubling and chaos for $f/f_0 \sim 0.11 – 1.64$</td>
<td>Period-doubling and chaos $f/f_0 \sim 0.16 – 1.76$</td>
<td>No chaos, nor period-doubling</td>
</tr>
<tr>
<td>1N4007</td>
<td>700</td>
<td>19</td>
<td>Period-doubling and chaos for $f/f_0 \sim 0.13 – 2$</td>
<td>Period-doubling and chaos $f/f_0 \sim 0.23 – 1.3$</td>
<td>No period doubling or chaos</td>
</tr>
<tr>
<td>1N5475B</td>
<td>160</td>
<td>82</td>
<td>Period-doubling and chaos for $f/f_0 \sim 0.66 – 2.2$</td>
<td>No chaos, nor period-doubling</td>
<td>No chaos, nor period-doubling</td>
</tr>
<tr>
<td>NTE610</td>
<td>45</td>
<td>16</td>
<td>Period-doubling and chaos for $f/f_0 \sim 0.14 – 3.84$</td>
<td>Period-doubling only for $f/f_0 \sim 1.17 – 3.25$</td>
<td>No chaos, nor period-doubling</td>
</tr>
</tbody>
</table>

The equation for the driving frequency $f_0$ is:

$$f_0 = \frac{1}{2\pi \sqrt{LC_j}}$$
Circuit Chaos: Rule of Thumb
Driven Nonlinear Diode Resonator

For lumped element nonlinear diode resonator circuits, Period Doubling and Chaos are observed for sufficiently large driving amplitudes when;

\[ \omega \sim \frac{\omega_0}{10} \leftrightarrow 4 \omega_0 \]
and

\[ \omega_0 < \frac{1}{10\tau_{RR}} \text{ to } \frac{1}{100\tau_{RR}} \]

where \( \omega_0 = \frac{1}{\sqrt{LC_j}} \) and \( C_j \) is the diode junction capacitance

\( \tau_{RR} \) is the “reverse recovery” time of the diode
Circuit Chaos
Driven Nonlinear Diode Resonator
(technical detail)

Several kinds of models of the driven RLD circuit show behavior consistent with experiment

Universal Feature of All Models:
All models display a “reverse-recovery-like” phenomenon, associated with a charge storage mechanism.

When $\omega >> 1/\tau_{ RR}$ period doubling and chaos are strongly suppressed
Piecewise Linear Capacitor has a “Reverse Recovery Time $\tau_{RR}$”

Diode 1N4007

$R = 25 \, \Omega$

PLC Model for the diode
Circuit Chaos
Driven Nonlinear Diode Resonator

The reverse recovery time of the diode is itself a nonlinear function of many parameters, including:

- History of current transients in the diode
- Pulse amplitude
- Pulse frequency
- Pulse duty cycle
- DC bias on the junction

These nonlinearities can expand the range of driving parameters over which period doubling and chaos are observed.
DC Bias Dependence of Reverse Recovery Time

Forward bias: broader charge distribution in p/n junction, and a longer recovery time

--- PSpice
___ NTE610

--- PSpice
___ 1N4007

Factor of 10 change in $\tau_s$
DC Bias Dependence of Period Doubling

+0.2 V (forward bias)

0.0 V

-0.4 V (reverse bias)

-0.6 V (reverse bias)

Reverse bias enhances period doubling in this case.

But reverse bias should reduce the C(V) nonlinearity!

Drop in $\tau_{RR}$ and $C_D$ with $V_{DC}$ makes $f \sim f_0 \sim 1/\tau_{RR}$.

RLD
NTE 610 diode
$R = 25 \, \Omega$, $L = 10 \, \mu\text{H}$
$f = 29 \, \text{MHz} > 1/\tau_{RR} = 22 \, \text{MHz} > f_0 = 12.3 \, \text{MHz}$

Reverse Voltage ($-V_v$)
Circuit Chaos
More Complicated Circuits

2-tone injection experiments:
(similar to Vavriv)

ω₀ ~ MHz
ω_LF ~ GHz
ω_HF ~ GHz

The ω_HF signal is rectified, introducing a DC bias on the p/n junction and increasing the circuit nonlinearity at ω_LF.

Our conclusion:
The combination of rectification and nonlinear dynamics in this circuit produces qualitatively new ways to influence circuit behavior by means of rf injection.
Two-Tone Injection of Nonlinear Circuits

Driven RLD Circuit

No change in period doubling behavior with or without RF

Driven RLD/TIA Circuit

RF injection causes significant drop in driving amplitude required to produce period-doubling!
RF Injection Lowers the Threshold for Chaos in Driven RLD/TIA

LF = 5.5 MHz + HF = 800 MHz

Period 1

Max. of Op-amp AC Voltage Output

No Incident Power

Low Frequency Driving Voltage $V_{LF}$ (V)

LF = 5.5 MHz + $V_{DC}$ Offset

No DC Offset

Low Frequency Driving Voltage $V_{LF}$ (V)
Two-Tone Injection of Nonlinear Circuits

In this case …

The combination of rectification, nonlinear capacitance, and the DC-bias dependence of $\tau_{RR}$ produce complex dynamics.

In general …

To understand the p/n junction embedded in more complicated circuits:

Nonlinear capacitance
Rectification
Nonlinearities of $\tau_{RR}$

All play a role!

$\Rightarrow$ More surprises are in store …
Chaos in the Driven Diode Distributed Circuit

A simple model of the ESD circuit on an IC

Delay differential equations for the diode voltage

1) \( 2 \ V_{inc}(t) = V(t) + Z_0 [gV + \frac{d}{dt}Q(V(t))] \)

2) \( V_{ref} = V(t) - V_{inc}(t) \)

3) \( V_{inc}(t) = V_{ref}(t-2T) + V_g(t-T) \)

\[
\frac{d}{dt}V(t) = \frac{\left(1 + Z_0g\right)}{Z_0C(V(t))}V(t) + \frac{\rho_g(1-Z_0g)}{Z_0C(V(t))}V(t-2T) + \frac{-\rho_gC(V(t))}{C(V(t-2T))} \frac{d}{dt}V(t-2T) + \frac{V_g\tau_g}{Z_0C(V(t))} \cos(\omega(t-T))
\]
Chaos in the Driven Diode Distributed Circuit

Simulation results

- \( V_g = 0.5 \text{ V} \) \quad \text{Period 1}
- \( V_g = 2.25 \text{ V} \) \quad \text{Period 2}
- \( V_g = 3.5 \text{ V} \) \quad \text{Period 4}
- \( V_g = 5.25 \text{ V} \) \quad \text{Chaos}
Chaos in the Driven Diode Distributed Circuit

\[ f = 700 \text{ MHz} \]
\[ T = 87.5 \text{ ps} \]
\[ R_g = 1 \Omega \]
\[ Z_0 = 70 \Omega \]
\[ \text{PLC, } C_r = C_f/1000 \]
Challenges for the Future

- Ten parameters to explore: $C_f, C_r, g, Z_o, R_g, V_g, \omega, T, V_f, V_{gap}$
- Experimental verification of numerical results

2D projection of chaotic orbit
Conclusions about Chaos in the Driven p/n Junction

• A history-dependent recovery/discharge time scale is the key physics needed to understand chaos in the driven RLD circuit

• Nonlinear Capacitor (NLC) models have a $\tau_{RR}$-like time scale

• Both the Hunt and NLC models have a history-dependent recovery time scale due to charge storage mechanisms

• Real diodes have strong nonlinearities of the reverse recovery time that are not captured in current models

• The addition of a TIA to the RLD circuit introduces a new way to influence nonlinear circuit behavior through rectification

• Embedding a diode in a distributed circuit offers new opportunities to induce chaos. See John Rodgers’ talk
Recent Papers on the Nonlinear Diode Resonator and Related Circuits:


DURIP 2004 proposal: “Nonlinear and Chaotic Pulsed Microwave Effects on Electronics” Anlage, Granatstein and Rodgers