Universal Field, Impedance, and S-Matrix Statistics of Metallic Enclosures

Students: Sameer Hemmady, X. Zheng,
Faculty: E. Ott, T. M. Antonsen and Steven M. Anlage

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Goal and Outline

To develop a quantitative statistical understanding of electromagnetic field distributions inside complicated enclosures, based upon minimal information about the system

• Motivation and description of the problem
• Approach to the solution – Random Coupling Model (RCM)
  • Generalized “port” concept
  • Impedance and Scattering matrices
  • Importance of Radiation Impedance
• Experimental tests of basic predictions of RCM
• Turning the theory into practical results
  • Field / Impedance Engineering
  • Minimum Requirements to predict Z, V, E PDFs
• Conclusions
Motivation:

Electromagnetic Coupling to Circuits

- Coupling of external radiation to computer chips is a complex process:
  - Apertures
  - Resonant cavities
  - Transmission Lines
  - Circuit Elements
  - System Size > Wavelength

What can we say about the nature of E fields inside such a cavity?

- Statistical Distribution using Wave Chaos!!
Statistical Measure of Field Distributions in Complex Cavities

We seek a statistical description of fields and currents in complicated metallic enclosures.

Prior work has established PDF of $|E|$ in complex cavities.

This description does not include the effects of coupling.

Here we bring the statistical electromagnetic predictions one step closer to reality by including the effects of coupling and losses.
The Ultimate Goal: Induced Field Distributions for an Arbitrary Enclosure

Random Coupling Model

Based on results and concepts from the field of Wave Chaos

The predictions are statistical in nature

What minimum information do we need to predict the range of voltages on port j because of 1 Watt injected through port i?
N-Port Description of an Arbitrary Enclosure

N Ports

- Voltages and Currents,
- Incoming and Outgoing Waves

\[
\begin{bmatrix}
V_1 \\
V_2 \\
\vdots \\
V_N
\end{bmatrix}
= [Z] \cdot
\begin{bmatrix}
I_1 \\
I_2 \\
\vdots \\
I_N
\end{bmatrix}
\]

\[
\begin{bmatrix}
V_1^- \\
V_2^- \\
\vdots \\
V_N^-
\end{bmatrix}
= [S] \cdot
\begin{bmatrix}
V_1^+ \\
V_2^+ \\
\vdots \\
V_N^+
\end{bmatrix}
\]

\[
S = (Z + Z_0)^{-1}(Z - Z_0)
\]

\[Z(\omega), S(\omega)\]

- Complicated Functions of frequency
- Detail Specific
Theoretical Work by Xing Zheng, Antonsen, Ott.

Statistical Model Impedance

\[ Z_{ij}(k) = -\frac{j}{\pi} \sum_n R_{Rad,i}^{1/2}(k_n) R_{Rad,j}^{1/2}(k_n) \frac{\Delta k_n^2 w_{in} w_{jn}}{k^2 (1 - jQ^{-1})} - k_n^2 \]

System Parameters
\[ \begin{align*} 
\Delta k_n^2 & \quad \text{mean eigenspacing} \\
Q & \quad \text{Quality Factor (Losses)} \\
k_n^2 & \quad \text{Eigenvalues} \\
w_{in} & \quad \text{Gaussian Random Variables} 
\end{align*} \]

Statistical Parameters
\[ \frac{k_n^2}{\Delta k_n^2 Q} \Rightarrow \text{Key Parameter} \sim \text{Im}(\omega_0)/\Delta\omega \]

Free Space Radiation Impedance:
\[ Z_{Rad}(\omega) = R_{Rad}(\omega) + jX_{Rad}(\omega) \]

Xrad comes in through the n cutoff of the sum. If k is small compared to the cutoff, there will be a net contribution to the sum given by the radiation reactance.

Steve Anlage, 7/9/2004
The Key Step: Forming the Normalized Impedance (z)

\[ Z_{\text{Cavity}} = R_{\text{Cavity}} + j X_{\text{Cavity}} \]

\[ Z_{\text{Rad}} = R_{\text{Rad}} + j X_{\text{Rad}} \]

Combine

\[ z = \frac{R_{\text{Cavity}}}{R_{\text{Rad}}} + j \frac{X_{\text{Cavity}} - X_{\text{Rad}}}{R_{\text{Rad}}} \]
Testable Predictions of the Random Coupling Model

Ideal World
Universal Properties
(Detail-Independent)

Coupling

Normalized impedance
\[
z = \frac{R_{\text{Cavity}}}{R_{\text{Rad}}} + j \frac{X_{\text{Cavity}} - X_{\text{Rad}}}{R_{\text{Rad}}}
\]

Experimental Data

PDFs of Re[z] and Im[z] are universal
\[
\sigma^2 \{\text{Re}[z]\} \approx \sigma^2 \{\text{Im}[z]\} \approx (1/\pi) \left( \frac{k^2}{\Delta k_n Q} \right)
\]

For TRS Systems

Normalized Scattering Parameter
\[
s = \frac{z - 1}{z + 1}
\]

|s| and Arg[s] are independent

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¼-Bow-Tie Resonator

EXPERIMENTAL SETUP:

- 2 Dimensional Quarter Bow Tie Wave Chaotic cavity
- Classical ray trajectories are chaotic - short wavelength - Quantum Chaos
- 1-port S and Z measurements in the 6 – 12 GHz range.
- Ensemble average through 100 locations of the perturbation
Antenna Detail

Coaxial Cable

Height (h)

Radius (a)

CAVITY LID

CAVITY BASE

Cross Section View

Implementation of “Radiation Boundary Conditions”

Antenna Entry Point

Microwave Absorber

Type: ARC Tech- DD 10017D

Operational Freq: 6-12 GHz

~ -25dB Reflection Loss
Experimental Tests of the Random Coupling Model

\[ z = \text{normalized impedance} \]
\[ s = \text{normalized scattering matrix} \]

- Single-parameter fits to PDF of \( \text{Re}[z] \), \( \text{Im}[z] \)
- Equivalence of variances of PDFs and single fitting parameter
- Insensitivity of \( \text{Re}[z] \) and \( \text{Im}[z] \) to irrelevant details
- Frequency, volume, loss dependence of \( \text{Re}[z] \) and \( \text{Im}[z] \) PDFs

- Single-parameter fits to PDF of \( |s| \), uniform distribution of \( \text{Arg}[s] \)
- Independence of \( |s| \) and \( \text{Arg}[s] \)
Testing the Effects of Varying Loss

Low Loss
Intermediate Loss
High Loss

\[ \frac{k^2}{\Delta k_n^2 Q} \sim \frac{\text{Im}[\omega_0]}{\Delta \omega} \]

\[ k^2 / \Delta k_n^2 Q = 7.6 \]
\[ k^2 / \Delta k_n^2 Q = 4.2 \]
\[ k^2 / \Delta k_n^2 Q = 0.8 \]

Probability Density

Re(z) Im(z)

\text{Freq. Range} : 7.2 \text{ to } 8.4 \text{ GHz}
\text{Cavity Height} : h = 7.87 \text{ mm}
\text{Antenna Diameter} : 2a = 1.27 \text{ mm}
\text{Statistics drawn from 160,200 pts.}
Testing Insensitivity to System Details

- Freq. Range: 9 to 9.75 GHz
- Cavity Height: h = 7.87 mm
- Statistics drawn from 100,125 pts.

\[
z = \frac{R_{\text{Cavity}}}{R_{\text{Rad}}} + j \frac{X_{\text{Cavity}} - X_{\text{Rad}}}{R_{\text{Rad}}}
\]
Testing Predictions on Variance of $z$ PDFs

RCM Prediction:

$$\sigma^2 \{\text{Re}[z]\} \approx \sigma^2 \{\text{Im}[z]\} \approx 0.32 / \frac{k^2}{\Delta k_n^2 Q}$$

$$\sigma^2 = \frac{0.33 \pm 0.02}{k^2 / \Delta k_n^2 Q}$$

Open: $\text{Re}[Z_{\text{norm}}]$  
Closed: $\text{Im}[Z_{\text{norm}}]$  

Given frequency, volume, losses ($Q$), we can predict all the statistical properties. 

- Frequency Range for loss cases: 7.2 to 8.4GHz 
- Antenna Diameter ($2a$) = 1.27mm
Insensitivity to System Coupling Details

- Frequency Range: 6 to 12 GHz
- Height of cavity: \( h = 7.87 \text{mm} \)
- Statistics drawn from 800,100 pts.

\[
P\{|S_{cav}|^2\} \\
2a=0.635\text{mm} \\
2a=1.27\text{mm}
\]

\[
P\{|s|^2\} \\
2a=0.635\text{mm} \\
2a=1.27\text{mm}
\]
Independence of $\phi_{\text{norm}}$ and $|s|$

- Frequency: 6 to 9.6 GHz
- Cavity Height: 7.87mm
- Antenna Diameter: 1.27mm

$$s = \frac{z - 1}{z + 1}$$
PDF of Reflection Coefficient with Varying Loss

Symbols: Data
- : RCM prediction

\( r = |s|^2 \)

Freq: 6.5 to 7.8 GHz
Experimental Tests of the Random Coupling Model

\[ z = \text{normalized impedance} \]
\[ s = \text{normalized scattering matrix} \]

✓ Single-parameter fits to PDF of Re[z], Im[z]
✓ Equivalence of variances of PDFs and single fitting parameter
✓ Insensitivity of Re[z] and Im[z] to irrelevant details
✓ Frequency, volume, loss dependence of Re[z] and Im[z] PDFs

✓ Single-parameter fits to PDF of |s|, uniform distribution of Arg[s]
✓ Independence of |s| and Arg[s]

Good Agreement!

These results are general, and their applicability is not limited in any way by the nature of our “bow-tie” cavity (dimensionality, shape, etc.)
Practical Implications for Real Life Problems
Bare Minimum Specifications for Field Statistics

Minimum Information:
Frequency, Volume
Losses
Radiation impedance of the ports

Determine the shape and scales of the $Z_{\text{Cavity}}$ and Field PDFs
Some Practical Implications
Predictions for V and E PDFs
Based on 1-port and 2-port Data

1-port  Specify Q, frequency ⇒ P(z)
 + Specify coupling (Z_{Rad}), P_{1}
\[ P(z) \rightarrow P(Z_{\text{Cavity}}), P(V_{1}), \text{PDF of voltage or field values near port 1} \]

2-port  Specify Q, frequency ⇒ P(\ddot{z})
 + Specify coupling (Z_{Rad1}, Z_{Rad2}), P_{1}
\[ P(V_{1}) \rightarrow P(Z_{\text{Cavity}}), P(E_{2}), \text{PDF of voltage or field values near port 2} \]
**E- and B-Field PDF Engineering**

**Radiation Impedance**

$Z_{\text{Rad}}$ can be calculated, computed, or measured for each port

**$Z_{\text{Rad}}$ analytical calculation**

Short dipole: $R_{\text{rad}} = 20\pi^2 \left( \frac{\ell}{\lambda} \right)^2 \Omega$

Annular current in 2D cavity:

$$R_{\text{Rad}} = \frac{kh\eta_0}{4} J_0^2(ka)$$

$$X_{\text{Rad}} = -\frac{kh\eta_0}{4} J_0(ka)Y_0(ka)$$

$k a \ll 1$

**$Z_{\text{Rad}}$ computation**

HFSS, etc.

**$Z_{\text{Rad}}$ measurement**

Bared coaxial probe in cavity:

![Graphs showing $R_{\text{rad}}$ and $X_{\text{rad}}$ vs. frequency for different values of $2a$.]
Application of RCM to a Real Problem
Induced Voltage PDFs in a Computer Enclosure and Room

Port 1: Bare Wire
Dia: 1.6mm
Port 2: Bus Line

Q= 5
1 W @ 5.3GHz

Port 1: Short Dipole Antenna
Port 2: PCB Bus Line

Q= 100
1 W @ 1 GHz
Cavity Impedance and Field PDF Engineering

**RCM Results**

\[ Z_{\text{cavity}} = R_{\text{cavity}} + i X_{\text{cavity}} \]

- **P(R_{\text{cavity}})**
  - High loss
  - Intermediate loss
  - Low loss

- **P(X_{\text{cavity}})**
  - Gaussian (high loss) \[ \Delta f_{3dB} \gg \text{spacing} \]
  - Lorentzian ↔ Gaussian (intermediate loss)
  - Std. Lorentzian (low loss) \[ \Delta f_{3dB} \ll \text{spacing} \]

- **R_{Rad}** sets the scale for **R_{cavity}**
- Low-loss case: \[ R_{\text{cavity}} < R_{Rad} \]
- Lossy case: \( \Rightarrow \) Gaussian distribution, width \( \sim \sqrt{Q} \)

- **X_{Rad}** sets the scale for **X_{cavity}**
- Low-loss case: broad tails, width \( \sim R_{Radiation} \)
- Lossy case: narrow distribution, width \( \sim \sqrt{Q} \)
Prescription to Engineer Cavities with Desired Electromagnetic Properties

Numerically Generated (z)
{Frequency, Q}

\[ Z_{cav} = jX_{rad} + z \cdot R_{rad} \]

\[ Z_{rad} = R_{rad} + jX_{rad} \]

- Freq : 6 to 9.6 GHz
- Antenna Dia (2a)= 1.27mm

![Diagram of cavity structure]

\[ P\{|S|^2\} \]

\[ P\{\phi_s\} \]
One-Port Complex Voltage PDFs

Simulated Results

Simulation Parameters:
1. Frequency : 9.2 - 9.6 GHz
2. \( \frac{k}{\Delta k_n^2} Q \)
   - High Loss: 4
   - Med Loss : 2.5
   - Low Loss : 0.07
3. Bared wire antenna diameter = 1.6 mm

1 Watt input power
Progress / Conclusions

We have created a Random Coupling Model to extend the statistical electromagnetic predictions to real enclosures by including the effects of coupling.

Experimental tests of many basic 1 port and 2-port predictions of the Random Coupling Model (RCM) have confirmed that the model is correct.

- Frequency, Volume
- Losses
- Radiation impedance of the ports

Determine the shape and scales of the E-field, Z, S PDFs

Clear strategies to engineer the field PDFs to prevent damage to circuits, components, etc.

OR

Clear statistical predictions of ‘effects’ given a minimum of assumptions about target

Preprints available at:  http://ArXiv.org/cond-mat/0403225
Future Work

• N-Port Measurements
  >> Other types of ports (e.g. slit/waveguide)
  >> Test the effects of non-reciprocity (magnetized ferrites)
  >> Case of interacting ports
  >> Create a database of port radiation impedances

• Three-Dimensional Cavities
  >> The high mode density makes experiment/theory simpler
  >> Test the RCM in the low-loss limit

• Comparison of RCM Predictions to Other Measurements
  >> Develop analytical interpolating functions for Z, E PDFs
  >> Compare our model predictions to “real” data

• Time-Domain Measurements
  >> Energy time-decay statistics
  >> Time and frequency domain correlations of Z, S, E-fields