Universal Field, Impedance, and S-Matrix Statistics of Metallic Enclosures

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MURI Review 2004

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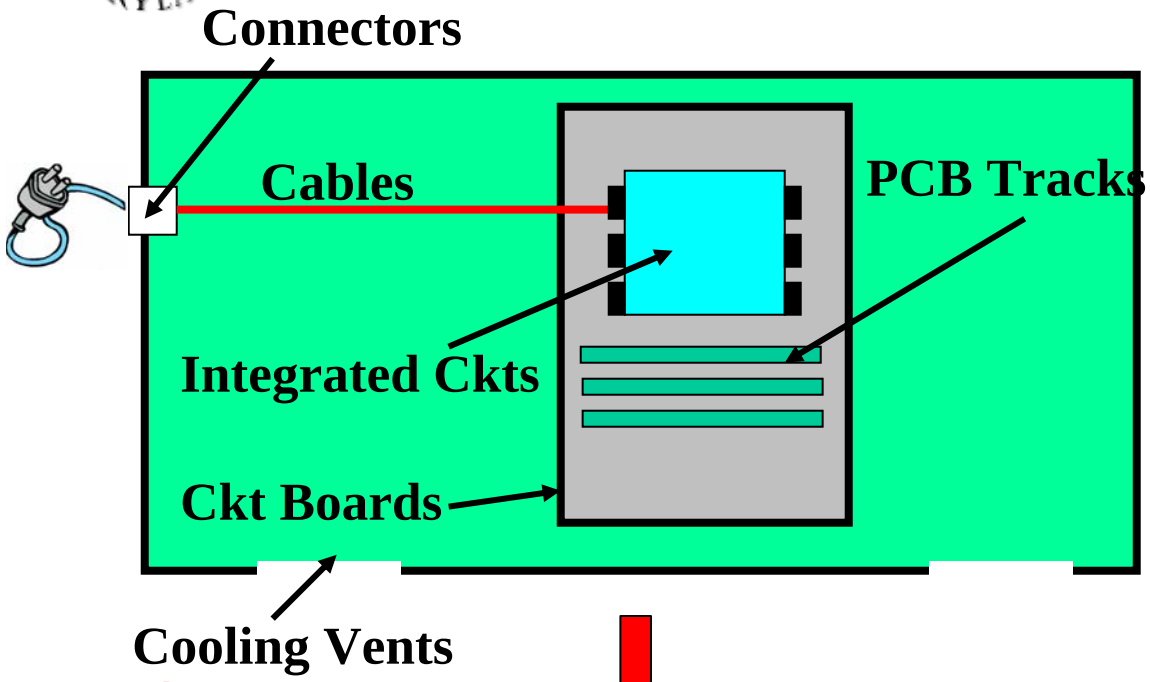
Goal and Outline

To develop a quantitative statistical understanding of electromagnetic field distributions inside complicated enclosures, based upon minimal information about the system

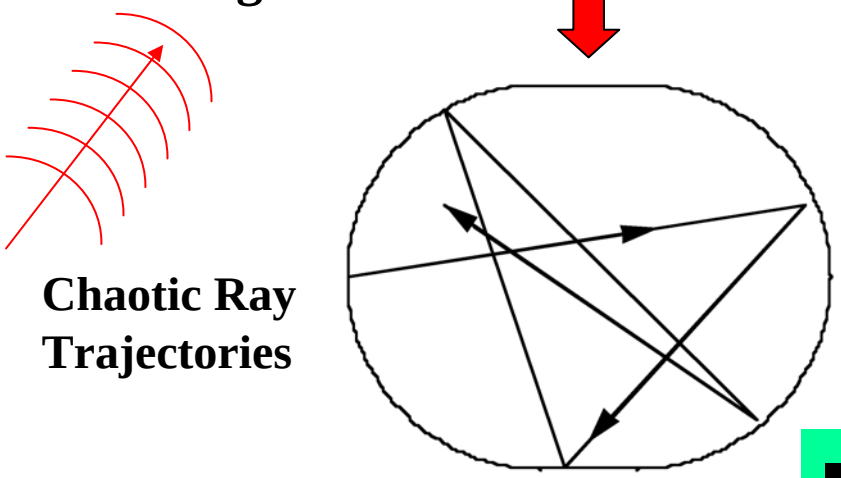
- Motivation and description of the problem
- Approach to the solution – Random Coupling Model (RCM)
  - Generalized “port” concept
  - Impedance and Scattering matrices
  - Importance of Radiation Impedance
- Experimental tests of basic predictions of RCM
- Turning the theory into practical results
  - Field / Impedance Engineering
  - Minimum Requirements to predict Z, V, E PDFs
- Conclusions

Motivation:

Electromagnetic Coupling to Circuits



- Coupling of external radiation to computer chips is a complex process:
  - Apertures
  - Resonant cavities
  - Transmission Lines
  - Circuit Elements
- System Size > Wavelength



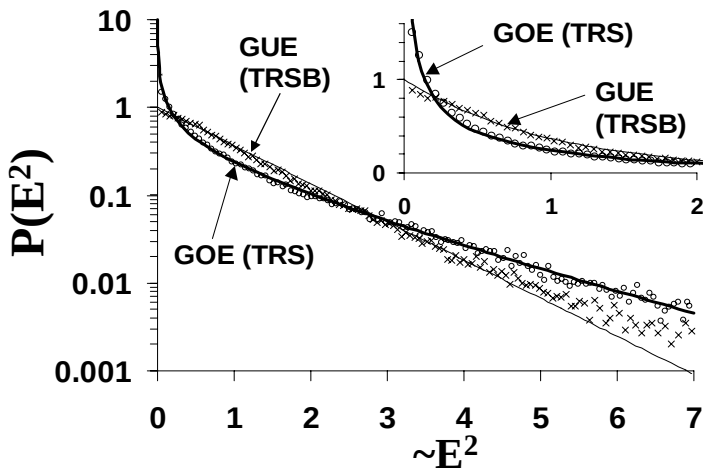
What can we say about the nature of E fields inside such a cavity?

- Statistical Distribution using Wave Chaos!!

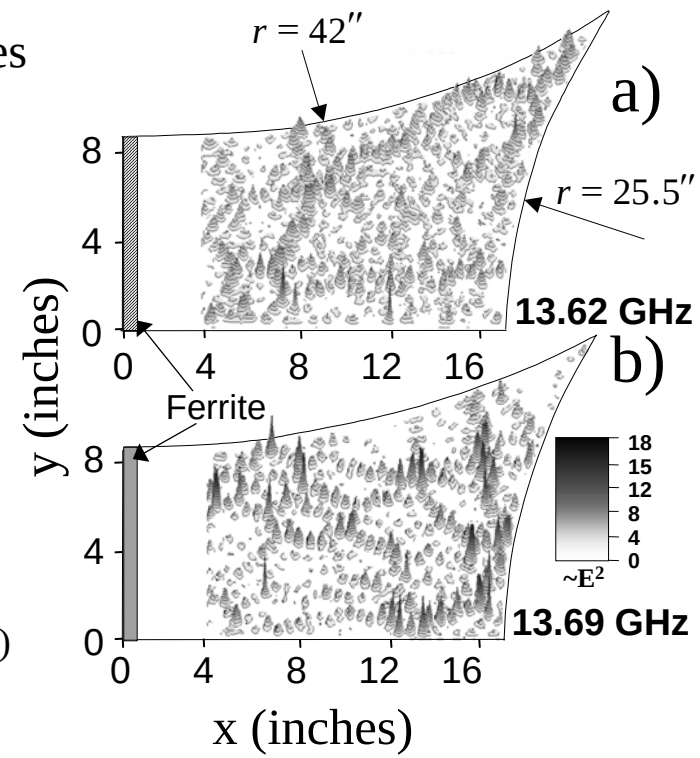
Statistical Measure of Field Distributions in Complex Cavities

We seek a statistical description of fields and currents in complicated metallic enclosures

Prior work has established PDF of |E| in complex cavities



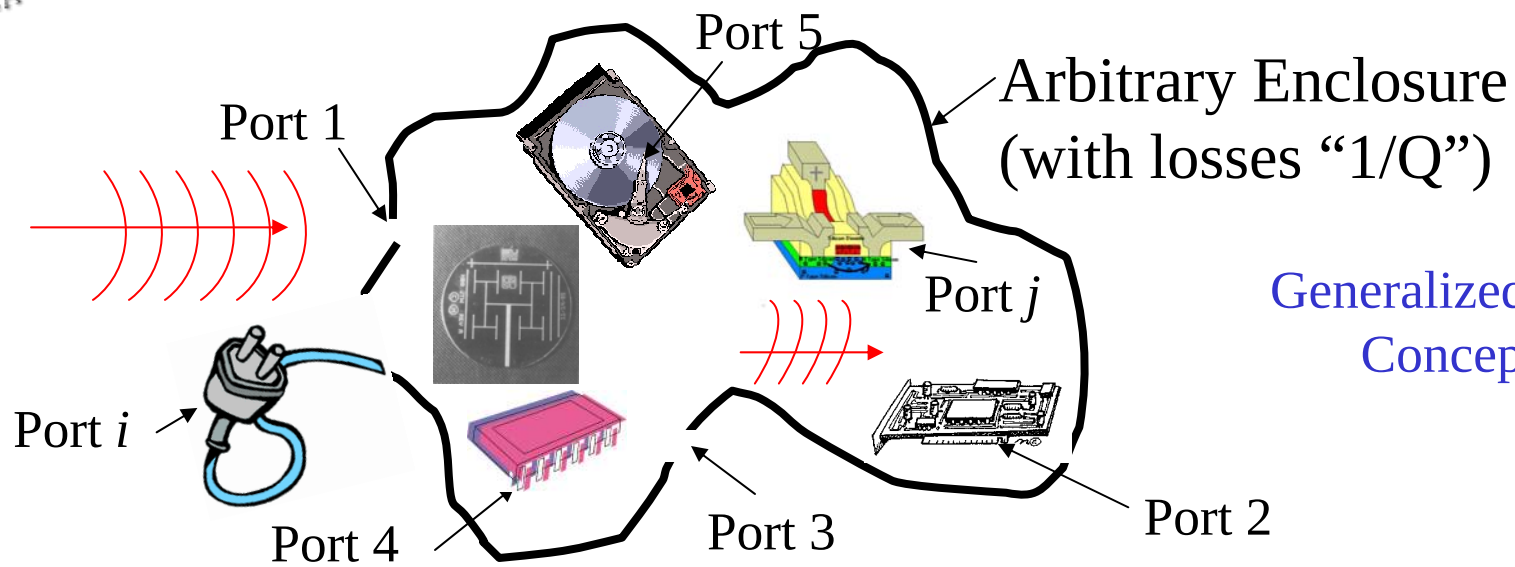
D. H. Wu, *et al.*, Phys. Rev. Lett. 81, 2890 (1998)



This description does not include the effects of coupling

Here we bring the statistical electromagnetic predictions one step closer to reality by including the effects of **coupling** and **losses**

The Ultimate Goal:
Induced Field Distributions for an Arbitrary Enclosure



Generalized Port Concept

What minimum information do we need to predict the range of voltages on port j because of 1 Watt injected through port i?

Random Coupling Model

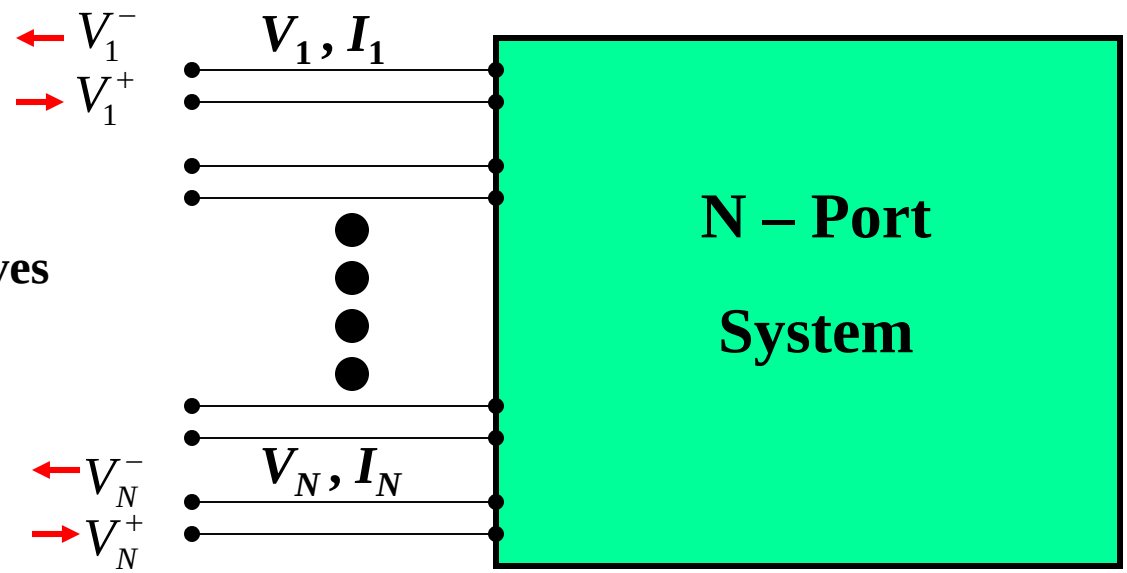
Based on results and concepts from the field of Wave Chaos

The predictions are statistical in nature

N-Port Description of an Arbitrary Enclosure

N Ports

- **Voltages and Currents,**
- **Incoming and Outgoing Waves**



Z matrix

$$\begin{bmatrix} V_1 \\ V_2 \\ \bullet \\ \bullet \\ \bullet \\ V_N \end{bmatrix} = [Z] \cdot \begin{bmatrix} I_1 \\ I_2 \\ \bullet \\ \bullet \\ \bullet \\ I_N \end{bmatrix}$$

S matrix

$$\begin{bmatrix} V^-{}_1 \\ V^-{}_2 \\ \bullet \\ \bullet \\ \bullet \\ V^-{}_N \end{bmatrix} = [S] \cdot \begin{bmatrix} V^+{}_1 \\ V^+{}_2 \\ \bullet \\ \bullet \\ \bullet \\ V^+{}_N \end{bmatrix}$$

$$S = (Z + Z_0)^{-1}(Z - Z_0)$$

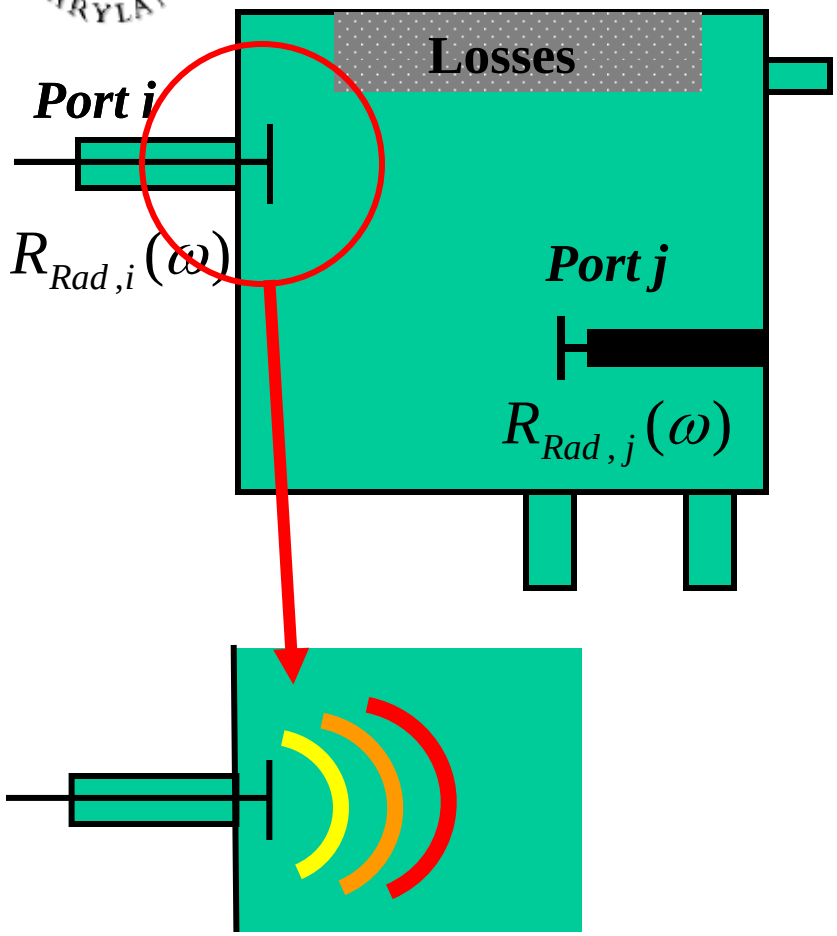
$$Z(\omega), S(\omega)$$

- Complicated Functions of frequency
- Detail Specific

SA1

Statistical Model of Z Matrix- Random Coupling Model

Theoretical Work by Xing Zheng, Antonsen, Ott.



Free Space Radiation Impedance:

$$Z_{Rad}(\omega) = R_{Rad}(\omega) + jX_{Rad}(\omega)$$

Statistical Model Impedance

$$Z_{ij}(k) = -\frac{j}{\pi}\sum_n R_{Rad\,i}^{1/2}(k_n) R_{Rad\,j}^{1/2}(k_n) \frac{\Delta k_n^2 w_{in} w_{jn}}{k^2(1-jQ^{-1}) - k_n^2}$$

System Parameters
- $R_{Rad,1}(\omega)$ **- Radiation Impedance**
- Δk_n^2 **-mean eigenspacing**
- Q **- Quality Factor (Losses)**

Statistical Parameters
- k_n^2 - Eigenvalues
- w_{in} -Gaussian Random Variables

$$\frac{k^2}{\Delta k_n^2 Q}$$ **=> Key Parameter**

~ Im(ω_0)/$\Delta\omega$

Similar (independent) work by L. K. Warne, *et al.*, IEEE Trans. Anten. Prop. **51**, 978 (2003)

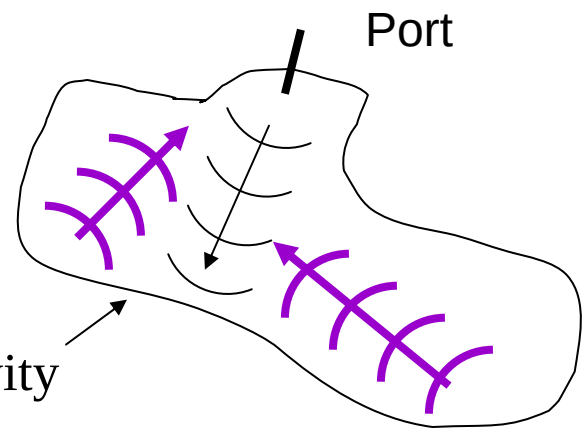
Slide 7

SA1 Xrad comes in through the n cutoff of the sum. If k is small compared to the cutoff, there will be a net contribution to the sum given by the radiation reactance.

Steve Anlage, 7/9/2004

The Key Step:
Forming the Normalized Impedance (z)

$\mathbf{Z_{Cavity}}$

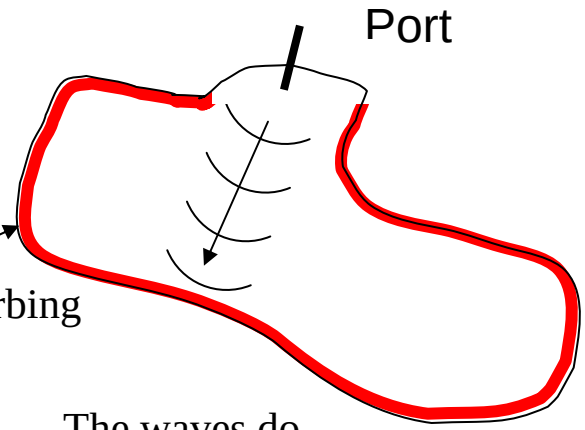


$$Z_{Cavity} = R_{Cavity} + j\,X_{Cavity}$$

Combine

$$z = \frac{R_{Cavity}}{R_{Rad}} + j\,\frac{X_{Cavity} - X_{Rad}}{R_{Rad}}$$

$\mathbf{Z_{Rad}}$



Perfectly absorbing boundary

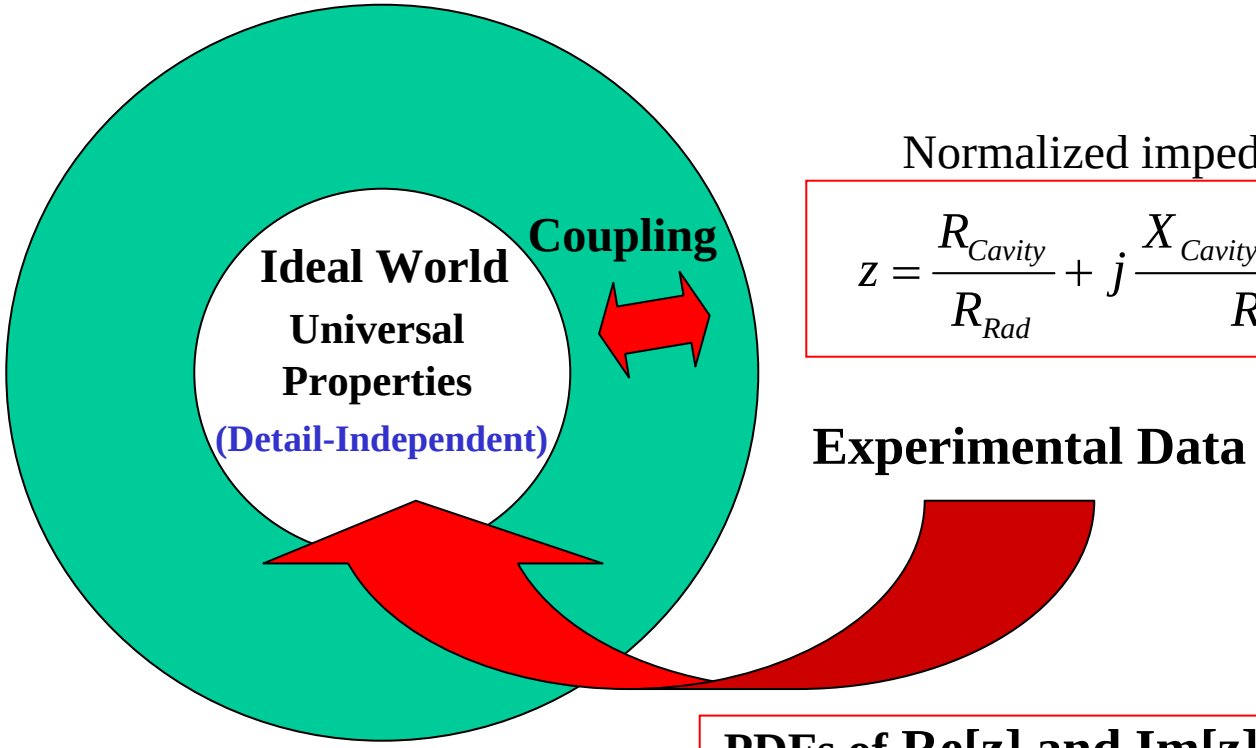
The waves do not return to the port

$$Z_{Rad} = R_{Rad} + j\,X_{Rad}$$

Radiation Losses

Reactive Impedance of Antenna

Testable Predictions of the Random Coupling Model



Normalized impedance

$$z = \frac{R_{Cavity}}{R_{Rad}} + j\frac{X_{Cavity} - X_{Rad}}{R_{Rad}}$$

PDFs of Re[z] and Im[z] are universal

$$\sigma^2\{\mathrm{Re}[z]\} \cong \sigma^2\{\mathrm{Im}[z]\} \cong (1/\pi)/\left(\frac{k^2}{\Delta k_n^2 Q}\right)$$ **For TRS Systems**

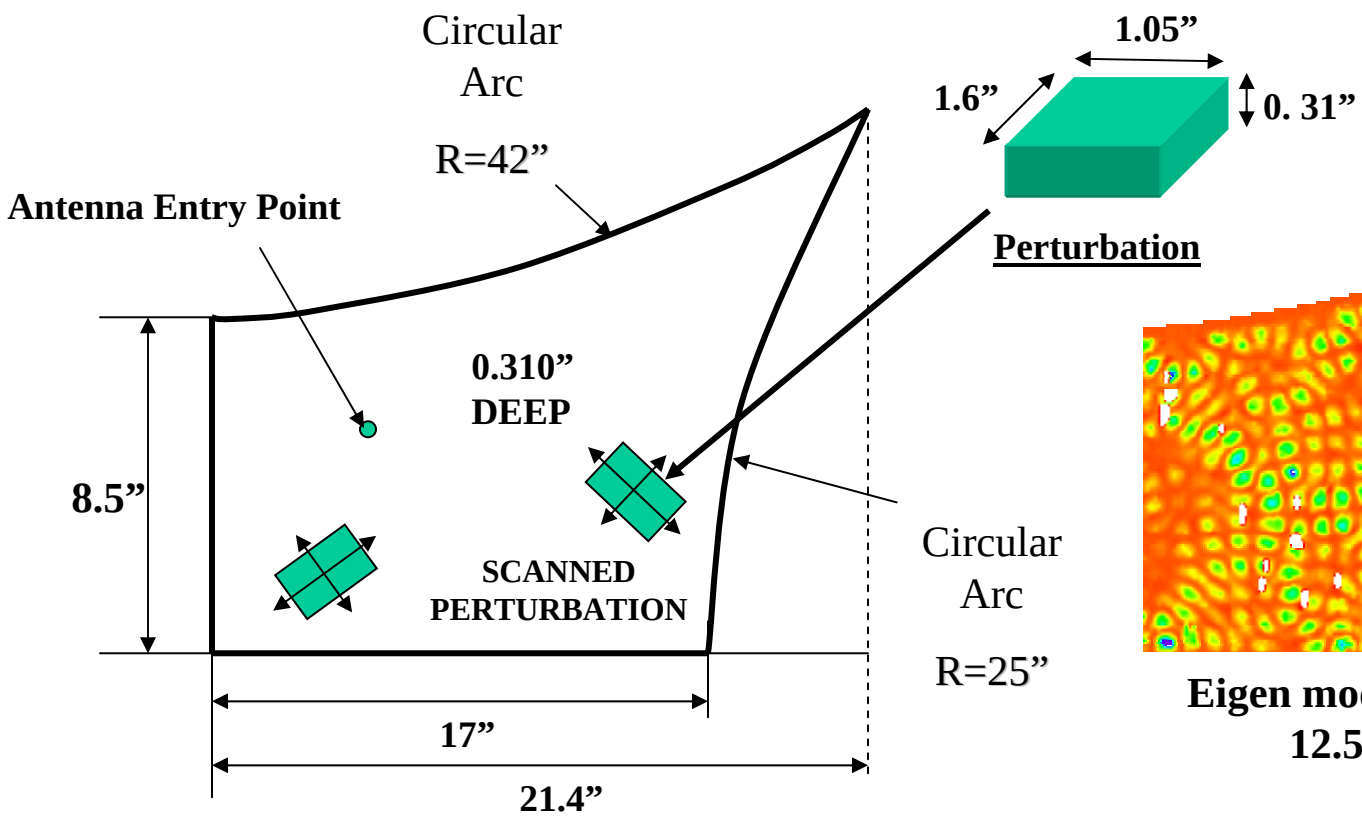
Normalized Scattering Parameter

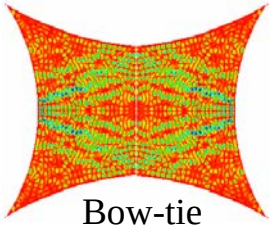
$$s = \frac{z-1}{z+1}$$

|s| and Arg[s] are independent

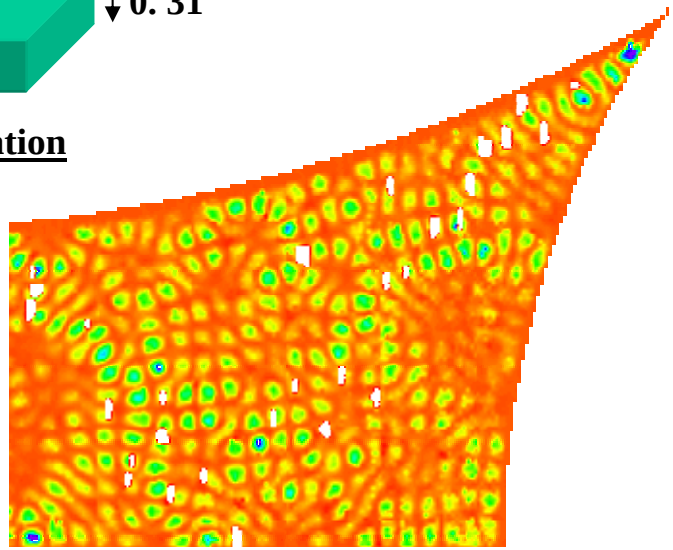
¼-Bow-Tie Resonator

EXPERIMENTAL SETUP:





Bow-tie resonator

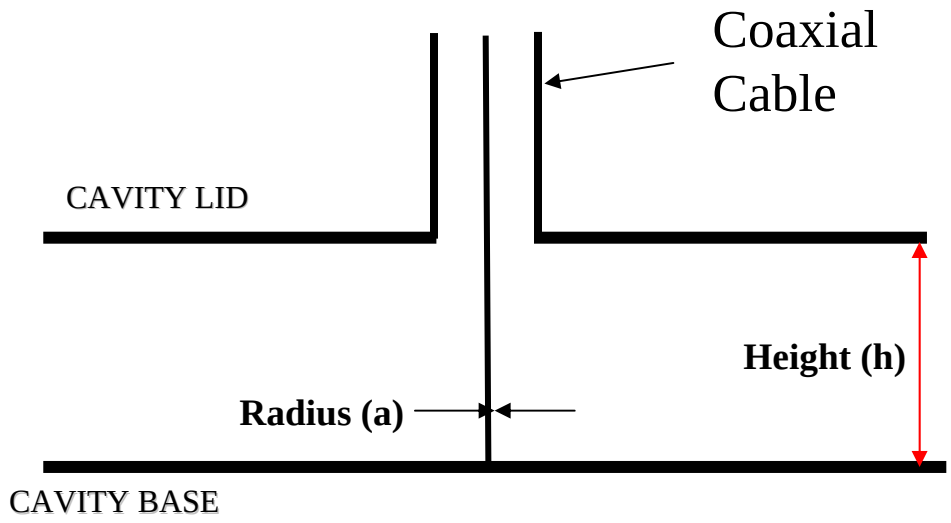


Eigen mode Image at 12.57GHz

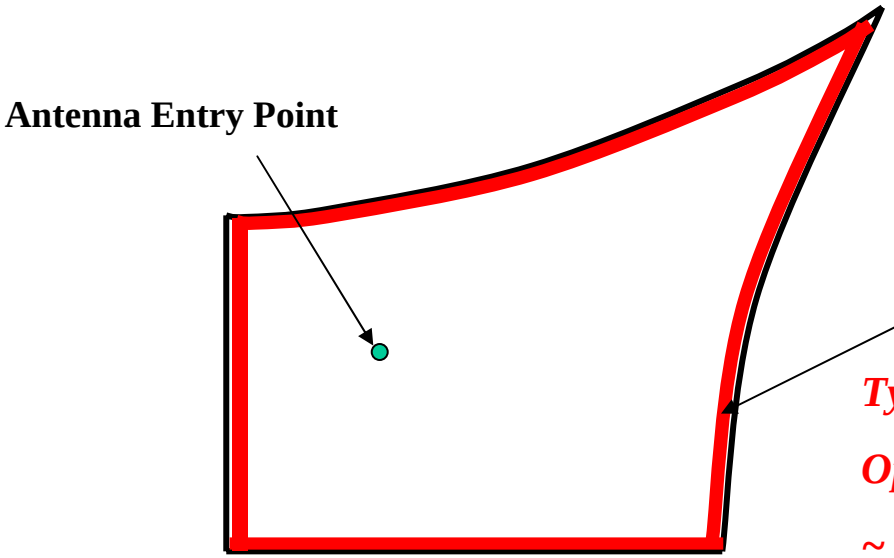
- 2 Dimensional Quarter Bow Tie Wave Chaotic cavity
- Classical ray trajectories are chaotic - short wavelength - Quantum Chaos
- 1-port S and Z measurements in the 6 – 12 GHz range.
- Ensemble average through 100 locations of the perturbation

Antenna Detail

Cross Section View



Implementation of "Radiation Boundary Conditions"



Microwave Absorber

Type: ARC Tech- DD 10017D

Operational Freq: 6-12 GHz

~ -25dB Reflection Loss

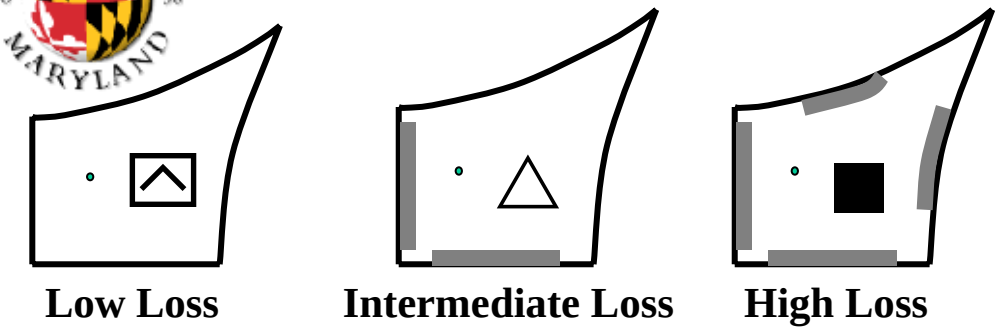
Experimental Tests of the Random Coupling Model

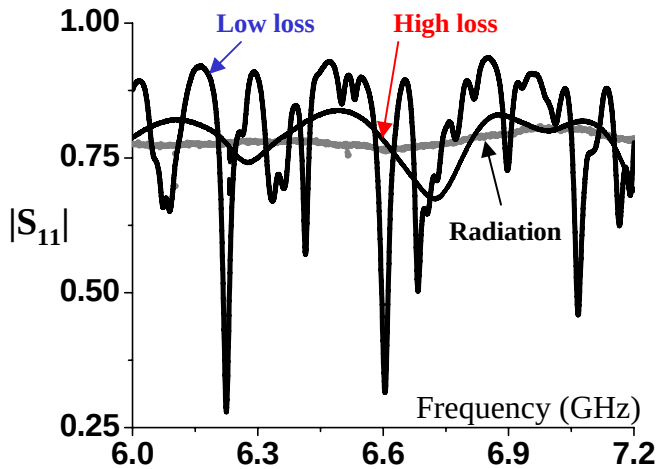
z = normalized impedance
s = normalized scattering matrix

- Single-parameter fits to PDF of Re[z], Im[z]
- Equivalence of variances of PDFs and single fitting parameter
- Insensitivity of Re[z] and Im[z] to irrelevant details
- Frequency, volume, loss dependence of Re[z] and Im[z] PDFs

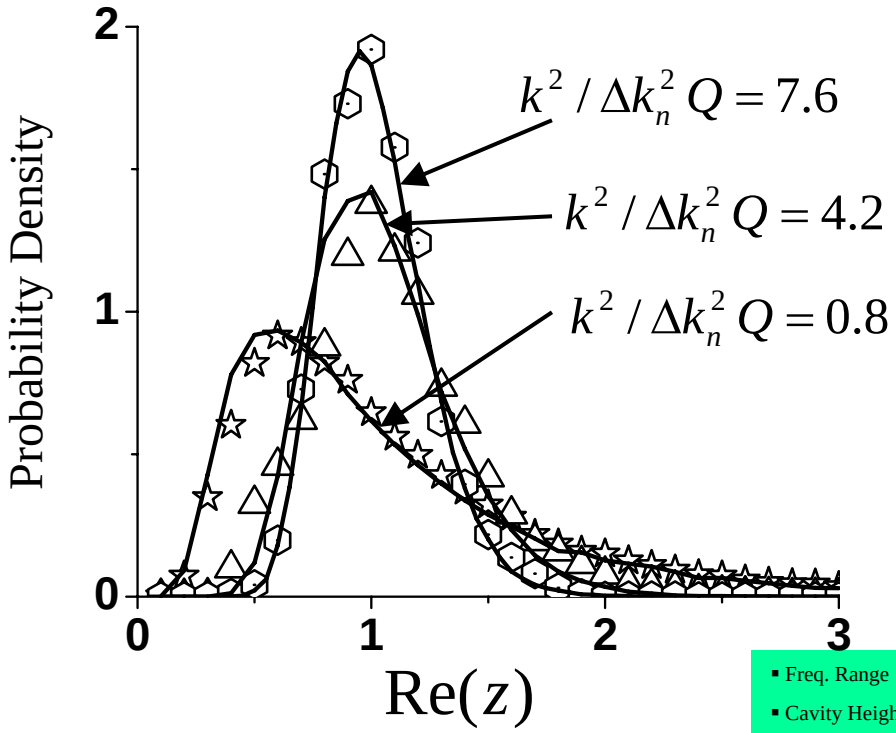
- Single-parameter fits to PDF of |s|, uniform distribution of Arg[s]
- Independence of |s| and Arg[s]

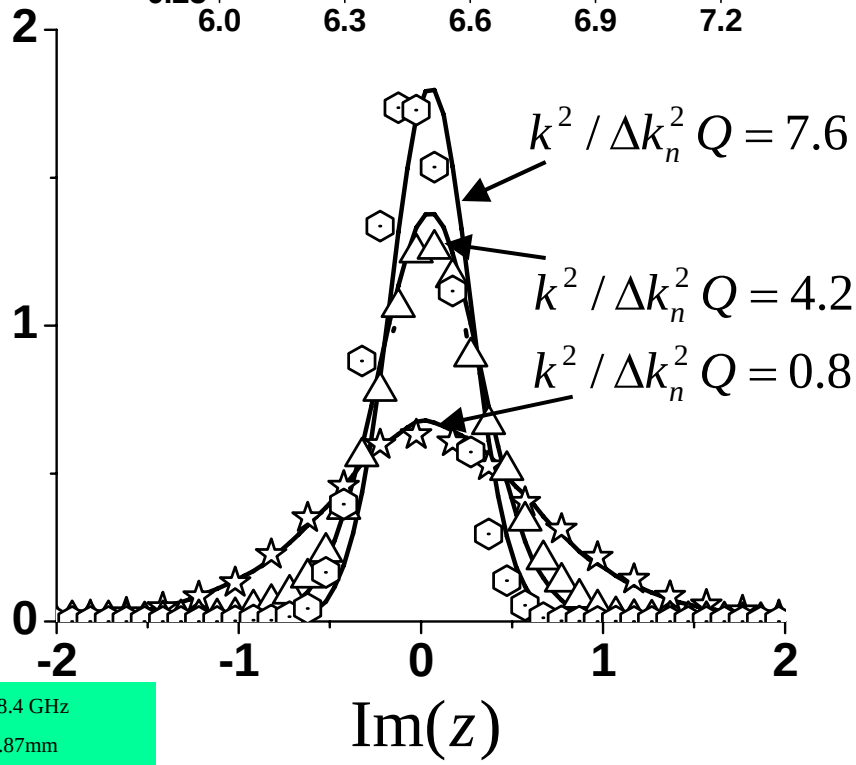
Testing the Effects of Varying Loss





$$\frac{k^2}{\Delta k_n^2 Q} \sim \frac{\mathrm{Im}[\omega_0]}{\Delta\omega}$$

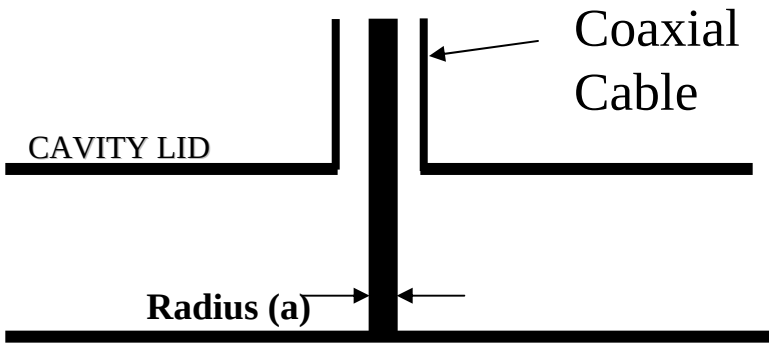




- Freq. Range : 7.2 to 8.4 GHz
- Cavity Height : h= 7.87mm
- Antenna Diameter : 2a=1.27mm
- Statistics drawn from 160,200 pts.

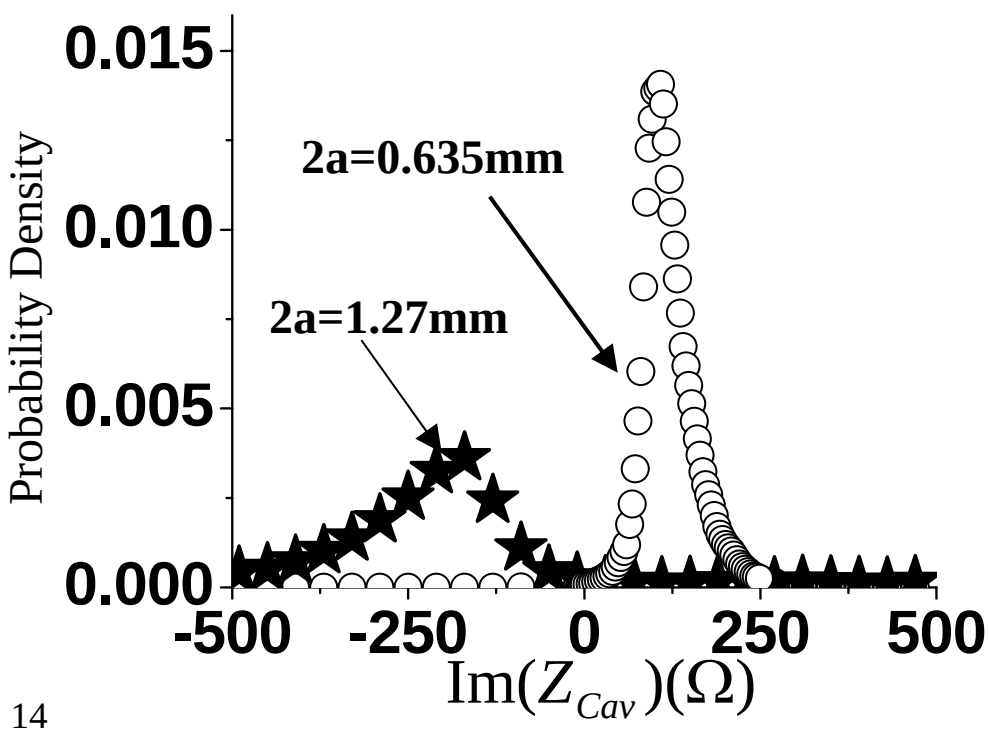
Testing Insensitivity to System Details

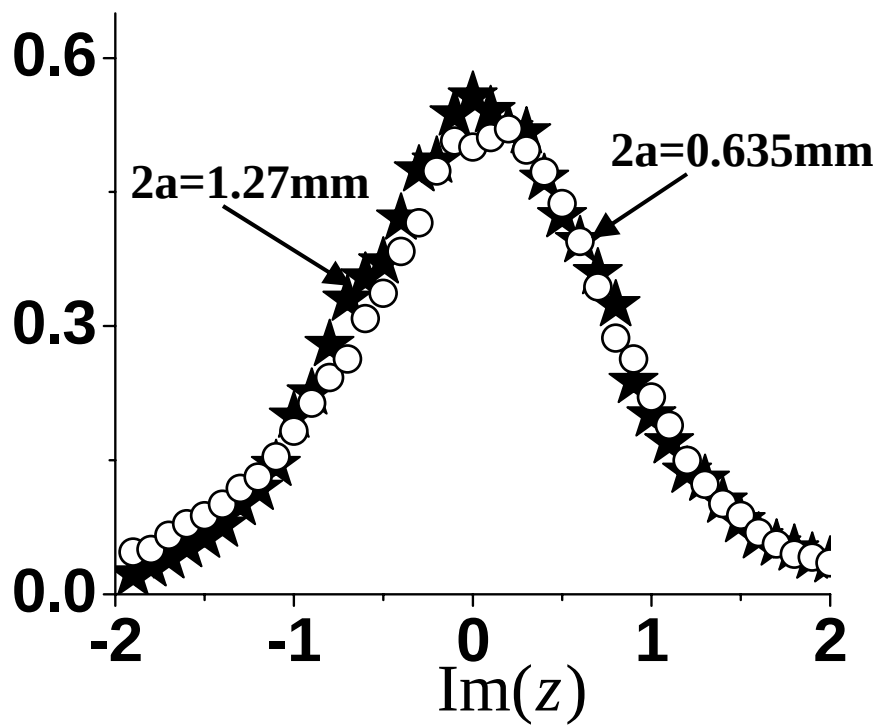
Cross Section View



- Freq. Range : 9 to 9.75 GHz
- Cavity Height : h= 7.87mm
- Statistics drawn from 100,125 pts.

$$z = \frac{R_{Cavity}}{R_{Rad}} + j\frac{X_{Cavity} - X_{Rad}}{R_{Rad}}$$





Testing Predictions on Variance of z PDFs

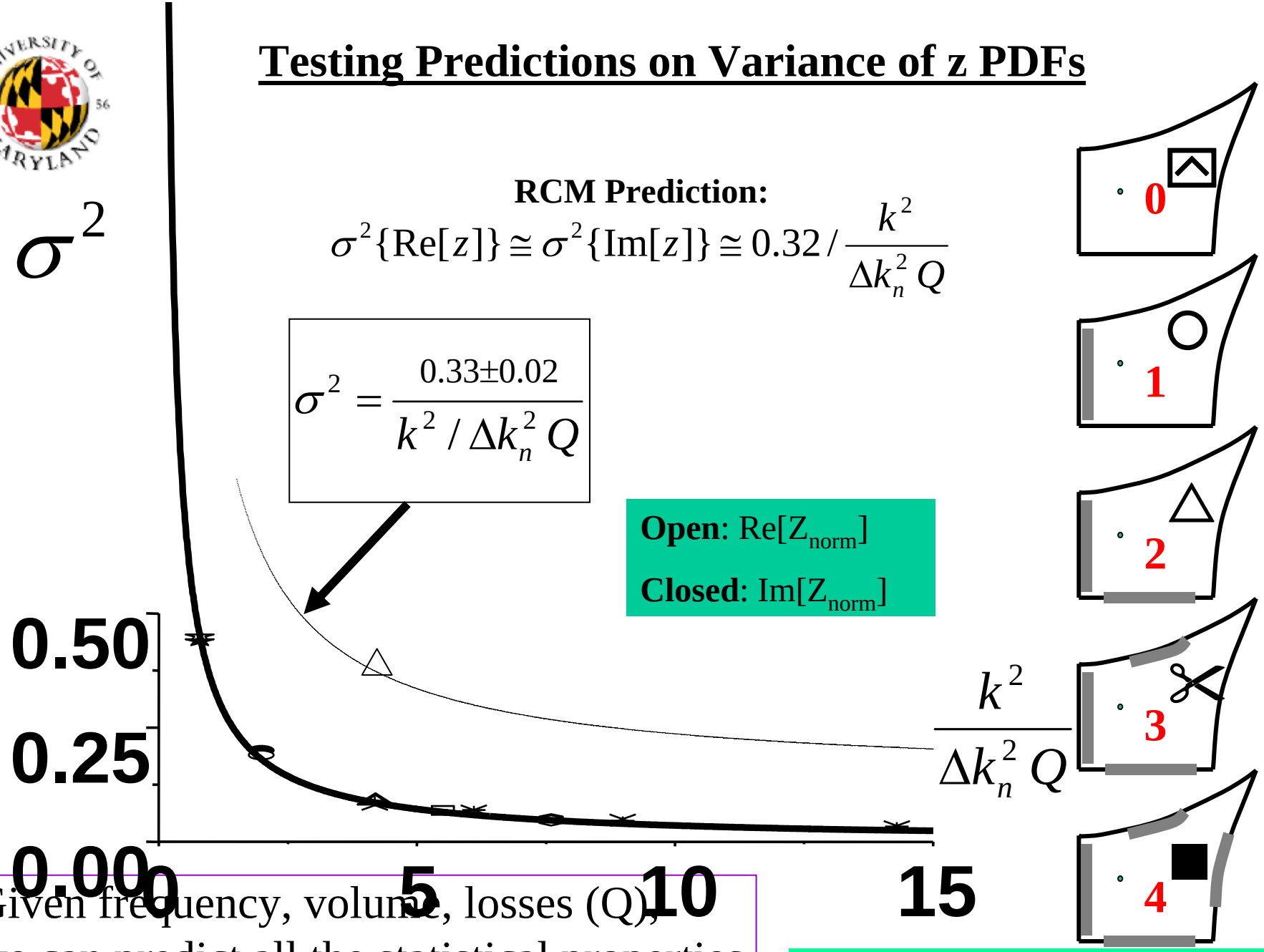
RCM Prediction:

$$\sigma^2\{\mathrm{Re}[z]\} \cong \sigma^2\{\mathrm{Im}[z]\} \cong 0.32 / \frac{k^2}{\Delta k_n^2 Q}$$

$$\sigma^2 = \frac{0.33 \pm 0.02}{k^2 / \Delta k_n^2 Q}$$

Open: $\mathrm{Re}[Z_{norm}]$

Closed: $\mathrm{Im}[Z_{norm}]$

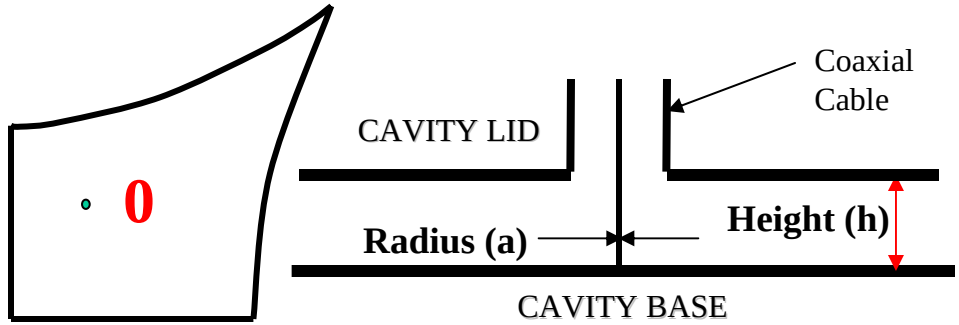


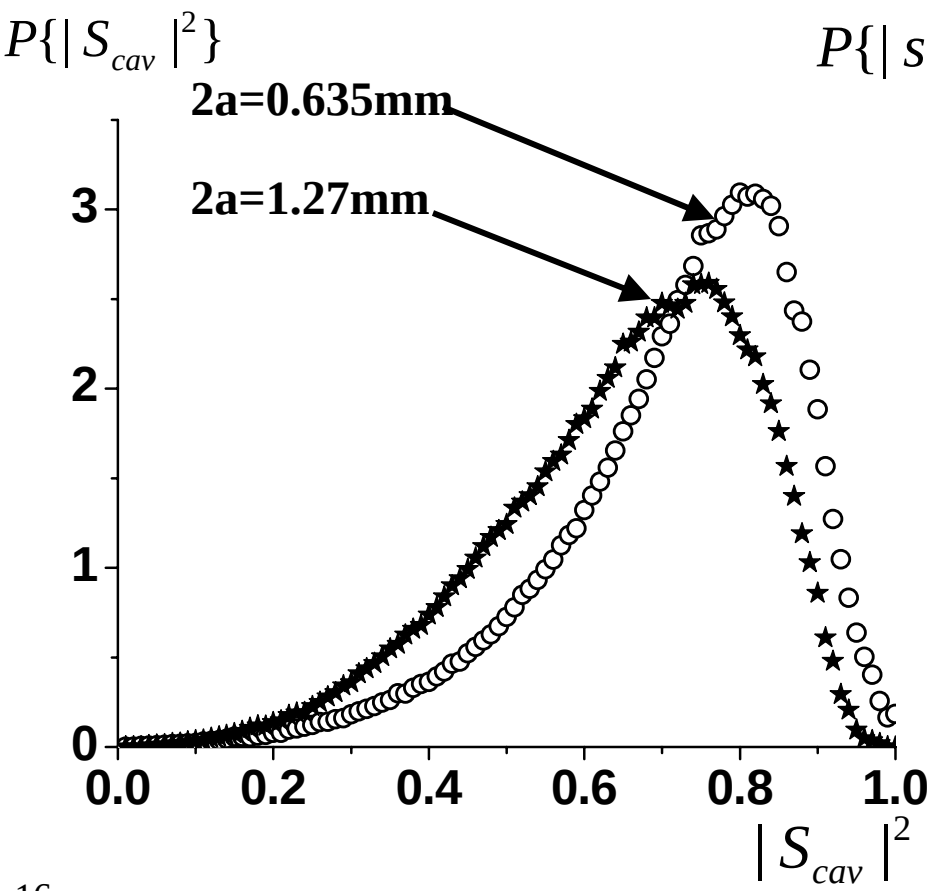
Given frequency, volume, losses (Q), we can predict all the statistical properties

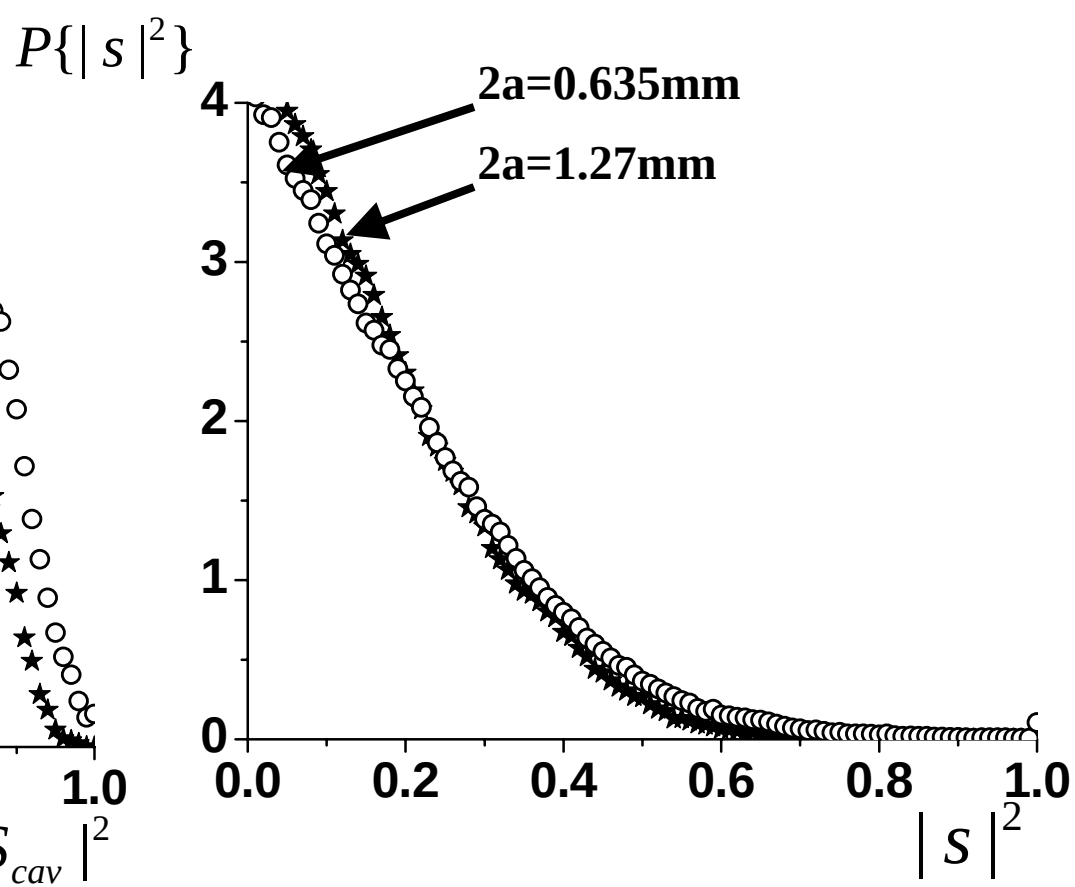
- Frequency Range for loss cases: 7.2 to 8.4GHz
- Antenna Diameter (2a) = 1.27mm

Insensitivity to System Coupling Details

- **Frequency Range: 6 to 12 GHz**
- **Height of cavity : h= 7.87mm**
- **Statistics drawn from 800,100 pts.**

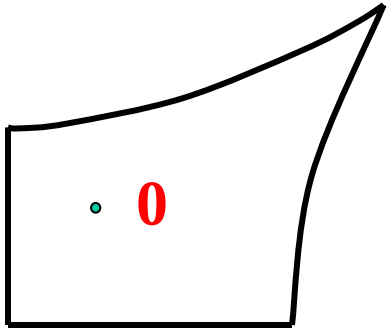




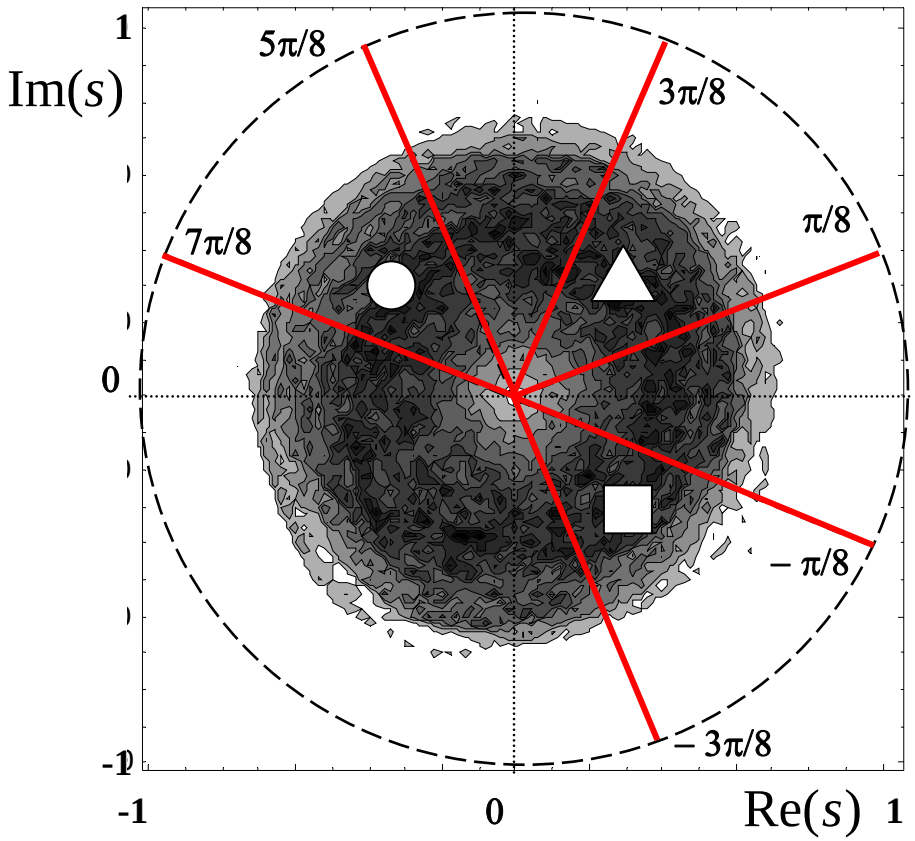


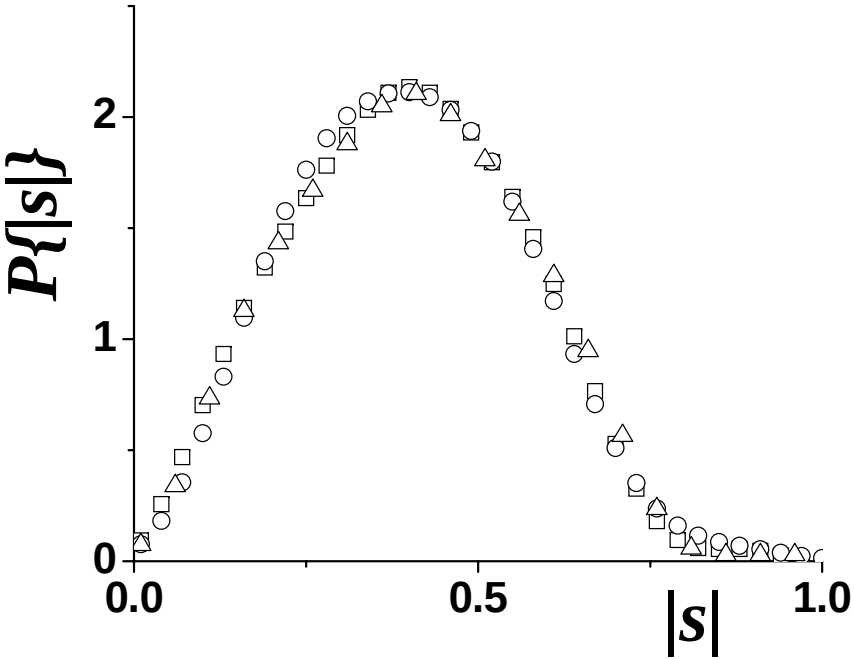
Independence of ϕ_{norm} and $|s|$

- Frequency: 6 to 9.6 GHz
- Cavity Height : 7.87mm
- Antenna Diameter : 1.27mm

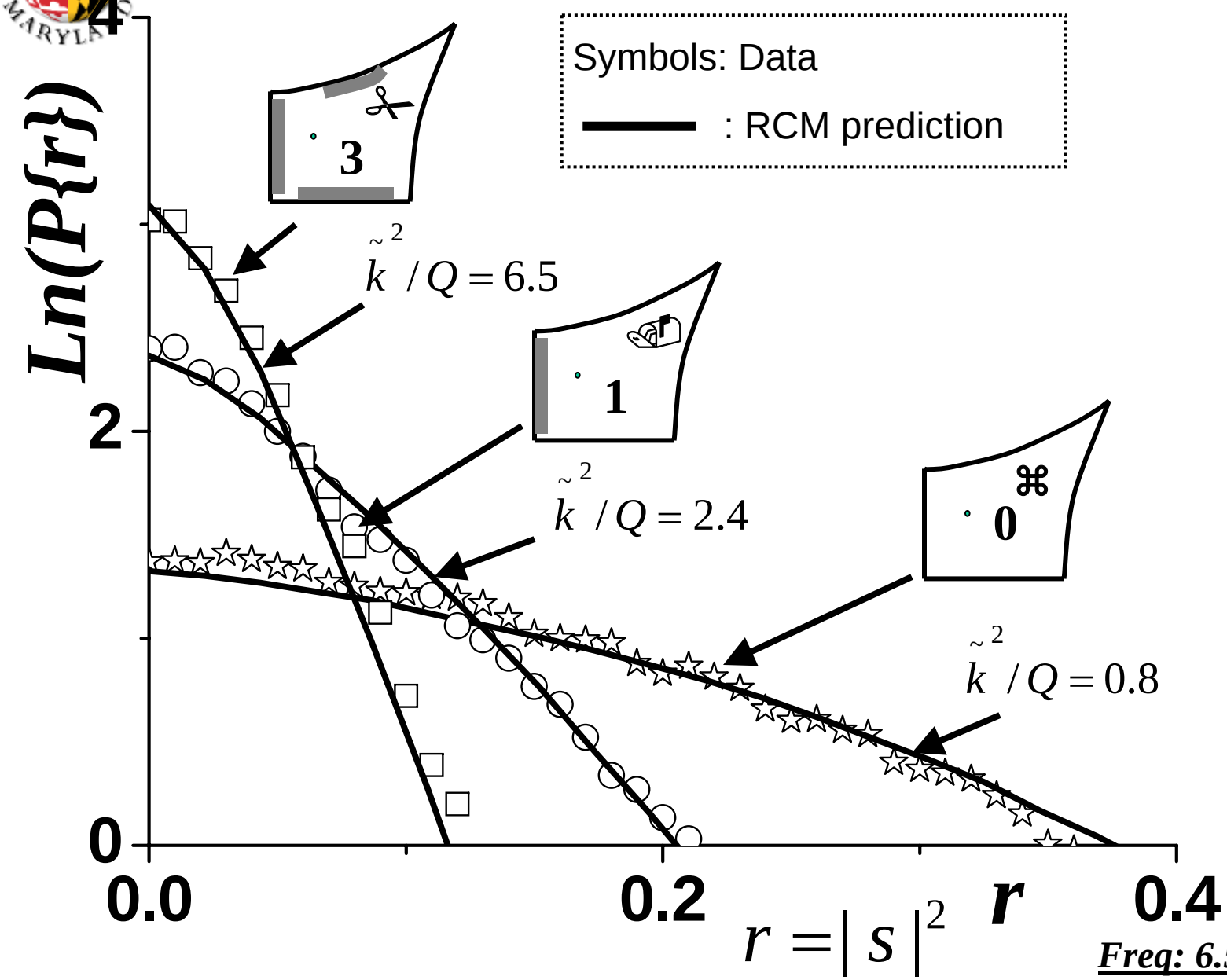


$$s = \frac{z-1}{z+1}$$





PDF of Reflection Coefficient with Varying Loss



Freq: 6.5 to 7.8 GHz

Experimental Tests of the Random Coupling Model

z = normalized impedance
s = normalized scattering matrix

- ✓ Single-parameter fits to PDF of Re[z], Im[z]
- ✓ Equivalence of variances of PDFs and single fitting parameter
- ✓ Insensitivity of Re[z] and Im[z] to irrelevant details
- ✓ Frequency, volume, loss dependence of Re[z] and Im[z] PDFs

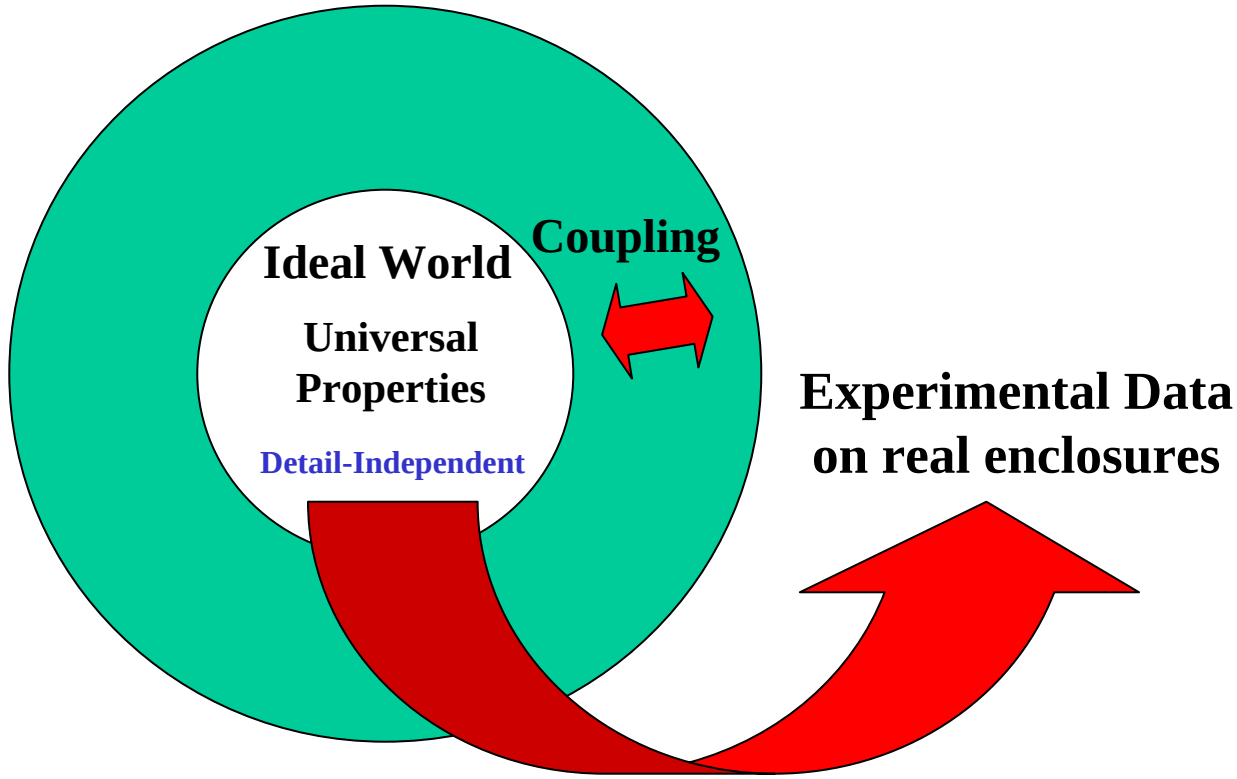
- ✓ Single-parameter fits to PDF of |s|, uniform distribution of Arg[s]
- ✓ Independence of |s| and Arg[s]

Good Agreement!

These results are general, and their applicability is not limited in any way by the nature of our "bow-tie" cavity (dimensionality, shape, etc.)

Practical Implications for Real Life Problems

Bare Minimum Specifications for Field Statistics



Minimum Information:

Frequency, Volume
Losses
Radiation impedance of the ports

} Determine the shape and scales of the Z_{Cavity} and Field PDFs

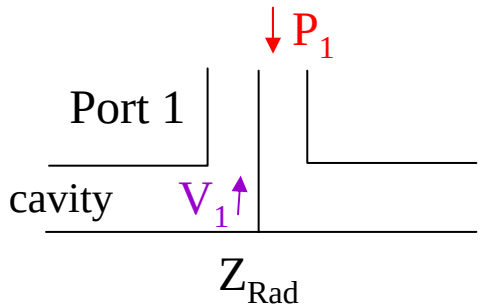
Some Practical Implications

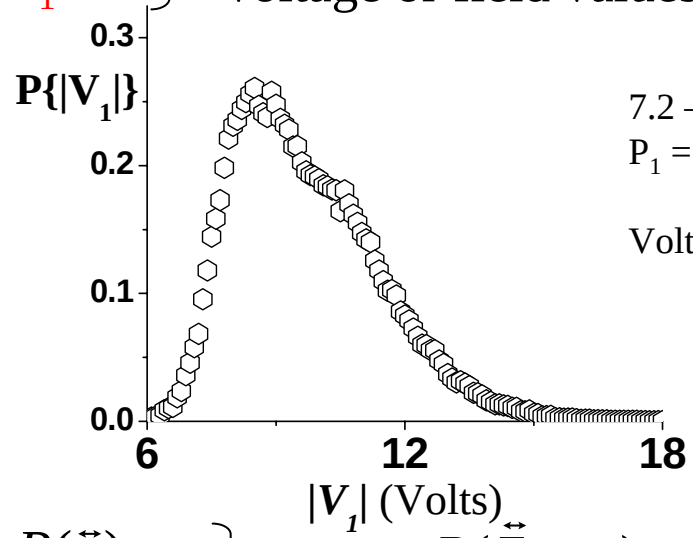
Predictions for V and E PDFs

Based on 1-port and 2-port <u>Data</u>

1-port Specify Q, frequency ⇒ P(z)
+ Specify coupling (Z_{Rad}), P_1

Yields $P(Z_{Cavity})$, $P(V_1)$, PDF of voltage or field values near port 1



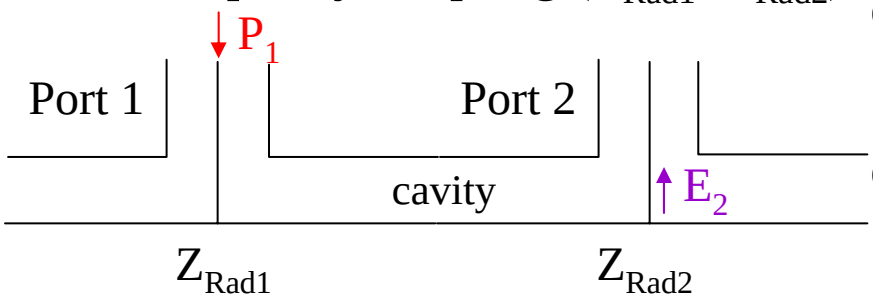


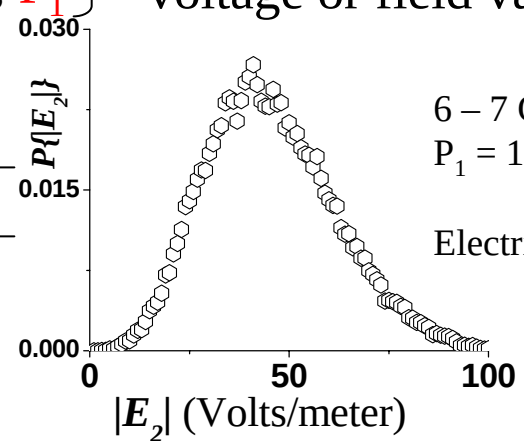
7.2 – 8.4 GHz
P_1 = 1 Watt

Voltage PDF

2-port Specify Q, frequency ⇒ $P(\vec{z})$
+ Specify coupling (Z_{Rad1}, Z_{Rad2}), P_1

Yields $P(\overleftrightarrow{Z}_{Cavity})$, $P(E_2)$, PDF of voltage or field values near port 2





6 – 7 GHz
P_1 = 1 Watt

Electric Field PDF

E- and B-Field PDF Engineering

Radiation Impedance

Z_{Rad} can be calculated, computed, or measured for each port

Z_{Rad} analytical calculation

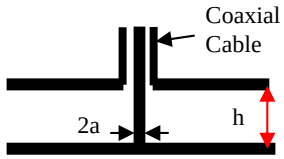
Short dipole: $R_{rad} = 20\pi^2 \left(\frac{\ell}{\lambda}\right)^2 \Omega$

Annular current in 2D cavity:

$$R_{Rad} = \frac{kh\eta_0}{4} J_0^2(ka)$$

$$X_{Rad} = -\frac{kh\eta_0}{4} J_0(ka) Y_0(ka)$$

$$ka << 1$$

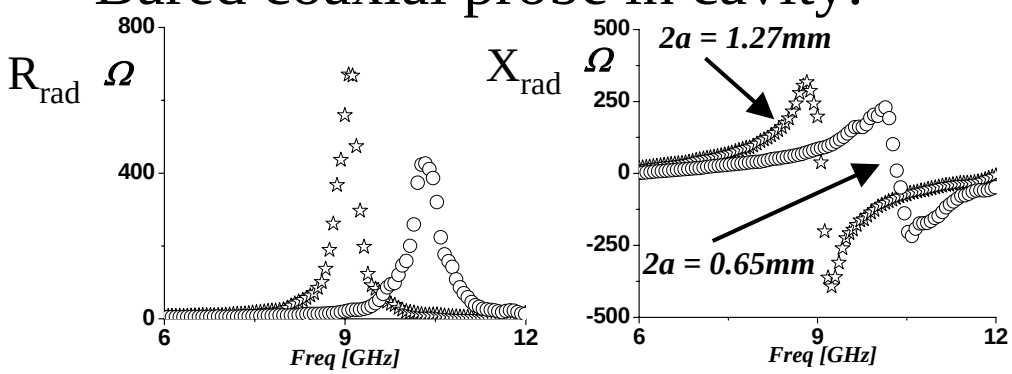


Z_{Rad} computation

HFSS, etc.

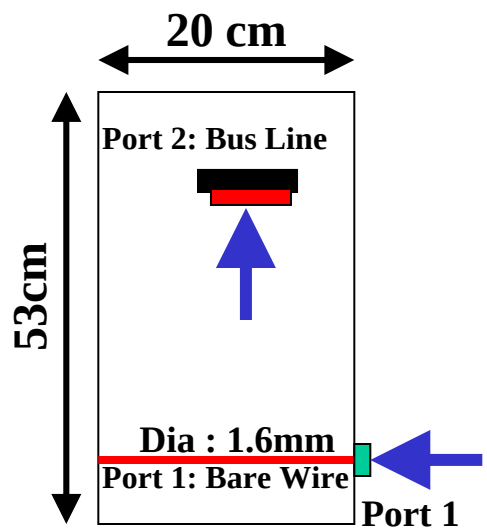
Z_{Rad} measurement

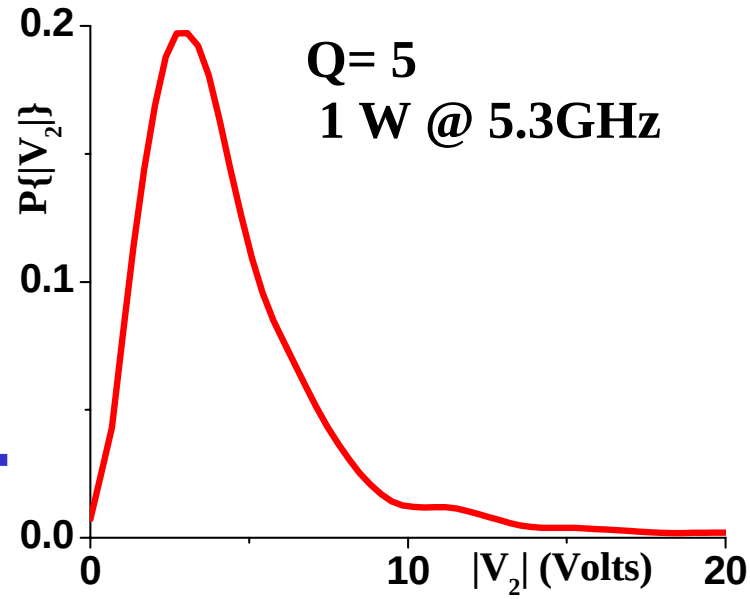
Bared coaxial probe in cavity:

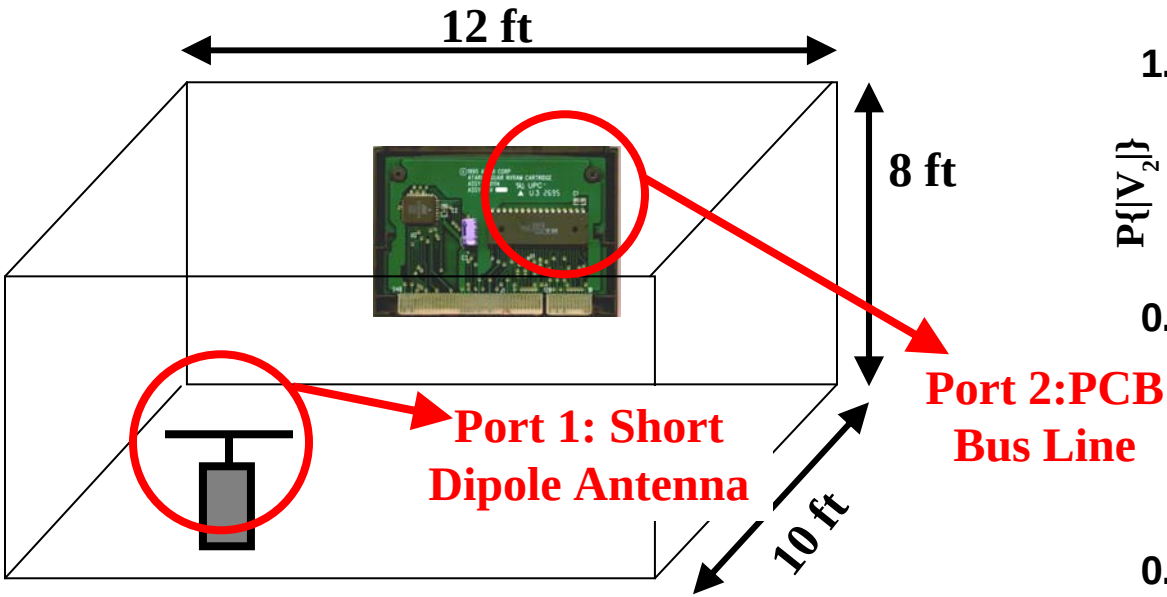


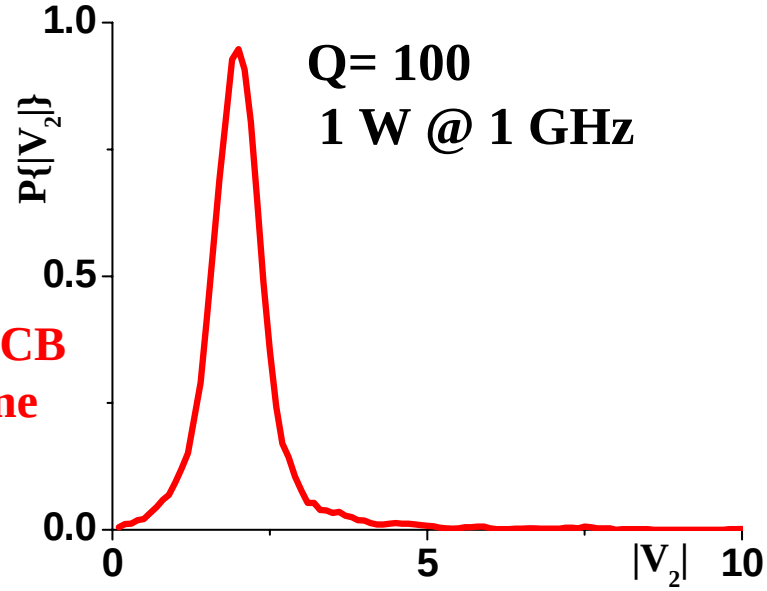
Application of RCM to a Real Problem

Induced Voltage PDFs in a Computer Enclosure and Room



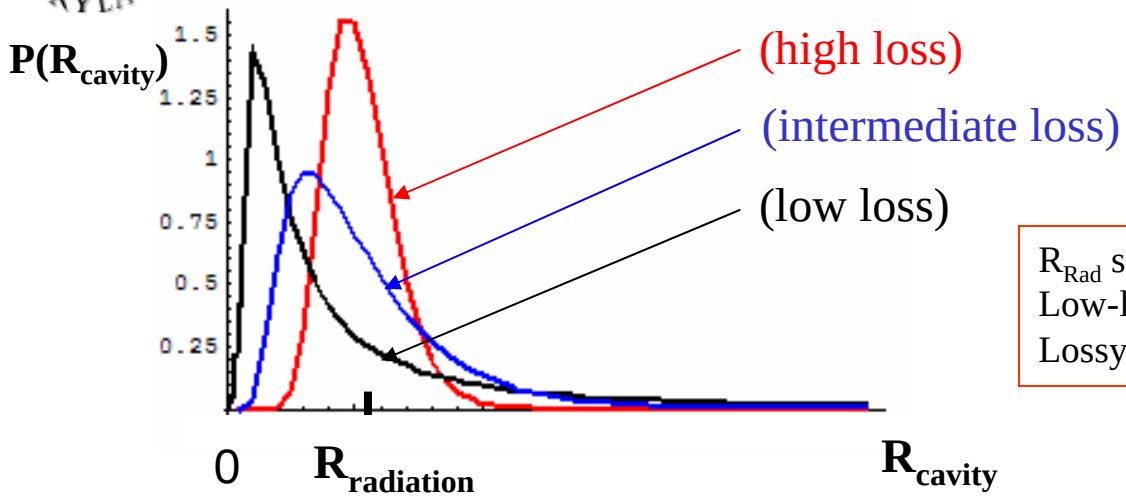






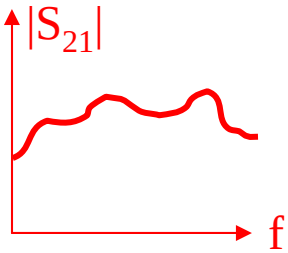
Cavity Impedance and Field PDF Engineering

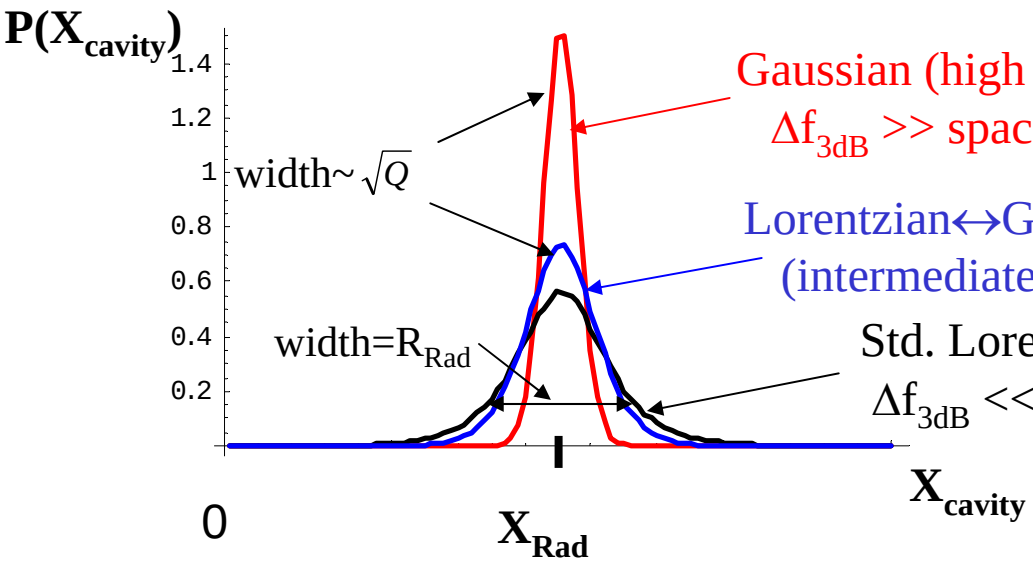
RCM Results

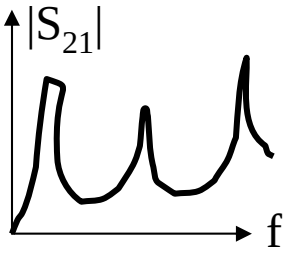


$$Z_{cavity} = R_{cavity} + i\, X_{cavity}$$

R_{Rad} sets the scale for R_{cavity}
Low-loss case: $R_{Cavity} < R_{Rad}$
Lossy case: $\Rightarrow$ Gaussian distribution, width ~ $\sqrt{Q}$







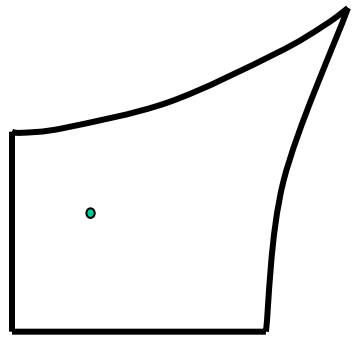
X_{Rad} sets the scale for X_{cavity}
Low-loss case: broad tails, width ~ $R_{Radiation}$
Lossy case: narrow distribution, width ~ $\sqrt{Q}$

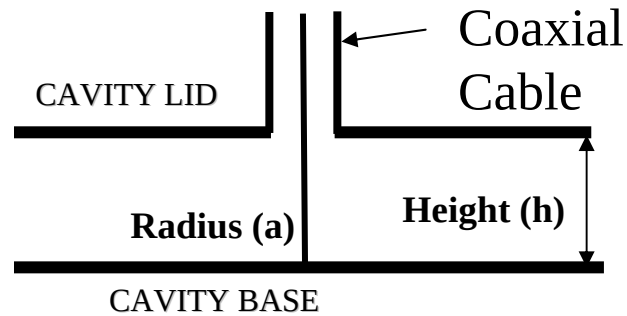
Prescription to Engineer Cavities with Desired Electromagnetic Properties

Numerically Generated (z)
{Frequency, Q}

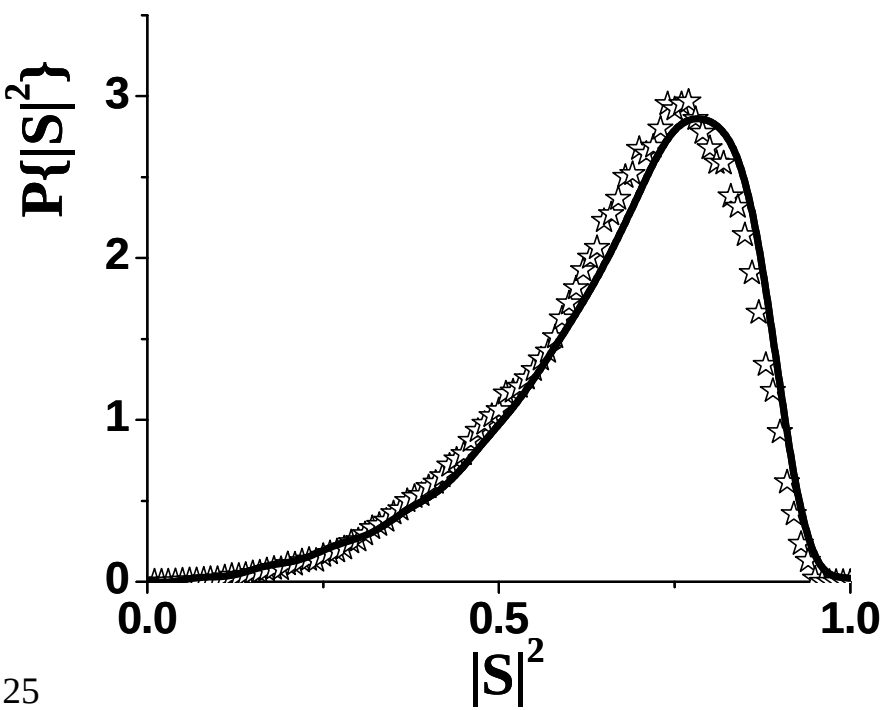
$$Z_{cav} = jX_{rad} + z \cdot R_{rad}$$

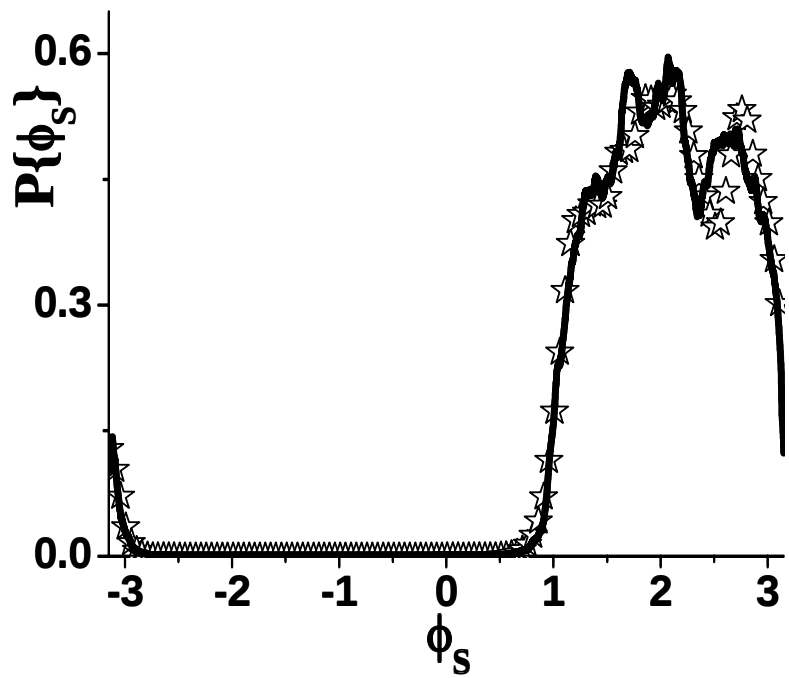
$$Z_{rad} = R_{rad} + jX_{rad}$$





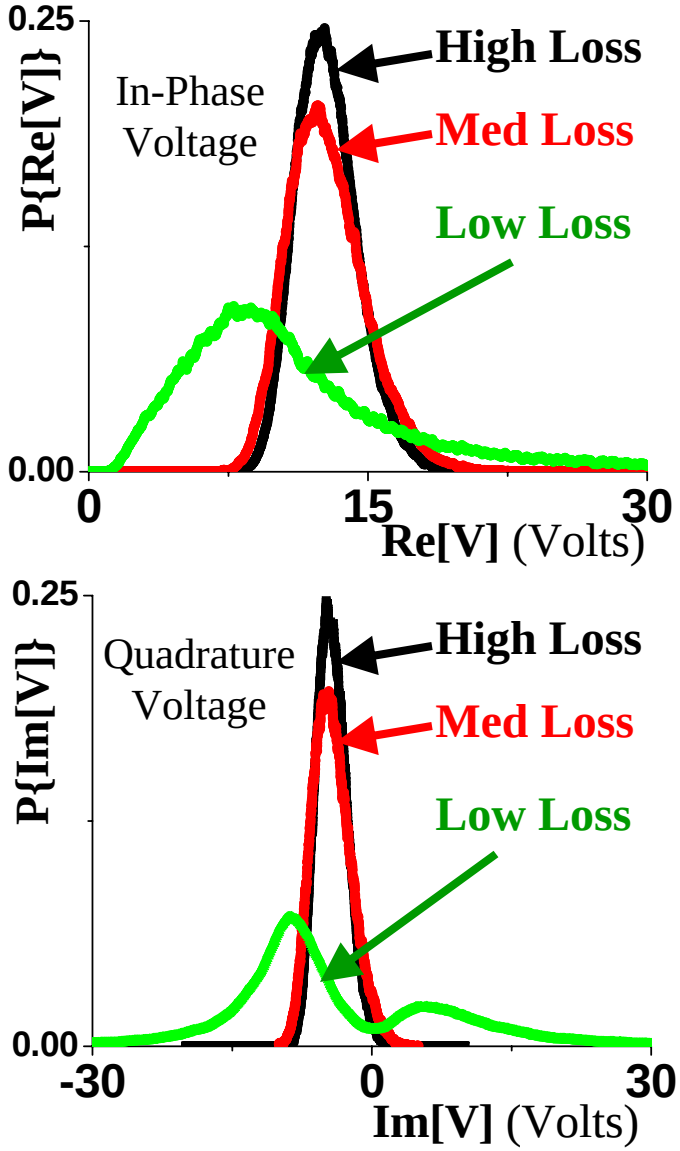
- **Freq : 6 to 9.6 GHz**
- **Antenna Dia (2a)= 1.27mm**





One-Port Complex Voltage PDFs

Simulated Results



Simulation Parameters:

1. **Frequency : 9.2 - 9.6 GHz**
2. $\tilde{k}^2 / \Delta k_n^2 Q$

   High Loss: 4

   Med Loss : 2.5

   Low Loss : 0.07
3. **Bared wire antenna diameter = 1.6 mm**

1 Watt input power

Progress / Conclusions

We have created a Random Coupling Model to extend the statistical electromagnetic predictions to real enclosures by including the effects of <u>coupling</u>

Experimental tests of many basic 1 port and 2-port predictions of the Random Coupling Model (RCM) have confirmed that the model is correct

Frequency, Volume
Losses
Radiation impedance of the ports

} Determine the shape and scales of the E-field, Z, S PDFs

Clear strategies to engineer the field PDFs to prevent damage to circuits, components, etc.

<u>OR</u>

Clear statistical predictions of 'effects' given a minimum of assumptions about target

Preprints available at: **http://ArXiv.org/cond-mat/0403225**
http://www.ireap.umd.edu/MURI-2001/publications.htm

Future Work

- N-Port Measurements
  - >> Other types of ports (e.g. slit/waveguide)
  - >> Test the effects of non-reciprocity (magnetized ferrites)
  - >> Case of interacting ports
  - >> Create a database of port radiation impedances

- Three-Dimensional Cavities
  - >> The high mode density makes experiment/theory simpler
  - >> Test the RCM in the low-loss limit

- Comparison of RCM Predictions to Other Measurements
  - >> Develop analytical interpolating functions for Z, E PDFs
  - >> Compare our model predictions to “real” data

- Time-Domain Measurements
  - >> Energy time-decay statistics
  - >> Time and frequency domain correlations of Z, S, E-fields