Comparison of a Low- to High-Confinement Transition Theory with Experimental Data from DIII-D

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(Received 7 August 2002; published 9 December 2002)

From our recent theory based on the generation of shear flow and field in finite β plasmas, the criterion for bifurcation from low to high confinement mode yields a critical parameter proportional to \( T_e/\sqrt{L_n} \), where \( T_e \) is the electron temperature and \( L_n \) is the density scale length. The predicted threshold shows very good agreement with edge measurements on discharges undergoing low-to-high transitions in DIII-D. The observed differences in the transitions with the reversal of the toroidal magnetic field are reconciled in terms of this critical parameter. The theory also provides an explanation for pellet injection \( H \) modes in DIII-D, thereby unifying unconnected methods for accomplishing the transition.

The initial observation of the low \((L)\) to high \((H)\) confinement transition in tokamaks in the axisymmetric divertor experiment (ASDEX) tokamak over two decades ago has promoted intense investigation of this phenomenon, because the improved confinement of the \( H \) mode significantly enhances the prospects for achieving fusion in magnetically confined tokamak plasmas [1–3]. Central to this enhanced confinement regime is the generation of localized zonal/shear flow, which is believed to be responsible for suppressing fluctuations, creating a transport barrier and thereby inhibiting transport. Although many theories for \( L-H \) transitions have been developed, the basic “trigger” mechanism for this transition has been elusive. Also many intriguing observations have added to the complexity of the problem. For single null divertor experiments, the heating power threshold for the discharges with the ion \( \nabla B \) drift away from the \( X \) point is about a factor of 2 higher than for those discharges with \( \nabla B \) drift towards the \( X \) point [4]. Another bewildering observation is that the injection of a pellet into the plasma induces an \( H \)-mode transition at heating power significantly below that of the conventional threshold [5]. Thus it is important to identify the physics-based critical parameter that controls these transitions in tokamaks.

An excellent review by Connor and Wilson [6] provides a comprehensive view of the experimental and theoretical studies devoted to \( L-H \) transitions in tokamaks. Computational modeling of \( L-H \) transitions in tokamak plasmas has also been an active area of research. Starting with the work of Guzdar et al. [7], Rogers et al. [8], and Xu et al. [9] edge models have been developed for studying the cause of anomalous transport in the \( L \) phase and the subsequent improvement in confinement in the \( H \) phase. Rogers et al. [8], in their finite-beta simulations of drift-resistive ballooning modes using the reduced set of Braginskii equations, have identified a two-dimensional parameter-space involving \( \alpha_{\text{MHD}} \) and \( \alpha_D \) in which the edge plasma displays dramatic changes in transport. The parameter \( \alpha_{\text{MHD}} = \beta q^2 R/L_n \), with \( q \) the safety factor, \( R \) the major radius, \( L_n \) the density gradient scale length, and \( \beta \) the ratio of the plasma pressure to the magnetic pressure, is the standard MHD parameter identified for the onset of ideal ballooning modes. The parameter \( \alpha_D \) is the ratio of the diamagnetic drift frequency \( \rho_i c_s/L_0 L_n \) to the ideal growth rate \( \frac{c_i}{\sqrt{RL_n/2}} \). Here \( \rho_i \) is the ion Larmor radius with the electron temperature and \( c_s \) is the ion acoustic velocity. In the dimensionless equations, a characteristic scale length \( L_0 \) is obtained by an optimal ordering which renders the inertial term, the curvature interchange term, and the parallel electron dynamics term in the vorticity equation comparable [7,8]

\[
L_0 = 2\pi q R (\nu_{ei} \rho_i/2\Omega_e R)^{1/2} (2R/L_n)^{1/4}.
\]

Here \( \nu_{ei} \) is the electron ion collision frequency, \( \Omega_e \) is the electron cyclotron frequency, and the other parameters have been defined earlier. The use of the Braginskii equations is justified, since the edge temperatures are low (40–50 eV) and the safety factor is large \((q > 3)\), making the mean-free path of electrons smaller than the connection length. For ions, the lowest order finite Larmor radius effects are included. This approximation is valid since the modes have wavelengths longer than the ion gyroradii.

From their simulations, Rogers et al. showed that for finite \( \alpha_D \), as \( \alpha_{\text{MHD}} \) was increased, a transition from a poorly confined state \((L \) mode\) to a good confined state \((H \) mode\) occurred. Also as the diamagnetic parameter was increased, the turbulence changed from resistive ballooning in character to more drift-wave–like with an adiabatic electron response. Subsequent comparison with data from ASDEX [10], C-Mod [11], and DIII-D [12] indicated that the numerically computed boundary for the \( L-H \) transition was in reasonable agreement with observations. Guided by the simulation results of Rogers et al., Guzdar et al. [13] developed a theory for the generation of shear/zonal flow \((\phi_s, \) the scalar potential)
and field \((A_{||}, \text{the vector potential})\) by finite \(\beta\) drift waves. The zonal flows are \(E \times B\) flows at zero frequency driven by a modulational instability of finite \(\beta\) drift waves. These investigations indicated that the important dimensionless parameter that determines the growth rate of the zonal flow and field is \(\bar{\beta} = \beta(qR/L_\omega)^2/2\). As a function of \(\bar{\beta}\) the growth rate for zonal flows has a minimum at \(\bar{\beta}_c\) which is identified as the threshold point for the onset of \(L-H\) transition. For \(\bar{\beta} > \bar{\beta}_c\), the shear/zonal flow stabilization, which leads to suppression of fluctuations and steepening of the density profile, would cause \(\bar{\beta}\) to increase, thereby increasing the growth rate of shear/zonal flow. This runaway situation for shear flow generation, suppression of turbulence, and subsequent steepening of the density would trigger the transition to \(H\) mode. Furthermore, by maximizing the growth rate over the radial mode numbers and the mode amplitude of the drift wave, the threshold condition could be written as

\[
(k_p\rho_s)^2\bar{\beta}_c = 2.0,
\]

where \(k_p = 2\pi/L_\omega\). This condition can be recast in terms of \(\alpha_D\) and \(\alpha_{\text{MHD}}\) to compare with the curve obtained in the simulations of Rogers et al. However, here we render it in terms of measurable plasma parameters and compare the predictions of the threshold with various discharges in the DIII-D tokamak. The onset criterion for \(L-H\) transitions can be written as

\[
\Lambda = \frac{T_e(k\text{eV})[R(m)A_i]^{1/6}}{L_n(m)^{1/2}B_T(T)^{2/3}Z_{\text{eff}}^{1/3}},
\]

and the critical value \(\Lambda_c = 0.45\). Here \(A_i\) is the ion mass relative to hydrogen and \(Z_{\text{eff}}\) is the effective ion charge. The local parameters \(T_e\) and \(L_n\) are their values at the location of the steepest part of the density gradient in the edge region. Thus for a given discharge, the parameter \(\Theta = T_e/L_n\), which varies in time, has to reach a critical value \(\Theta_c\) just prior to the observed transition. The critical value is defined as

\[
\Theta_c = 0.45\frac{B_T(T)^{2/3}Z_{\text{eff}}^{1/3}}{[R(m)A_i]^{1/6}}.
\]

It is expressed in terms of known quantities for the discharge. We have assumed that \(Z_{\text{eff}} = 1\) for all the cases considered.

We compare this parameter with edge plasma data from the DIII-D tokamak. The density gradients are very steep in the edge region and the high resolution data on DIII-D allows for comparison with theoretically derived scalings which depend on plasma scale lengths. The edge electron parameters were obtained from the DIII-D multipoint, multitime Thomson scattering system. The local parameters used in the expressions above were derived from fits of a hyperbolic tangent density profile at the location of the steepest gradient [14]. We have used half the value of this computed density scale length. This is because in DIII-D the density and temperature determined by Thompson scattering are measured along a vertical chord of the laser. This would map into a steeper density on the outboard side of the discharge, where our local theory with the strong ballooning approximation is valid. In this work we have used a three-point back-average to remove the rapid point-to-point fluctuations in the data. This short time scale averaging was done to determine the average value of the plasma parameter required for computing the thresholds. Thus we were able to follow the time evolution of these parameters during the \(L-H\) transition, which occurred on time scales only marginally slower than the fluctuations in the data.

In Figs. 1(a)–1(d) the temporal evolution of the measured onset parameter \(\Theta\) is plotted (solid line) as a function of time for four different discharges: shot No. 84044 with toroidal field \(B_T = 2.11\) T, plasma current \(I_p = 1.33\) MA and line average density \(\bar{n} = 3.7 \times 10^{19}\) m\(^{-3}\); shot No. 78161 with \(B_T = 1.51\) T, \(I_p = 1.00\) MA, and \(\bar{n} = 3.7 \times 10^{19}\) m\(^{-3}\); shot No. 102025 with \(B_T = 2.07\) T, \(I_p = 1.57\) MA, and \(\bar{n} = 4.7 \times 10^{19}\) m\(^{-3}\); and shot No. 102015 with \(B_T = 2.12\) T, \(I_p = 1.08\) MA, and \(\bar{n} = 2.5–3.6 \times 10^{19}\) m\(^{-3}\). The horizontal dot-dashed line is the critical value \(\Theta_c\) given by Eq. (4). The vertical dashed line is the location of the first data point in the time history identified as an \(H\) mode as indicated by the \(D\)-alpha signal. For the first two discharges (Figs. 1(a) and 1(b)), with \(\nabla B\) towards the \(X\) point and sufficient beam heating, \(\Theta\) undergo a rapid increase above the critical value \(\Theta_c\) just prior to the observed transition. For shot No. 102025, \(\nabla B\) was reversed away from the \(X\) point at \(t = 3450\) ms. The beam power was

![FIG. 1. \(\Theta\) (solid line), the critical value \(\Theta_c\), (dot-dashed) versus time for DIII-D discharge (a) 84044, (b) 78161, (c) 102025, and (d) 102015. The vertical dashed line indicates onset of \(H\) mode.](image-url)
respectively. The line labeled as ‘‘?”’’ points (cyan circles), ‘‘?”’’ points (green circles). See text.

insufficient to cause the transition. The transition at \( t = 4100 \) ms was triggered by a pellet injection. Thus three very different discharges made a spontaneous transition into the \( H \) mode at the critical value \( \Theta_c \) given by Eq. (4). However, for shot No. 102015 with \( \nabla B \) drift away from the \( X \) point (Fig. 1(d)), the transition did not occur and \( \Theta \) stayed below the critical value \( \Theta_c \) even with a late-time pellet injection.

From Eq. (4) we define a critical temperature \( T_{ec} \), which depends on the instantaneous density scale length. In Fig. 2 we plot the observed electron temperature \( T_e \) versus the calculated \( T_{ec} \) for all points in the time-series for twenty-one discharges. Fourteen of these discharges were used recently to parameterize \( L \) mode and \( H \) mode plasma states using a pattern recognition algorithm [15]. The full set of discharges had plasma current scans from 1–2 MA, line average density scans from \((1–4) \times 10^{19} \) m\(^{-3}\) and toroidal field variation from 1.1–2.18 T. The red points are all the points in these discharges which were clearly identified to be in the \( H \) mode, while the blue ones are the \( L \) mode points. The small subset of points labeled as ‘‘?”’’ and ‘‘?”’’; which could not be identified unambiguously as \( L \) or \( H \), are the cyan and green circles, respectively. The line \( T_{ec} = T_{ec} \) is the solid black line while the solid magenta line is \( T_{ec} = 1.15T_{ec} \). There is a clear separation between the \( L \) mode and \( H \) mode data points almost along the diagonal line \( T_{ec} = T_{ec} \). The line \( T_{ec} = 1.5T_{ec} \) is a better fit to the separation of the \( L-H \) data since it minimizes the number of \( H \) mode points below it and the number of \( L \) mode points above it.

An alternate, yet instructive, way of displaying the data is by computing the value of \( \Lambda \) defined by Eq. (3). In Fig. 3 is shown the histogram of the parameter \( \Lambda \) for the \( L \) mode points (white) and the \( H \) mode points (black) for the same data set as in Fig. 2. Here we have not included the questionable (‘‘?”’’ and ‘‘?”’’) data points. In the vicinity of the critical point \( \Lambda_c = 0.45 \) the number of data points is at a minimum. In fact the real minimum occurs at \( 1.15\Lambda_c = 0.517 \) as is to be expected from the best-fit curve \( T_{ec} = 1.15T_{ec} \) in Fig. 2. This behavior at the critical point occurs because the \( L \) mode discharge becomes unstable at that point and subsequently evolves to the new stable \( H \) mode. Incidentally, if \( Z_{eff} \) was taken to be 1.5, the multiplicative constant of (1.15) for the best-fit could be accounted for.

To further investigate the role of \( \nabla B \), we use the data from the two discharges studied by Carlstrom et al. [4]. For shot No. 96338 the \( \nabla B \) drift was towards the \( X \) point. For shot No. 96348 it was reversed while all other plasma parameters were the same. In Fig. 4 we plot (dashed) the parameter \( \Theta \) versus time for shot No. 96338. Also plotted is the critical value \( \Theta_c \) (dot-dashed) of this parameter for both these discharges. The first indication by the D-alpha signal that this discharge has evolved into an \( H \) mode occurs at \( t = 3.068 \) s (vertical dotted line). Here again \( \Theta \)
crossed the critical value \( \Theta_c \) in the vicinity of the drop in the 
\( D \)-alpha signal. Also the quality of the \( H \) mode was
low and the \( D \)-alpha signal showed strong oscillations.
This is reflected in the large oscillations in the parameter \( \Theta \). After that,
since the neutral beam power was reduced below threshold, the discharge slipped back into an \( L \)
mode at \( t = 3.55 \) s indicated by the vertical dotted line.
We observe that \( \Theta \) decreases below the critical value just
prior to the indicated transition. What is interesting to
note is that the \( H-L \) transition occurs at the same value of
the parameter \( \Theta_c \). Unlike the power, there is no hysteresis
in this onset parameter for the \( L-H \) and \( H-L \) transitions.
The solid line trace is \( \Theta \) for shot No. 96348 with the \( \nabla B \)
field away from the X point. The transition to the \( H \)
mode (at much higher power) as indicated by the \( D \)-alpha signal
(vertical dotted line) occurs at \( t = 3.831 \) s. The transition
occurs when the onset parameter exceeds the same critical
value \( \Theta_c \). For these two discharges, an examination of the
time history of the electron temperature and the
density scale length in the \( L \) mode phase shows that the
temperatures are very similar, while the density scale
length is larger for shot No. 96348 almost by a factor of
1.5 to 2. This could be due to a change in the anomalous
transport, a change in the particle fueling, as discussed by
Boedo et al. [16], or a combination of these two effects.
Thus for both discharges, the transition occurs when the
parameter of choice \( \Theta \) exceeds the same critical value.
This indicates that the underlying physics for the onset
is the same for both these cases.

Finally, we investigate two shots studied by Gohil et al.
[5], in which the transition to the \( H \) mode was induced by
pellet injection. In shot No. 96559 the pellet was injected at
\( t = 3624.7 \) ms, while for shot No. 100162 it was
injected at \( t = 4257.9 \) ms. The time evolution of \( \Theta \) for
these two shots are the solid lines in Figs. 5(a) and 5(b),
respectively. The critical value \( \Theta_c \) is the dot-dashed line
in each of these plots. The vertical dashed line is the last
point in the time-series identified as an \( L \) mode, and the
vertical dotted line is the first point in the time history
indicated as an \( H \) mode. Once again there is clear evidence
of the occurrence of the transition when the onset
parameter \( \Theta \) crosses the critical value \( \Theta_c \), and this occurs
in the time interval within one data point of the last \( L \)
and first \( H \) mode points. In this case the injection of the pellet
causes a sharp reduction in the density scale length, and
even though the electron temperature drops, the critical
value of \( \Theta \) is exceeded to induce the transition.

Thus based on our theory of shear flow and field
generation by finite-beta drift waves in tokamak edge
plasmas, we have identified a critical trigger parameter
\( \Lambda_r \) or \( \Theta_c \) defined by Eqs. (3) and (4), respectively, for the
onset of \( L-H \) transition in tokamak plasmas. The transition
occurs at the same critical value of the parameter for
discharges with oppositely directed \( \nabla B \) drifts. The difference
in either the anomalous turbulent transport, particle
fueling, or a combination, causes the local density scale
length to be different for the two cases. Finally, the
pellet-induced \( H \) modes, which occur for power levels
significantly below the conventional values, also do so for
the same value of the onset parameter. Thus seemingly
different transition mechanisms are unified by identifying
the relevant trigger parameter for all such transitions.

This work was supported by the U.S. Department of
Energy under Grant No. DE-FG02-93ER54197 at UMD,
Contract No. DE-AC03-99ER54463, and Grant No. DE-
FG02-93ER54197 at GA.

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