Sensor Platform Motion Control

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ONR MURI Review - Exploiting Novel Dynamics for Nonlinear Sensor Networks

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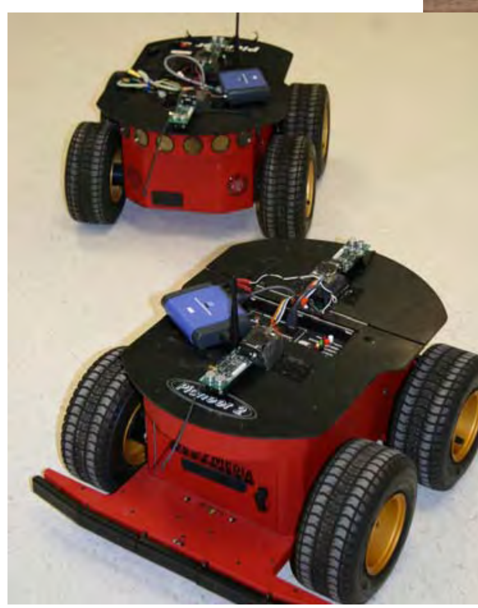
Motivation

Synthesize and understand feedback laws for motion pattern generation in networks of sensor platforms.

Dragoneye UAV



Outline

- Geometry for Cooperative Control

  Curvature, patterns, and feedback

- Pursuit Laws and Cooperative control

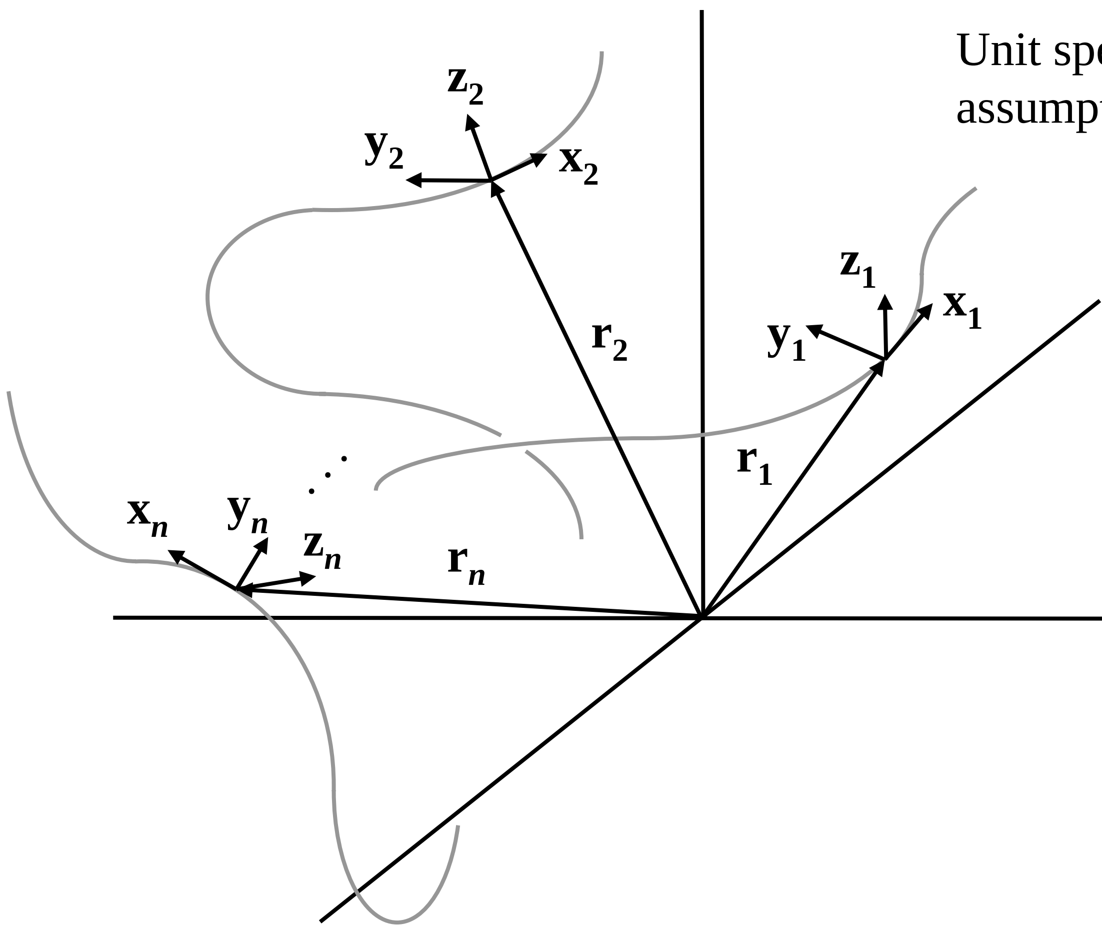
  Pursuit manifolds, accessibility and cohesion

- Mutual Pursuit and a Hamiltonian System

  Symmetry and reduction

1. Geometry for Cooperative Control
(of sensor platforms)

Platforms as Interacting Particles in 3D



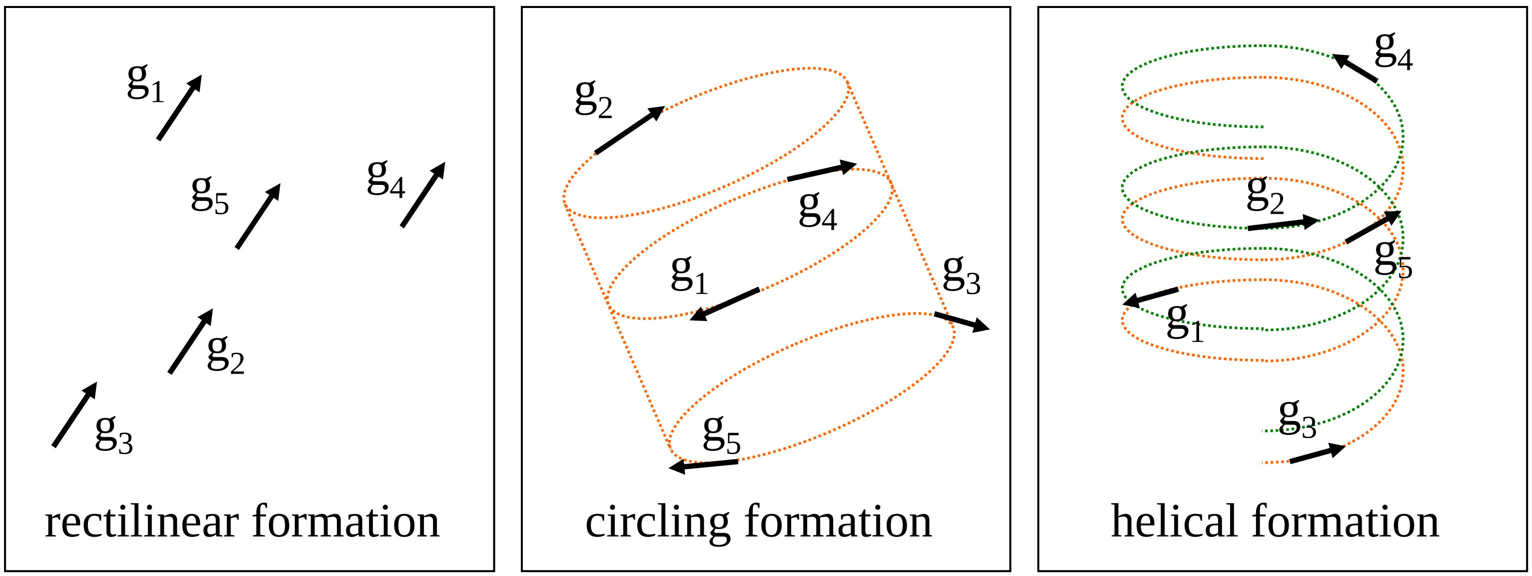
Unit speed assumption

$$
\begin{aligned}
\dot{\mathbf{r}}_1 &= \mathbf{x}_1 \\
\dot{\mathbf{x}}_1 &= \mathbf{y}_1 u_1 + \mathbf{z}_1 v_1 \\
\dot{\mathbf{y}}_1 &= -\mathbf{x}_1 u_1 \\
\dot{\mathbf{z}}_1 &= -\mathbf{x}_1 v_1 \\
\dot{\mathbf{r}}_2 &= \mathbf{x}_2 \\
\dot{\mathbf{x}}_2 &= \mathbf{y}_2 u_2 + \mathbf{z}_2 v_2 \\
\dot{\mathbf{y}}_2 &= -\mathbf{x}_2 u_2 \\
\dot{\mathbf{z}}_2 &= -\mathbf{x}_2 v_2 \\
&\vdots \\
\dot{\mathbf{r}}_n &= \mathbf{x}_n \\
\dot{\mathbf{x}}_n &= \mathbf{y}_n u_n + \mathbf{z}_n v_n \\
\dot{\mathbf{y}}_n &= -\mathbf{x}_n u_n \\
\dot{\mathbf{z}}_n &= -\mathbf{x}_n v_n
\end{aligned}
$$

The **natural curvatures** are controls. In general, time-dependent speeds are dictated by propulsive/lift mechanisms. Here we fix speed =1 for simplicity.

3D Equilibrium Shapes

- Control laws are assumed to be invariant under rigid motions.
- Shape variables capture relative distances and angles between particles.
- Shape equilibria correspond to steady-state formations.



rectilinear formation | circling formation | helical formation

Other spatial patterns?

Interaction (Feedback) Law for 3D

Natural curvatures for particle #1:

$$u_1 = -\eta\left(\frac{\mathbf{r}}{|\mathbf{r}|}\cdot\mathbf{x}_1\right)\left(\frac{\mathbf{r}}{|\mathbf{r}|}\cdot\mathbf{y}_1\right) + f(|\mathbf{r}|)\left(\frac{\mathbf{r}}{|\mathbf{r}|}\cdot\mathbf{y}_1\right) + \mu\mathbf{x}_2\cdot\mathbf{y}_1$$

$$v_1 = -\eta\left(\frac{\mathbf{r}}{|\mathbf{r}|}\cdot\mathbf{x}_1\right)\left(\frac{\mathbf{r}}{|\mathbf{r}|}\cdot\mathbf{z}_1\right) + f(|\mathbf{r}|)\left(\frac{\mathbf{r}}{|\mathbf{r}|}\cdot\mathbf{z}_1\right) + \mu\mathbf{x}_2\cdot\mathbf{z}_1$$

Natural curvatures for particle #2:

$$u_2 = -\eta\left(\frac{\mathbf{r}}{|\mathbf{r}|}\cdot\mathbf{x}_2\right)\left(\frac{\mathbf{r}}{|\mathbf{r}|}\cdot\mathbf{y}_2\right) - f(|\mathbf{r}|)\left(\frac{\mathbf{r}}{|\mathbf{r}|}\cdot\mathbf{y}_2\right) + \mu\mathbf{x}_1\cdot\mathbf{y}_2$$

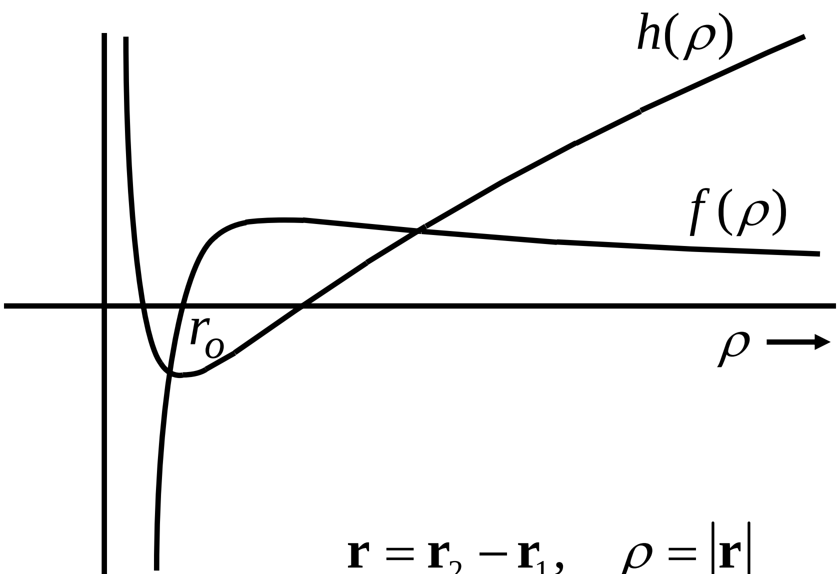
$$v_2 = \underbrace{-\eta\left(\frac{\mathbf{r}}{|\mathbf{r}|}\cdot\mathbf{x}_2\right)\left(\frac{\mathbf{r}}{|\mathbf{r}|}\cdot\mathbf{z}_2\right)}_{\text{Baseline alignment}} \underbrace{- f(|\mathbf{r}|)\left(\frac{\mathbf{r}}{|\mathbf{r}|}\cdot\mathbf{z}_2\right)}_{\text{Collision avoidance}} \underbrace{+ \mu\mathbf{x}_1\cdot\mathbf{z}_2}_{\text{Heading alignment}}$$

Lyapunov Function

$$V = \underbrace{-\ln\left(1+\mathbf{x}_2\cdot\mathbf{x}_1\right)}_{\text{Penalize heading-direction misalignment}} + \underbrace{h(|\,\mathbf{r}_2-\mathbf{r}_1\,|)}_{\text{Penalize inter-particle distances which are too large or small}}$$

Penalize heading-direction misalignment

Penalize inter-particle distances which are too large or small



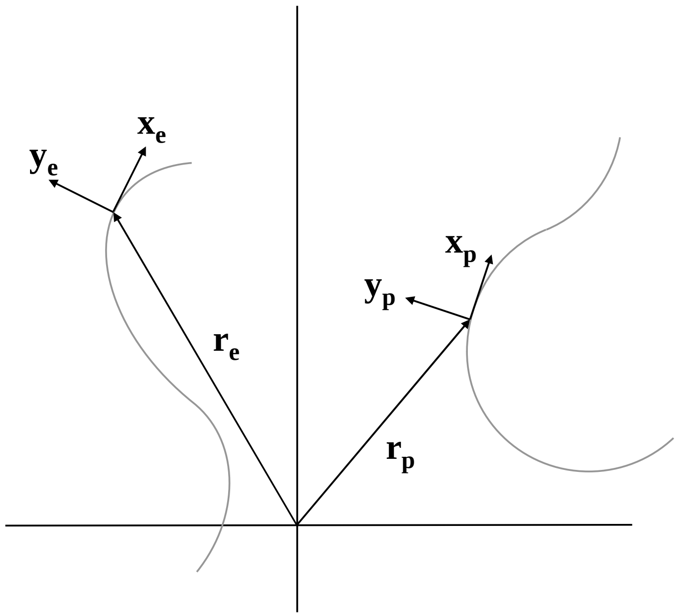
$$\mathbf{r} = \mathbf{r}_2 - \mathbf{r}_1, \quad \rho = |\mathbf{r}|$$

- V depends only on shape variables.
- Idea: show that for suitable choice of control law, $dV/dt \leqslant 0$.
- Justh-PSK (2003-2006).

2. Pursuit Laws and Cooperative Control

Modeling Pursuit in 2D

Here we specialize the models of section 1 to the plane. The speed ratio is given by ν, assumed constant and less than 1 in what follows.

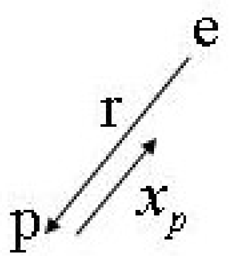


$$\begin{aligned} \dot{\boldsymbol{r}}_p &= \boldsymbol{x}_p \\ \dot{\boldsymbol{x}}_p &= \boldsymbol{y}_p u_p \\ \dot{\boldsymbol{y}}_p &= -\boldsymbol{x}_p u_p \end{aligned} \qquad (1)$$

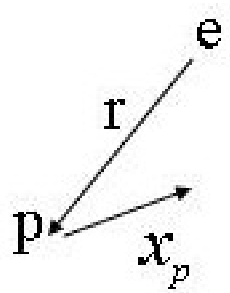
$$\begin{aligned} \dot{\boldsymbol{r}}_e &= \nu \boldsymbol{x}_e \\ \dot{\boldsymbol{x}}_e &= \nu \boldsymbol{y}_e u_e \\ \dot{\boldsymbol{y}}_e &= -\nu \boldsymbol{x}_e u_e \end{aligned} \qquad (2)$$

$r = r_p - r_e$ *is the baseline*

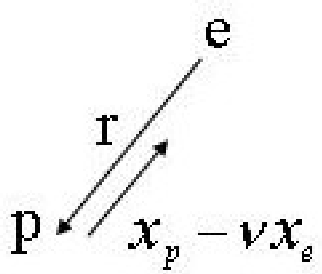
Three Pursuit Manifolds



Classical Pursuit



Constant Bearing



Motion Camouflage

A Distance Function

Let $$\Gamma \triangleq \left(\frac{d}{dt}|r|\right) \Big/ \left|\frac{dr}{dt}\right| \qquad (3)$$

$$= \frac{r}{|r|} \cdot \frac{\dot{r}}{|\dot{r}|}$$

well-defined on non-collision states.

Observe $-1 \le \Gamma \le 1 \quad , \quad 1-\nu \le |\dot{r}| \le 1+\nu$

and $$1-\Gamma^2 = \frac{|w|^2}{|\dot{r}|^2}$$

Driving Γ to ± 1 corresponds to reducing distance to motion camouflage manifold

As $\Gamma \to +1$ baseline lengthening

As $\Gamma \to -1$ baseline shortening

Finding a Feedback Law

Under the hypothesis that $|u_e|$ is bounded, one can justify a simple control law.

$$u_p = -\mu\left(\frac{r}{|r|}.\dot{r}^{\perp}\right) \qquad (4)$$

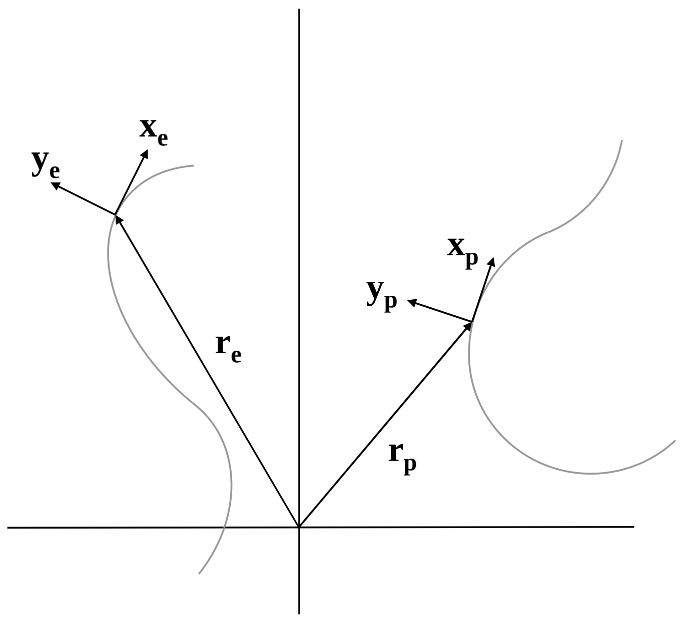
Definition

For the pursuit-evader system (1), (2) with Γ defined by (3), we say that motion camouflage is <u>accessible in finite time</u> if for any $\varepsilon > 0$, there exists a time $t_1 > 0$ such that

$$\Gamma(t_1) \leq -1 + \varepsilon$$

E.W. Justh and P. S. Krishnaprasad (2006), *Proc. R. Soc. A*, 462:3629-3643.
P.V. Reddy, E.W. Justh and P. S. Krishnaprasad (2006), *45th IEEE CDC*, pp.3327-3332.

Proposed Cohesion Law from Pursuit



$$\begin{aligned} \dot{\boldsymbol{r}}_p &= \boldsymbol{x}_p \\ \dot{\boldsymbol{x}}_p &= \boldsymbol{y}_p u_p \\ \dot{\boldsymbol{y}}_p &= -\boldsymbol{x}_p u_p \end{aligned} \qquad (1)$$

$r = r_p - r_e$ *is the baseline*

$$\begin{aligned} u_p &= -\mu\left(\frac{r}{|r|}.\dot{r}^{\perp}\right) = u \\ u_e &= -\frac{\mu}{\nu}\left(\frac{r}{|r|}.\dot{r}^{\perp}\right) \end{aligned}$$

$$\begin{aligned} \dot{\boldsymbol{r}}_e &= v\boldsymbol{x}_e \\ \dot{\boldsymbol{x}}_e &= v\boldsymbol{y}_e u_e \\ \dot{\boldsymbol{y}}_e &= -v\boldsymbol{x}_e u_e \end{aligned} \qquad (2)$$

3. Mutual Pursuit and a Hamiltonian System

Dynamics in Mutual Pursuit

$$\dot{r} = g$$

$$\dot{g} = uh$$

$$\dot{h} = -ug$$

Here $\quad g = x_p - \nu x_e \quad$ *and* $\quad h = y_p - \nu y_e.$

Let $\quad \lambda = \frac{r}{|r|} \bullet h \quad \gamma = \frac{r}{|r|} \bullet g \quad$ *and* $\quad \rho = |r|.$

Note $\quad u = -\mu\lambda.$

Symmetry & Reduction

$$\dot{\rho} = \gamma$$

$$\dot{\gamma} = (\frac{1}{\rho} - \mu)(\delta^2 - \gamma^2)$$

Here we have used the conservation law $\gamma^2 + \lambda^2 \equiv \delta^2$.

Discrete Symmetry

The system is reversible under the involution

$$(\rho, \gamma) \mapsto (\rho, -\gamma)$$

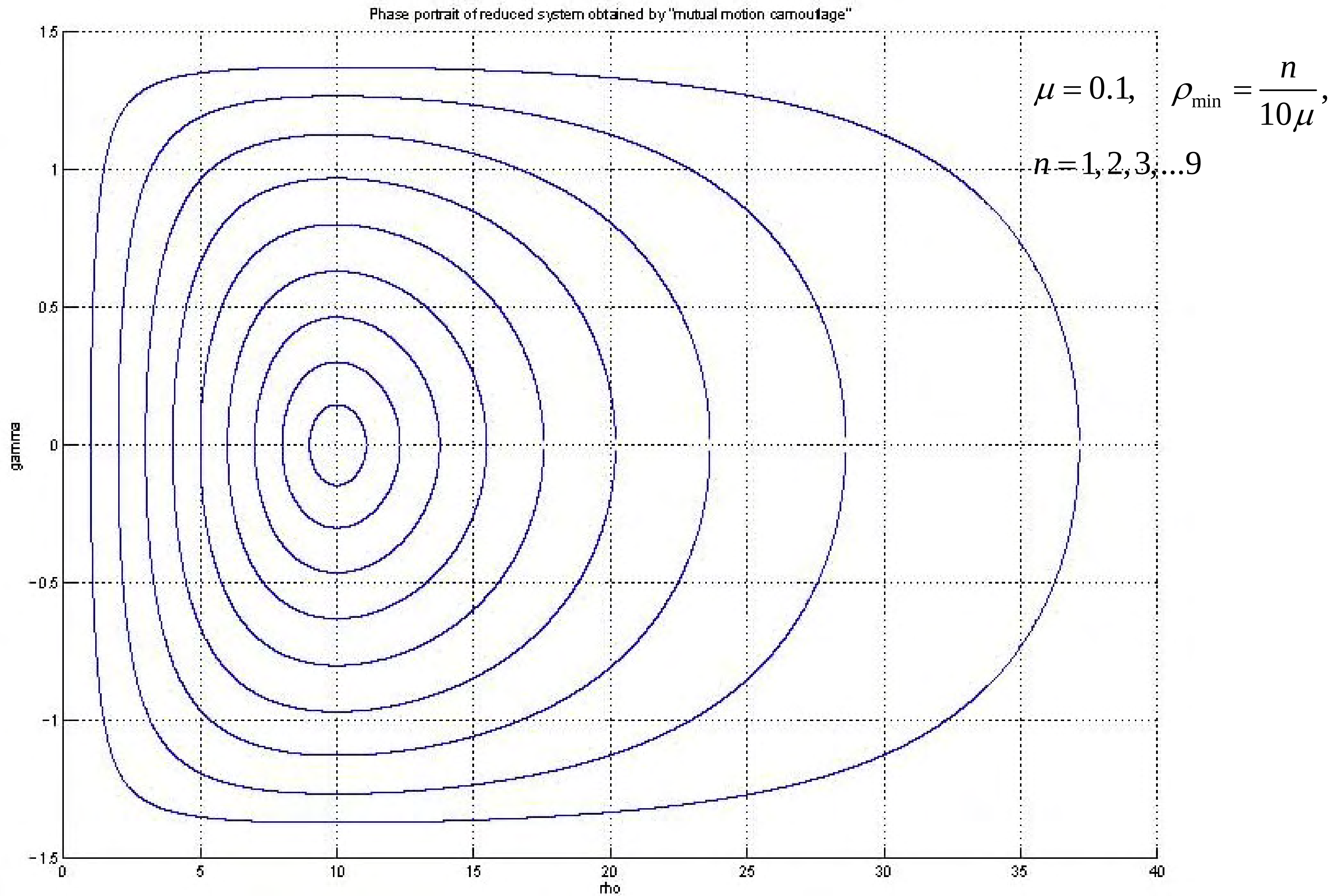
In fact, Birkhoff's theorem applies, and with the exception of two collision/escape manifolds all orbits are periodic. The "energy integral"

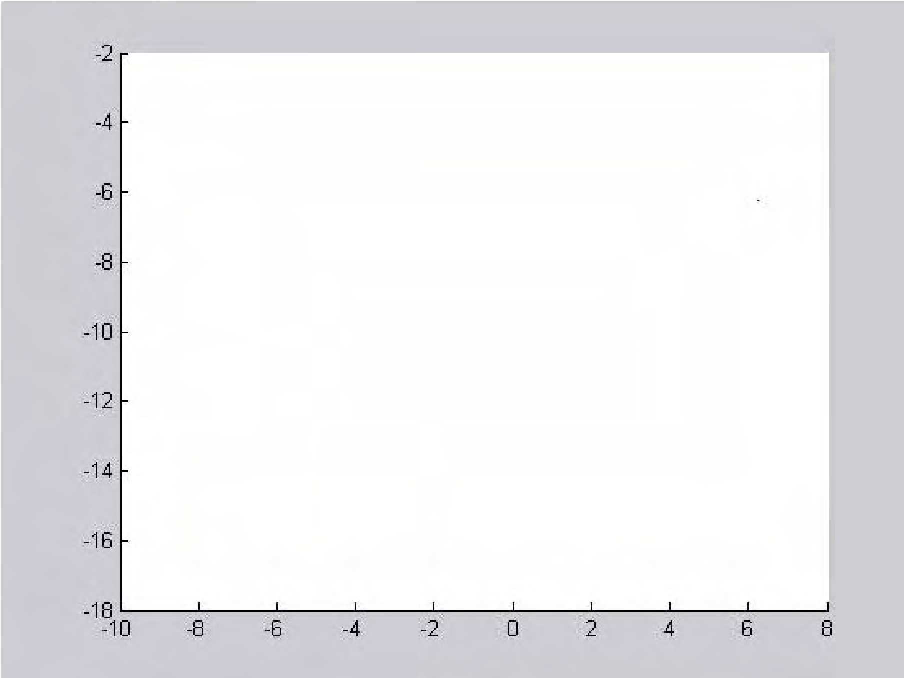
$$E(\rho, \gamma) = \rho^2(\delta^2 - \gamma^2)\exp(-2\mu\rho)$$

implies a Poisson bracket

$$\{\rho, \gamma\} = -\frac{\exp(2\mu\rho)}{2\rho^2}$$

Phase portrait of reduced system

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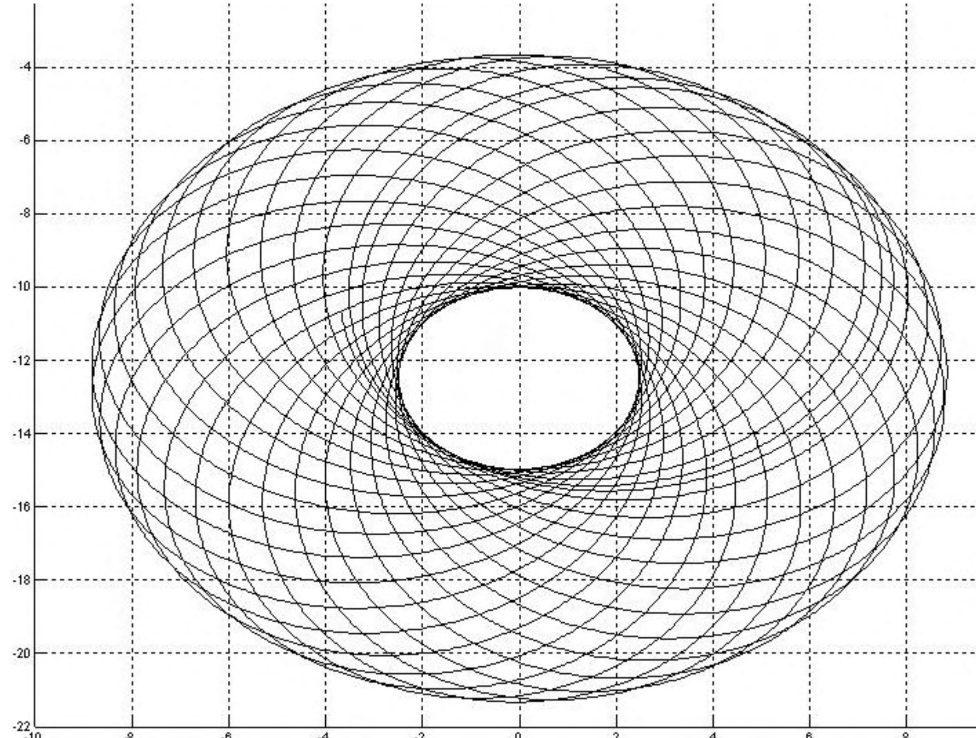
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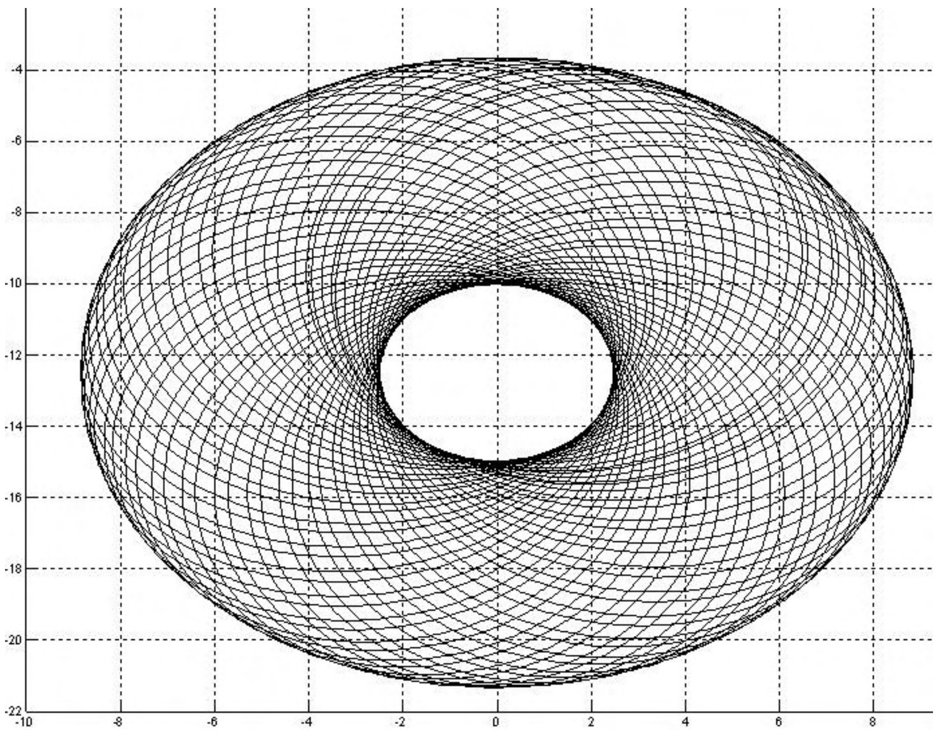
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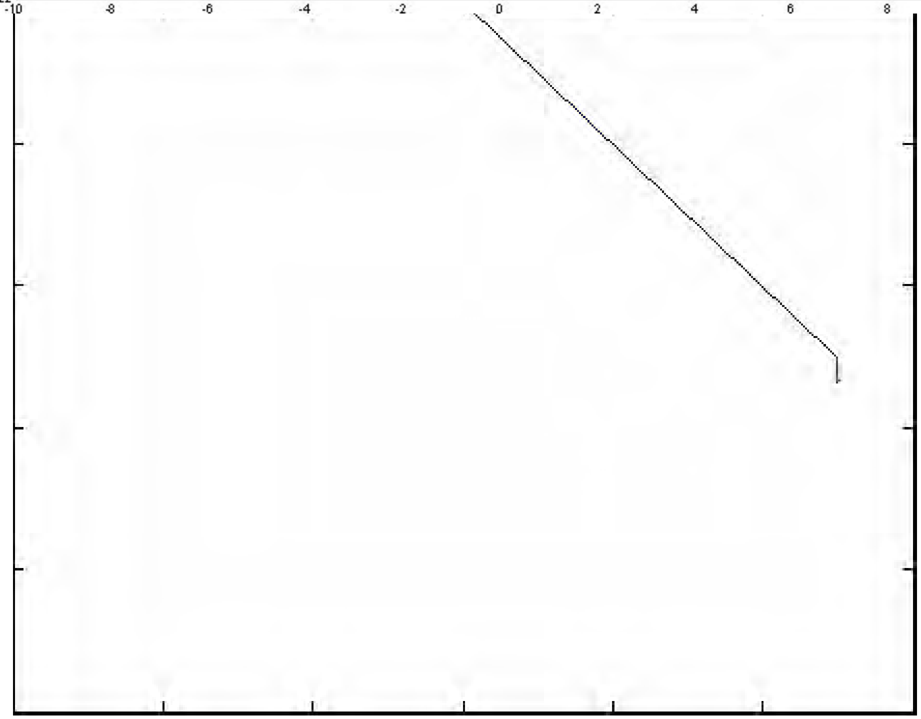

Ongoing Work, Collaborations, and Support

- Dynamics of center of mass, dissipation, and stabilizing specific periodic orbits (Matteo Mischiati, G3) – see poster
- Many-body network coupling (Dr. Eric Justh, NRL), and relationship to earlier feedback laws for cohesion
- Alternative mutual pursuit mechanisms

  (Kevin Galloway, G5) – see poster

See also poster of Dr. Arash Komaee on stochastic control over free space optical links

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