Supersymmetric Quantum Mechanics and Reflectionless Potentials

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Abstract

In the following presentation I will review some of the consequences of supersymmetric (SUSY) quantum mechanics (QM). This includes a discussion of supersymmetric Hamiltonian formalism, as well as discussion of scattering properties and how symmetric potential functions can share these properties; results from using this formulism with the particle in a box and quantum oscillator examples were studied. Examples of super potential functions that correspond with non-trivial reflectionless potentials are given, in addition to a general discussion of supersymmetry and topological boundary modes.

General Hamiltonian Formalism

...for some Hamiltonian (H_1) let...

$$H_1\psi_0(x) = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\psi_0(x) + V_1(x)\psi_0(x) = 0$$

...where...

$$\frac{\hbar^2}{2m}\frac{\psi_0''(x)}{\psi_0(x)} = V_1(x) \qquad H_1 = A^\dagger A$$

therefore

$$H_2 : \left(\frac{\;^2}{2\,m} \frac{d^2}{dx^2} \; V_2(x) \right)$$

$$A = \frac{\hbar}{\sqrt{2m}}\frac{d}{dx} + W(x)$$

where $W(x)$ is the Super Potential

let ...

$$: \frac{\;}{\sqrt{2\,m}}$$

... now since

$$A : \left(\frac{d}{dx} \; W(x) \right)$$

we know

$$A^\dagger : \left(\frac{d}{dx} \; W(x) \right)$$

so

$$H_1 : \;^2 \frac{d^2}{dx^2} \; V_1(x) : \left(\frac{d}{dx} \; W(x) \right) \left(\frac{d}{dx} \; W(x) \right)$$

after some simplification ...

$$V_1 : W^2 \; \frac{dW}{dx}$$

Our first Hamiltonian's super partner

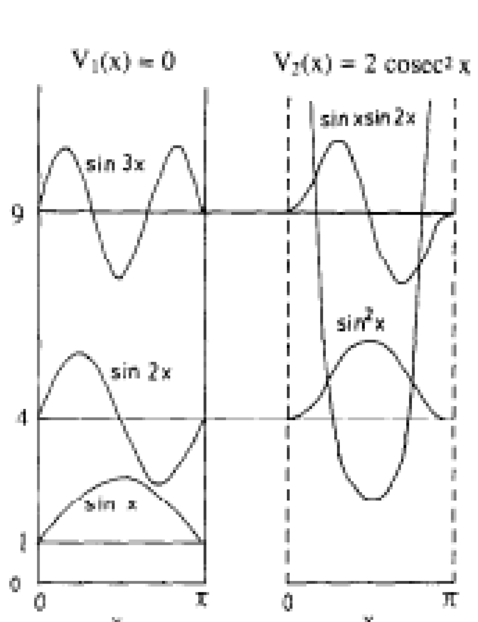
$$H_2 = AA^\dagger$$

the Infinite square well example

$$V_1 = W^2 - \frac{\hbar}{\sqrt{2m}}W'$$

$$V_2 = W^2 + \frac{\hbar}{\sqrt{2m}}W'$$

all solutions are the same except the ground states for each potential well.



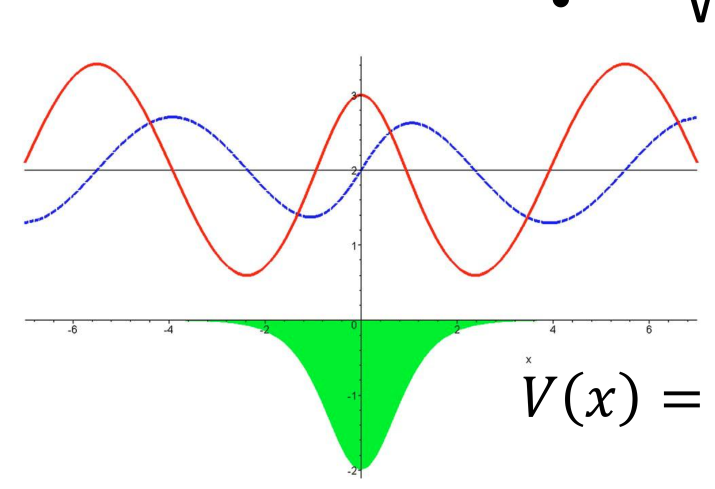
from
Supersymmetry and quantum mechanics; by Freed Cooper, Avinash Khare, and Uday Sukhatme

Introduction

The realization of supersymmetry in Quantum Mechanics has allowed for the application of the more subtle and abstract aspects of quantum mechanics in several different areas of science and engineering. This summer I had the fortune of working under University of Maryland Professor, Dr. Victor Galitski; during this time I was challenged by my mentor to begin learning the fundamentals of Supersymmetry and Quantum mechanics. Despite having no prior exposure to modern algebra or quantum mechanics I happily (and somewhat recklessly) accepted his challenge . In the following presentation I will review some of the basic consequences of supersymmetric quantum mechanics. In addition, I will give a general discussion of more cutting edge applications of supersymmetric quantum mechanics.

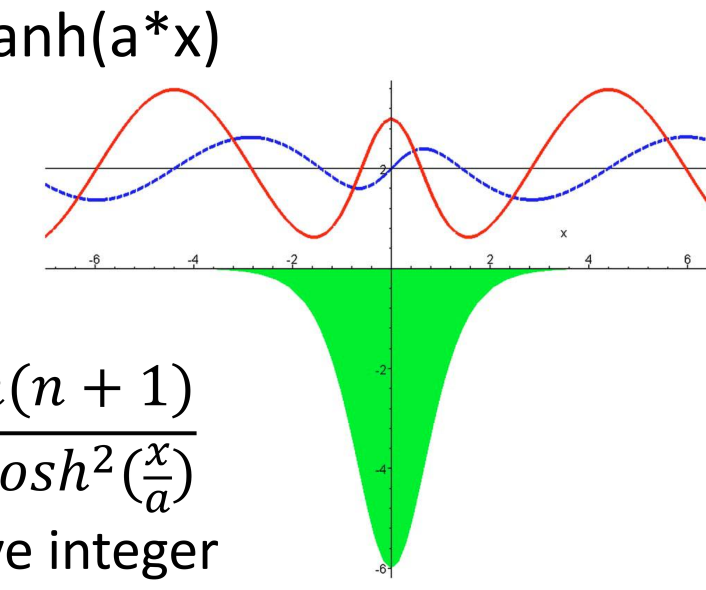
Reflectionless Potentials

- Even constant potential functions can have supersymmetric partner's
- In some cases this leads to potential barriers allowing complete transmission of matter waves
- These potentials are often classified by their super potential function
  - One class of reflectionless potential is :
    - W(x)= A*tanh(a*x)



$$V(x) = -\frac{\hbar^2}{2ma^2}\frac{n(n+1)}{cosh^2(\frac{x}{a})}$$

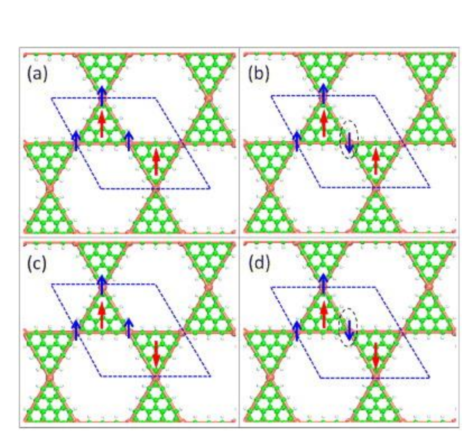
Where n is a positive integer



from:
Reflectionless eigenstates of the sech² potential ; by: John Lekner

More cutting edge applications

It has been recently found that certain correlations between the supersymmetric algebra of a classical lattice structure and a quantum lattice exist. C. Kane and T Lubensky showed this in their paper on "Topological Boundary Modes in isostatic Lattices". In their paper they were able to establish a connection between the topological mechanical modes and the topological band theory of electronics systems, thus allowing them to "predict new topological bulk mechanical modes with distinct boundary modes".



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