ABSTRACT

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Background and theory for the geodynamo is presented, along with a review of current research in this area being conducted by D. P. Lathrop in his laboratory at the University of Maryland. An experimental apparatus which the author helped design and fabricate is described, and results are outlined.

DYNAMO THEORY AND EXPERIMENT

By

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Chapter 1: Background

Ferromagnetism in rocks ("lodestones") and its effect on paramagnets like aluminum (or sodium) has been a familiar phenomenon for thousands of years, as has the Earth's intrinsic magnetic field. Naturally, early observers concluded the two phenomena were connected. However, as temperature estimates of the Earth's interior improved it became apparent that the mantle and core temperatures, upwards of 5000 Kelvin[1], lie well above the Curie temperature of all known materials (*e.g.*, ~1000 K for iron.) Hence ferromagnetism may be ruled out as the source of the Earth's global magnetic field, usually referred to as the 'main field.'

It was Joseph Larmor who first[2] hit upon the currently accepted explanation when, in 1919, he proposed that convection currents of conducting material in the Earth's core are responsible for the slowly varying magnetic dipole we observe at and above Earth's surface. He initially proposed this as an explanation for the Sun's magnetic field, but eventually it was extended to the Earth as well.

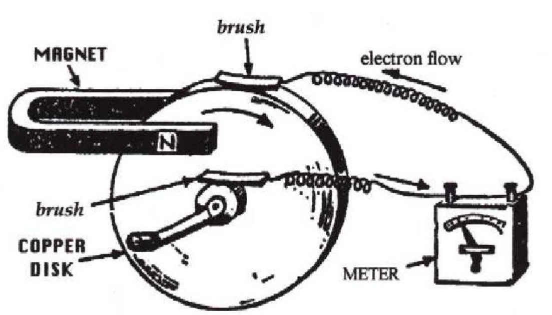
As it unfolded, geodynamism proved to be a quantitatively challenging area of research. In order to positively confirm Larmor's hypothesis, a successful dynamo would have to be realized either as theoretical model or as practical experiment. In this paper we begin with an overview of the subject, followed by a more detailed look at a series of experiments conducted by Daniel P. Lathrop in the Non-Linear

[1] D. Alfè, *NATURE*, vol. 401, 30 Sept 1999
[2] Sir Joseph Larmor, "Possible rotational origin of magnetic fields of Sun and Earth", *Elect. Rev. 85*, 512, 26 Sept 1919b.

Dynamics laboratory at the University of Maryland, College Park, under the auspices of the Institute for Research in Electronics and Applied Physics.

Chapter 2: Faraday's Disk Dynamo

In order to understand the mechanism of magnetic field generation in the Earth's outer core, we start by considering a device known as a disk dynamo. This device was conceived by Michael Faraday and exploits the principle that bears his name. Essentially it converts mechanical into electric power with the aid of large, stationary magnets. It involves only one moving part, a conducting copper plate that rotates through a fixed magnetic field. Just as a changing magnetic field will, by Faraday's principle, induce an electric current in a conductor, so a conductor moving differentially through a fixed magnetic field leads to a current. The direction of that current produces a magnetic field opposing the change, as required by Lenz' Law. Here is a sketch of a dynamo apparatus:



Faraday's disk dynamo - for producing continuous (pure) dc voltage. This was the world's first electrical generator.

Figure 1a. From Archer Enterprises' StarDrive Engineering website, http://www.stardrivedevice.com/faradisk.html

Let's look at a simplified model of the disk dynamo (Figure 1b). The axle and disk rotate in the positive direction ω as seen from above. Motion in this direction is maintained by a torque τ on the axle, not shown.

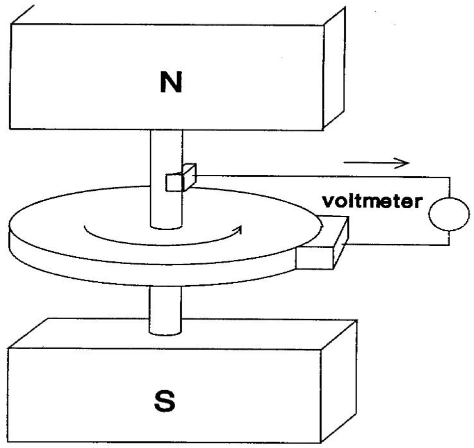


Figure 1b. Schematic drawing of a disk dynamo.

A hypothetically uniform magnetic field B points downward from North to South. The disk itself must be fabricated of a good conductor, usually copper. Inside the copper there is a high density of charge carriers. These are the conduction electrons which carry a negative charge.

As the disk spins, the conduction electrons are carried along in the azimuthal direction with velocity vector $v = r\omega = r\dot{\varphi}\hat{\varphi}$, where r is an electron's radial coordinate. They experience a Lorentz force

$$F = q\breve{v} \times B = (-er\omega B)\hat{\varphi} \times \hat{z} = er\omega B\hat{r}.$$

Thus the electrons in this schematic diagram experience a radial push outward, thereby accounting for the flow of current radially inward as displayed. Comparing this picture with Faraday's dynamo depicted in figure 1b, we see that the direction of rotation is reversed relative to the applied field, thereby reversing the flow of current to the radially outward direction.

Needless to say this current provides power and therefore power, both mechanical and magnetic, must be put into the system in order to maintain its motion. This is the reason we must apply a torque τ on the axle, otherwise the dynamo will run down due to Ohmic heating. But suppose we wind the current-carrying wire around the axle in such a way that the resulting magnetic field reinforces the external field already present? A stronger field should increase the Lorentz force and thus increase the current, which in turn creates an even stronger field, which increases the current yet again, and so forth.

This positive-feedback loop is known as the "dynamo effect." It has been shown[3] that under the right conditions the induced field will completely supplant the external field, so that only mechanical energy need be supplied. When stable equilibrium between the mechanical input and the induced electric current is reached, we have a self-sustaining magnetic dynamo.

This was the explanation put forth by Larmor for planetary magnetism. However, at about the same time his theory was published, evidence began to accrue for reversals in the Earth's field. This evidence came in the form of the paleomagnetic record, particularly from lava flows that provide a series of snapshots of the main field over the last several million years. The simple disk dynamo, however, is dynamically stable, and can only reverse its field if we reverse the direction of rotation. How then do we account for the oddly-spaced reversals in the

[3] T. Rikitake, "Electromagnetism and the Earth's Interior," Elsevier, Amsterdam, 1966.

Earth's paleomagnetic record? We will investigate this question further in the next section.

Chapter 3: The Earth's Magnetic Field

The Earth's magnetic field appears on its surface to be roughly dipole, with North and South currently aligned about 11° away from the axis of rotation. The most recent dipole reversal is estimated at 780,000 years ago, but reversals occur on average every 200,000 years or so[4]. As noted in the last section, the single-disk dynamo is innately incapable of such field reversals. On the other hand, the double-disk dynamo, in which the field of one dynamo is applied to a second dynamo and *vice-versa*, can exhibit sudden reversals and chaotically unstable field behavior. However, such a model is highly unrealistic in the context of Earth's interior.

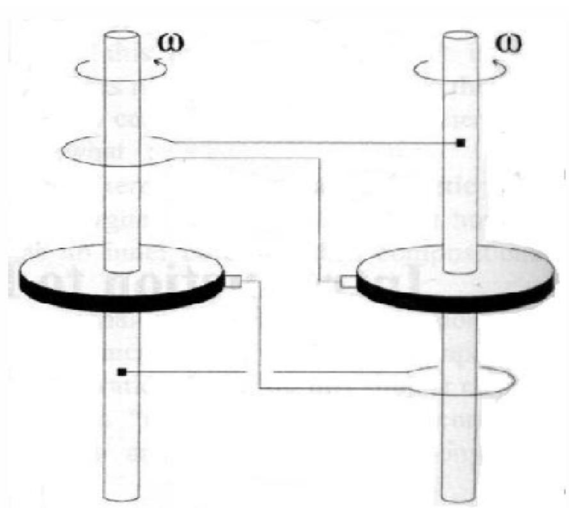


Figure 2. A double disk dynamo, taken from R. Merrill.

[4] Roberts and Glatzmaier, "Geodynamo Theory and Simulations," *Review of Modern Physics*, vol. 72, no. 4, Oct 2000.

Before offering a plausible explanation, let us examine some of the characteristics of the Earth's interior, which may be conveniently divided into four concentric shells:

1) An outer shell of negligible thickness called the crust.
2) The mantle, a relatively stable solid capable of plastic flow; it extends about halfway down.
3) The liquid outer core, composed of molten iron (90%-95%) and some other trace elements, including nickel and sulfur.
4) The solid inner core, similar in composition to the outer core, but at greater pressure.

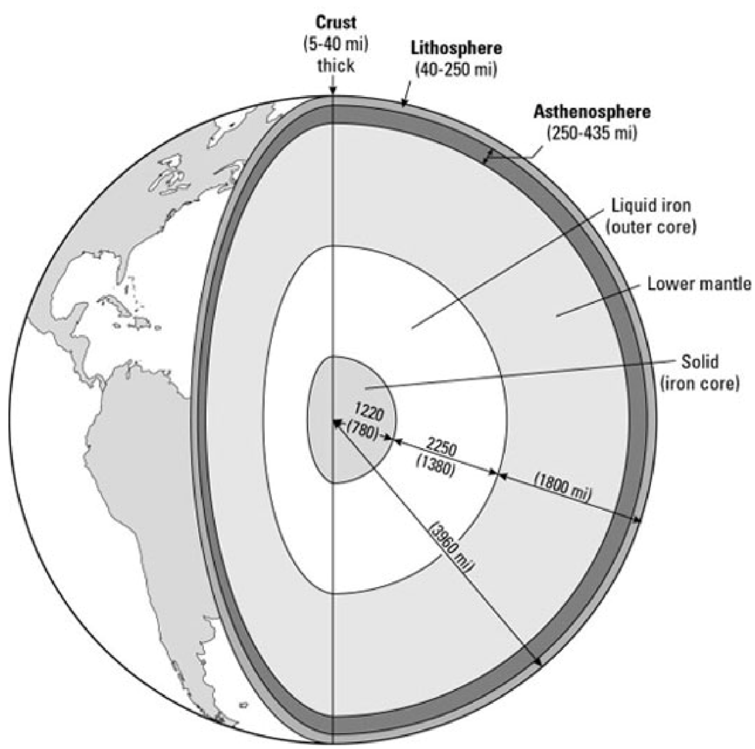


Figure 3. Cutaway view of the Earth's interior.
Adapted from Geography for Dummies, © 2002 Wiley Publishing.

The outer shell certainly contributes to our planetary field through ferromagnetism, but nowhere near enough to account for the main field. The mantle's composition and temperature preclude significant ferromagnetism, and the dynamics of the mantle are too slow to be electrodynamically significant.

This leaves the core, both parts of which, at around 5000 K, are well above the Curie temperature of iron from which they are predominantly formed. The inner core undergoes solid body rotation, hence cannot generate a magnetic field by itself. However, it imparts turbulence through various mechanisms to the molten iron encasing it, namely the liquid outer core. Thus we have a situation in the outer core where a fairly good conductor, molten iron with impurities, is being driven turbulently. Such a system is *capable* of generating a self-sustaining magnetic dynamo, although precise quantification of the parameters necessary for dynamo action proves to be a very difficult problem. It is this very problem which has led to D. Lathrop's series of liquid sodium experiments at the University of Maryland.

Before examining how this problem is being attacked experimentally, let us review some of what we know from theoretical considerations, together with generally accepted features of the outer core. The geophysics community has long accepted that a geodynamo in the outer core is the source of the main field for a number or reasons. First, if there were no self-sustaining dynamo action then the main field would dissipate through Ohmic heating in about 10,000 years[5]. But the paleomagnetic record indicates the presence of a magnetic field over some billions of years, necessitating a self-regenerative source like a dynamo. The paleomagnetic record also indicates numerous reversals over geological history, again consistent with theoretical fluid dynamo behavior. Yet further evidence is the presence of spherical harmonics far higher than the $l=1$ dipole. It now seems clear Joseph

[5] David Stevenson, "Planetary magnetic fields," *Earth and Planetary Science Letters* 6523 (2002) 1-11

Larmor's hypothesis was correct: the cause of Earth's main magnetic field is the turbulent flow of conducting material deep in the interior, which can only mean the outer core.

Once we accept this theory, a simple question arises: What is the energy source that stimulates this turbulent flow, and has been doing so for billions of years? About this question there remains considerable uncertainty. Direct observation of the fluid core is impossible, so all models must fall back on speculation as to the actual character of the fluid flow. One possible candidate is buoyancy driven convection. We may identify two types of convection: thermal and compositional. Compositional convection occurs as matter solidifies on the surface of the inner core, freeing lighter elements to rise in convective buoyancy currents. On the other hand, thermal convection is due to steep temperature gradients that contribute energy to drive the geodynamo. The origin of these gradients is the subject of current research, and has been variously assigned to the latent heat of fusion at the inner core boundary, the residual heat of formation, or even radioactivity. In the next section we examine some of the specifics related to this overall model of the Earth's internal dynamo.

Chapter 4: Recent and Ongoing Experiments

Given this model for dynamo action, it is possible to write down the equations of motion for spherical Couette flow. Since we are describing fluid motion with an electromagnetic component, the relevant equation is Navier-Stokes with an additional Lorentz force component[6]:

$$\frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \vec{\nabla})\vec{u} = -\frac{1}{\rho}\vec{\nabla}p + \nu\nabla^2\vec{u} + \frac{1}{\rho\mu_0}\left(\vec{\nabla} \times \vec{B}\right) \times \vec{B}$$

Equally important is the Induction Equation:

$$\frac{\partial \vec{B}}{\partial t} = \vec{\nabla} \times \left(\vec{u} \times \vec{B}\right) + \eta\nabla^2\vec{B}$$

Here u is the velocity, p is pressure, ρ is density, and B magnetic field. There are two material parameters that enter into the equations, v and η. The first one is kinematic viscosity, the second one magnetic diffusivity. We can recast these equations in a slightly different form that includes only dimensionless parameters: the Reynolds number, the magnetic Reynolds number, and the interaction parameter (see Table 1.) Writing them in that form allows us to compare dynamos on widely differing scales.

[6] Following D. Sisan, W. Shew, D. P. Lathrop, “Lorentz force effects in magneto-turbulence”, section 3, *Physics of the Earth and Planetary Interiors* 135 (20003) 137-159.

Reynolds Number $\text{Re} = \frac{uL}{\nu}$ u = mean fluid velocity L = length scale ν = kinematic viscosity	The ratio of inertial to viscous forces that determines whether a flow will be laminar or turbulent.
Magnetic Reynolds Number $\text{Re}_M = \frac{uL}{\eta}$ η = magnetic diffusivity	A number that describes the comparison of advection to magnetic diffusion.
Interaction parameter $N = \frac{B^2 L}{\eta \rho \mu_0 u}$ ρ = density μ_0 = magnetic permeability of free space	Compares inertial to Lorentz forces.

Table 1. Dimensionless numbers important to magnetic dynamo action.

Unfortunately, as these equations involve non-linearities they are often not soluble analytically. Numerical attempts at modeling core dynamics have been only moderately successful. It is for these reasons that we turn to experimental models, in the hope that they shed some light on the fluid action necessary for producing a magnetic dynamo. With this in mind we offer an outline of a research line being carried out by Dan Lathrop at the University of Maryland. Among the many goals of his research, one is to produce a self-sustaining dynamo in the laboratory that reasonably models the Earth's interior, something that has so far eluded science.

There are many practical problems to overcome in designing a system like this. First, it is not practical to heat iron to over 1000 K and then spin it around inside a laboratory vessel. It turns out that liquid sodium, with a melting point of ~100° C, is a good substitute. Sodium is an excellent liquid metal conductor and has other material properties that make it a good candidate. For example, an important dimensionless parameter of a magnetic dynamo is the magnetic Prandtl number of the working fluid. The Prandtl number is the ratio of kinematic viscosity to magnetic diffusivity, and for sodium above the melting point it is approximately 10^{-5}.

A number of liquid sodium experiments in Lathrop's non-linear dynamics lab are either being built, running, or have already been carried out, all with an eye toward gaining some insight into the transition to dynamo action. The first experiment, titled Dynamo I, employed a saucer-shaped apparatus 20.5 cm in diameter, constructed of titanium, and holding approximately 1.5 liters of liquid sodium. Heaters were placed alongside the equator of the vessel, and there was a cooling system inside the axle. This set up a thermal gradient in the sodium which broadly resembled that of the Earth's interior, but reversed. However, radial acceleration is also reversed as the device rotates at up to 200 Hz. These two reversals in effect cancel each other in the temperature and momentum equation, producing a flow similar in outline to the Earth's.

In order to probe for a self-generating dynamo, the apparatus was encircled by two electromagnetic Helmholtz coils which pulsed a magnetic field through the system periodically. This pulse acted as a seed field: if there were dynamo action, the seed field would grow into a self-sustaining field of greater strength. Such an effect would be detected by a ring of Hall probes placed around the vessel for that purpose. No such field was detected, although the decay time of the seed field indicated progress toward a laboratory dynamo. Complete results are available in Peffley *et al*, "Characterization of Experimental Dynamos," *Geophys. J. Int.* (2000) 142, 52-58.

The next experiment, dubbed Dynamo II, employed a slightly different geometry. The overall vessel was slightly larger and spherical, with a diameter of 30 cm. In the center of the vessel were two independent drive shafts. A variety of different mechanical devices were attached to the drive shafts in order to initiate and control flows, including propellers and a simple copper sphere. Although direct observation of a dynamo did not occur, the experiment produced several interesting results. Among these was the first laboratory observation of the Magneto-Rotational Instability, which we shall not discuss here[7] but which has been empirically observed in various astronomical objects. In addition, some quantifiable progress towards a dynamo was made in terms of a retarded decay time. For details, see Dan Sisan's Ph.D. dissertation, "Hydromagnetic Turbulent Instability in Liquid Sodium Experiments", University of Maryland, 2004.

Closely following this was Dynamo III-a, which was larger than its predecessors (D = 60 cm) and more closely modeled the Earth's interior. Inside the outer sphere and concentric with it was a conducting copper inner sphere 20 cm in diameter. This approximates the Earth's core, which has a 2.85:1 ratio of core-mantle boundary to inner-outer core boundary. The whole apparatus rotated together at rates up to 30 Hz.

In addition there were two axially mounted Helmholtz coils. These are included in order to kick-start a dynamo effect if there is one, or to mimic one if there is not. Heaters on the outside and a cooling tower in the center generate convective buoyancy in the liquid sodium. This experiment was directed less at generating a dynamo than re-creating the convective and zonal fluid flows of the Earth's liquid outer core. Results show that the zonal velocity scales as $U_\phi \sim \Omega D \alpha \Delta T$ and

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[7] D. Sisan, N. Mujica, *et al*, "Experimental observation and characterization of the magneto-rotational instability," *Physical Review Letters* 93 (2004).

convective velocity as $U_\phi \sim \Omega D\sqrt{\alpha \Delta T}$ where Ω is rotation rate, D is a length scale of 20 cm, α is the thermal diffusivity, and ΔT is the temperature difference between outer and inner sphere surfaces. These results helped characterize hitherto unknown attributes of the fluid outer core. For further discussion see W. Shew's Ph.D. dissertation "Liquid Sodium Model of Earth's Outer Core," University of Maryland, 2004.

The next dynamo experiment, Dynamo III-b, marks a return to the search for the dynamo effect. The apparatus fabricated by Shew for his convection experiments was modified to allow for independent rotation of both inner and outer spheres. It is known that the solid inner core of the Earth rotates slightly faster than the planet's surface. However, planetary dynamos come in all different shapes and sizes, and differential rotation expands the potential field of exploration considerably. In the next section we will discuss the design, fabrication, and operation of this experiment in detail, as this was the present author's primary involvement with this research line.

Chapter 5: Experimental Apparatus Dynamo III-b

The apparatus for this experiment is a hollow titanium outer sphere 60 cm in diameter enclosing a solid copper inner sphere 20 cm in diameter. The cavity lying between them is filled with molten liquid sodium. Both spheres are mounted on bearings and may be rotated independently by two 7.5 HP electric motors. The mechanical action of the experiment involves either counter- or co-rotation of the two spheres at rotation rates between –50 and 50 Hz, inducing turbulent flow in the sodium known as *spherical couette flow.* This simple model is intended to approximate the earth's solid rotating parts, namely the inner core and crust, and the molten iron-nickel outer core where the geomagnetic field is believed to originate.

This system by itself requires several subordinate systems in order to run smoothly. The sphere itself sits inside a stainless steel tub, providing both scaffolding for subordinate systems and a safety containment vessel in case of sodium leakage. The mechanical energy put into the sodium by the two 7.5 kW electric motors dissipates as heat, causing the sodium to rise in temperature approximately 7 K per minute. Initially a kerosene-based cooling system was integrated into the design. However, an exhaust fan in the outer vessel draws cool air across the sphere, and this convective cooling allows the experiment to run repeatedly within the 110° C to 125° C range.

Before any scientific data is taken, the sodium must be brought up to its 98° C melting point and slightly beyond. This is accomplished by an array of high-resistance heater bulbs that blanket one quarter of the sphere. It takes a little over an hour to bring 225 lbs of sodium and 400 lbs of titanium and copper to a working temperature of 110° C. A thermocouple extending from the inner shaft 1 cm into the sodium is used for monitoring temperature inside the vessel.

The sphere is embraced both vertically and horizontally by pairs of electromagnetic coils. The larger set of coils, nick-named Boris & Natasha, are set up axially and generate a magnetic field in the vertical direction of approximately 40 Gauss. Boris & Natasha are programmed to produce a square-wave pulse of 1.75 seconds, followed by a quiescent period of 5 seconds during which we observe the magnetic field decay. Two smaller coils are mounted horizontally like wheels on the ends of an axle. They generate approximately 10 Gauss of magnetic field in a sinusoidal pattern. The period of this sinusoid is synchronized with the period of rotation of the larger sphere, and hence of the sodium itself. A dipole magnetic field is set up equatorially in the sodium, gaining field strength with each pass by the two fixed coils. All of this activity is coordinated by a LabView program that simultaneously sets the motorized rotation rates and triggers a magnetic pulse in either the *x*- or *z*-direction.

The detection elements have undergone some modification since the apparatus first went on line. Originally there were three Hall probes used to acquire magnetic data while the sphere was spinning. They were mounted on ¼ " copper tubing which served the dual purposes of structurally supporting the probes and maintaining them at a constant temperature of about 10° C. The probes, labeled *x, y, & z,* were located as follows:

- The *x* probe was mounted horizontally at the equator, roughly at the center of one of the horizontal electromagnetic coils. It was positioned to detect an equatorial dipole.
- The *y* probe was also mounted horizontally at the equator, 90° away from the *x* probe.
- The *z* probe was mounted as close to the apex of the sphere as the mechanical apparatus would allow, pointing vertically down onto the pole. Thus the *z* probe was very near the center of Natasha, the topmost vertical coil, and measures the axial magnetic field.

All probes are mounted as close to the sphere as is practically possible without mechanically interfering with rotation.

A typical run starts with bringing the sodium up to the melting point and beyond. Once that is achieved, the motors are turned on and the sphere begins rotating. Various differential rotation rates are explored on different runs. At any rate, once the sphere is spinning a brief magnetic pulse in the negative *z*-direction is applied to the apparatus, setting up a magnetic field in the sodium. The magnetic field decay curve is then observed using the Hall probes and recorded digitally as an ASCII file for later analysis. Outer sphere rotation rates of up to 30 Hz were achieved, with inner sphere rates twice that, but eventually mechanical problems required shutting down the experiment for repair.

As of this writing, repairs are nearly complete and the apparatus should be ready to run in the next few weeks. While the apparatus was down, experimenters took the opportunity to extend the Hall probe array and increase the magnetic field strength of the Helmholtz coils. There are now Hall probes at 0°, 45°, 60° and 90° around the equator, and a series of 21 probes along a meridian and oriented in the axially radial position. That is to say, they measure the magnetic field in the horizontal direction (with respect to the floor of the lab) as it exits the sphere. Any measurement component in the *z*-direction will be drowned out by the strong magnetic pulse of the Helmholtz coils, and there is a separate *z* probe for the purpose of measuring that applied field.

Chapter 6: Results and Future Work

Multiple runs of data were acquired in the 2005-2006 time frame. While the quality of the data appears to be good, no single clear message has emerged from the analysis. A dynamo was not produced. Nevertheless, there are some interesting features of the data worthy of comment. We present a compendium of these below.

Our first observation upon taking data with the 60 cm device was a sudden onset of oscillations in the magnetic field concurrent with the application of a *z*-pulse from the Helmholtz coils. Although puzzling at first, these appear to be inertial waves in the fluid. Inertial waves are large-scale waves associated with turbulence in fast-rotating systems. A peculiar hallmark of these waves is their failure to obey Snell's law of reflection. Instead they reflect symmetrically about the axis of rotation, regardless of the angle of incidence. Inertial waves have been detected in the Earth's interior using a gravimeter[8], presumably triggered by an earthquake immediately preceding the observation.

Of more direct interest to the geodynamo problem were some features of the mode spectrum. When we take the signal picked up by one of the equatorial detectors and decompose it into spherical harmonic modes, we see that at certain rotation rates some modes are much more pronounced than others. In empirical terms this translates as an increase in power for those modes. In these experiments the outer sphere rotation rate is held constant at $\Omega = 30$ Hz. As rotation rate of the inner sphere increases, one or two modes clearly dominate the power spectrum. We do not offer

[8] Melchior, P. & Ducarme, B. *Phys. Earth planet. Interiors* 42, 129−134 (1986)

an explanation for this phenomenon, but it may be of relevance to the spectral decomposition of Earth's main field.

Currently the 60 cm is being re-fitted with considerably larger electromagnets, along with some other mechanical adjustments to smooth out the operation. Another series of runs is expected to occur in the summer of 2006. While interesting in its own right, this experiment also serves as a useful small-scale model for the largest dynamo experiment yet attempted, a sphere 3 meters in diameter that will rotate at speeds up to 10 Hz. As the magnetic Reynolds number scales with the diameter of the sphere, this will greatly improve the chances of observing a dynamo or near-dynamo.

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