

# Entrainment of Weakly Coupled Oscillators by External Driving

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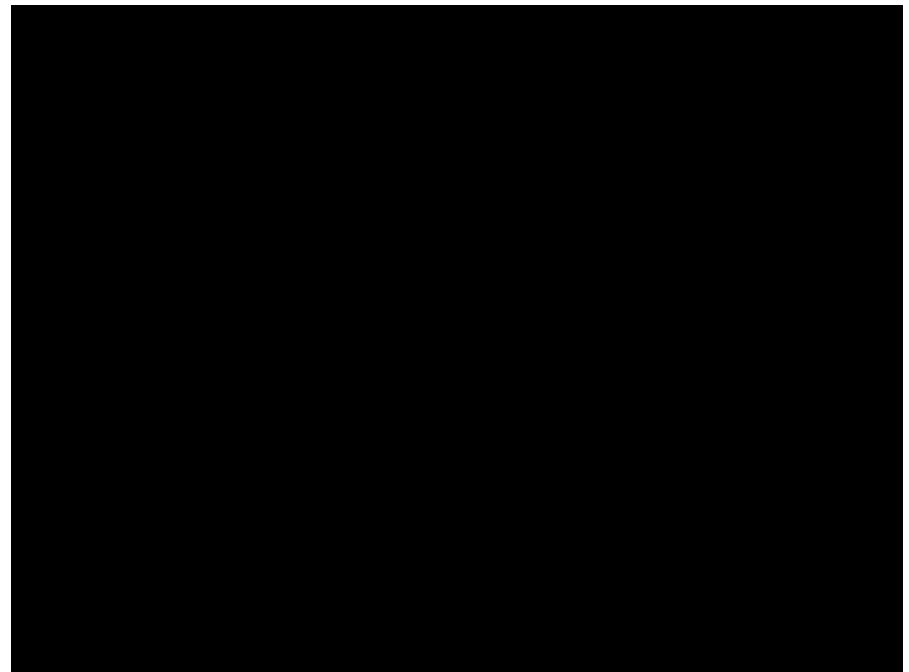
# Introduction

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What is the effect of external driving on many weakly coupled oscillators?

- Systems that are modeled as weakly coupled oscillators include
  - The heart's pacemaker cells
  - Josephson junctions
  - The flashing of certain species of fireflies
  - The Suprachiasmatic Nucleus (SCN) in the hypothalamus

The Suprachiasmatic Nucleus



# Undriven Case: Kuramoto's Model (1975)

$$\frac{d\theta_i}{dt} = \omega_i + (K/N) \sum_{j=1}^N \sin(\theta_j - \theta_i)$$

$\theta_i$  – phase angle of oscillator  $i$

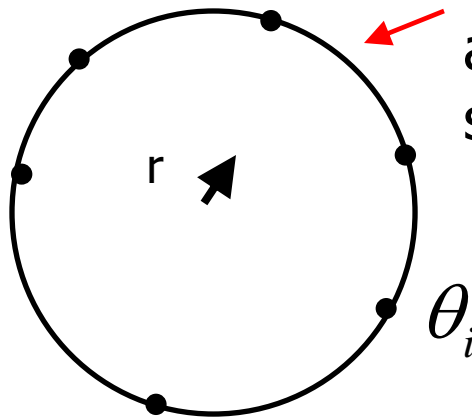
$K$  – coupling strength

$\omega_i$  – natural frequency of oscillator  $i$

$N$  – number of oscillators

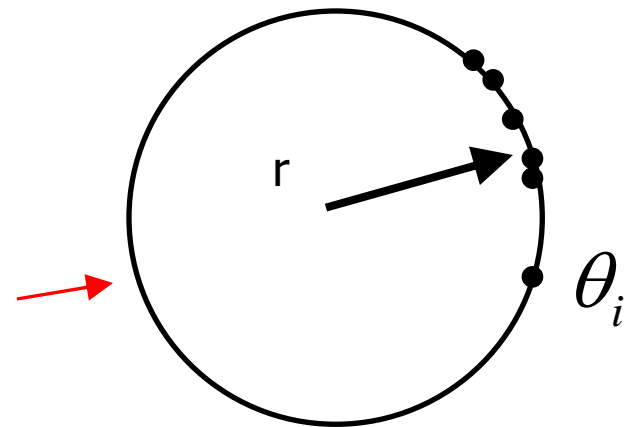
Order Parameter:

$$r = \left| \frac{1}{N} \sum_j e^{i\theta_j} \right|$$



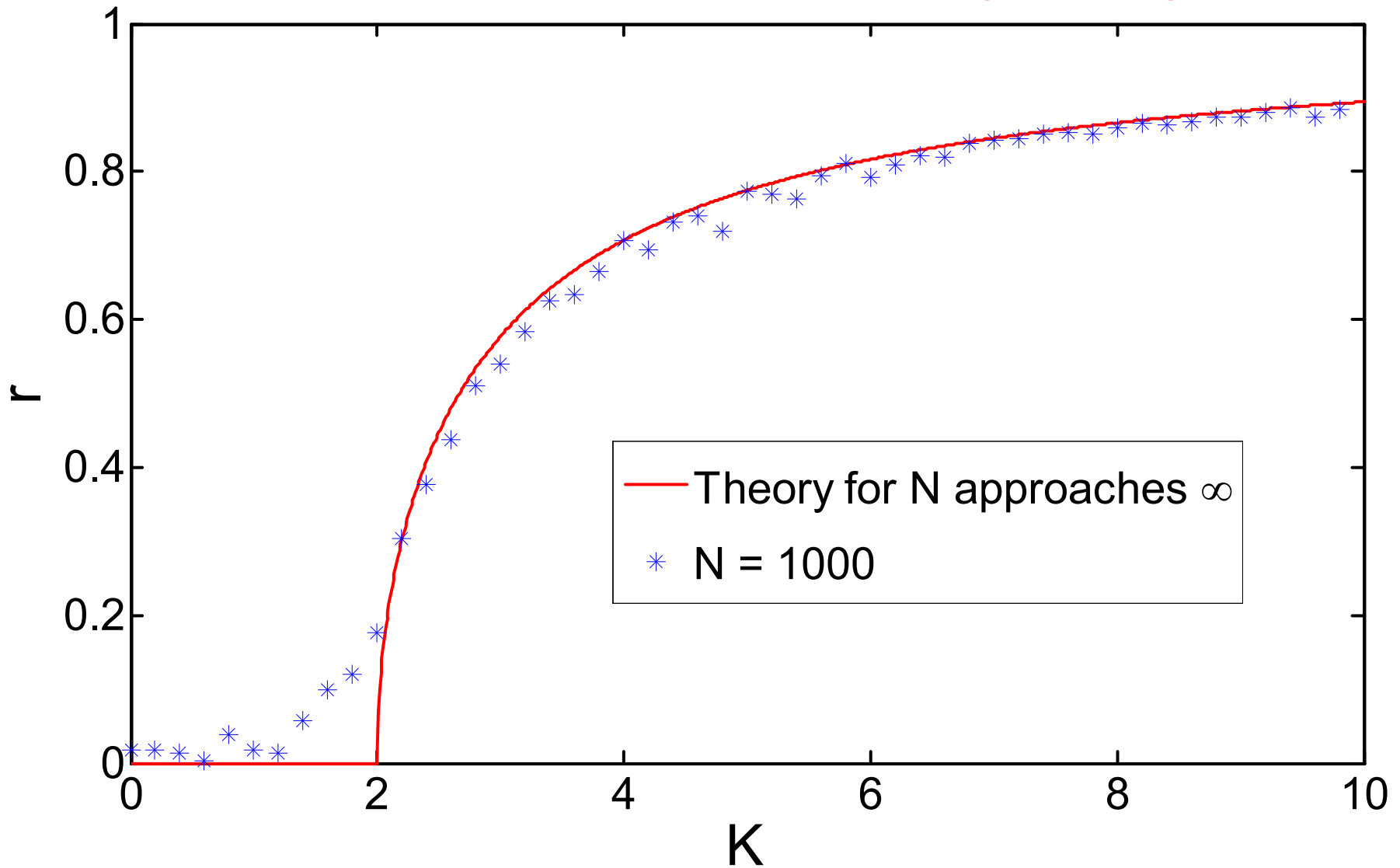
If oscillator phases are random,  $r$  is small

If oscillators are nearly in phase,  $r \rightarrow 1$



# Kuramoto's Model (1975)

## Order Parameter vs. Coupling Strength



# Our Work: Driving the Kuramoto Model

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$$\frac{d\theta_i}{dt} = \omega_i + (K / N) \sum_{j=1}^N \sin(\theta_j - \theta_i) + \underbrace{M \sin(\Omega t - \theta_i)}_{\text{(Drive)}}$$

Does the driving cause entrainment?

**Entrainment:** The response of the oscillator system varies periodically at the period  $\frac{2\pi}{\Omega}$  of the drive.

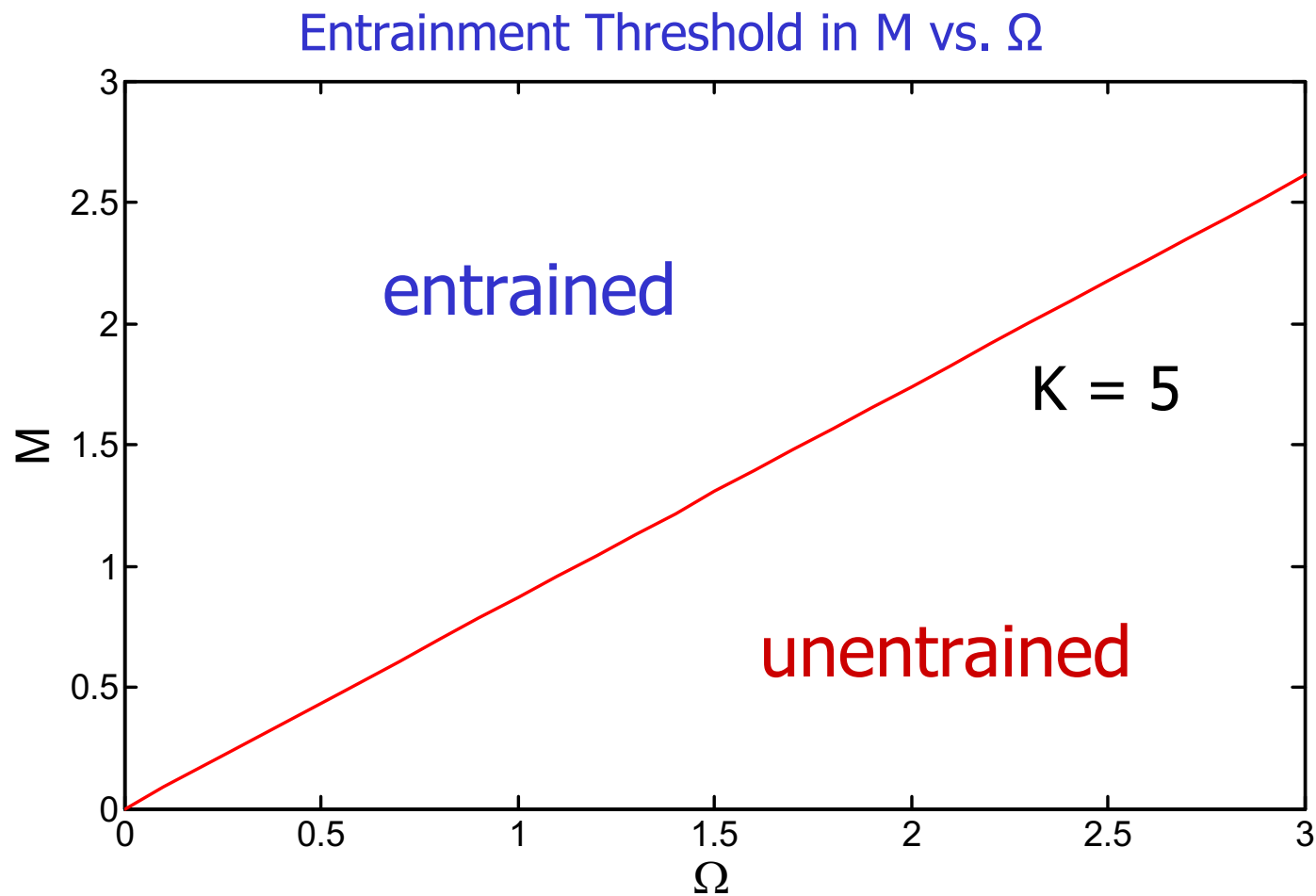
- SCN – The system is synchronized to the drive signal (daylight). The pattern of wakefulness/sleepiness is the same each day.

**Unentrainment:** Response does not vary periodically with the drive period.

- SCN – The pattern of wakefulness/sleepiness is different each day.

# Our Result

There is a **threshold** for entrainment.

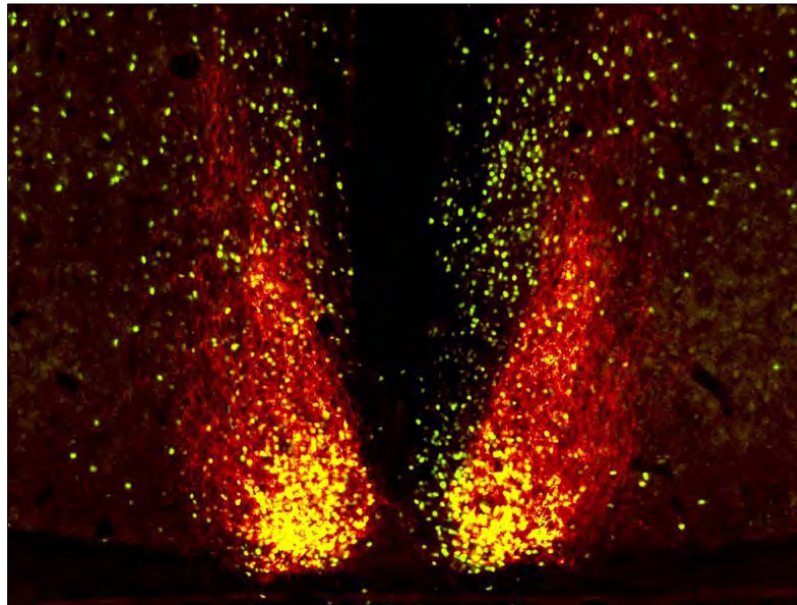


# Summary

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- Entrainment by a periodic drive occurs for drive amplitudes above a **threshold**  $M > M_c(K, \Omega)$ .
- When this condition is not met, the oscillator system responds to the drive in a nonsynchronous manner (in the SCN case, this could lead to complex irregular rhythms).

## The SCN



[www.well.ox.ac.uk/foster/circadian.shtml](http://www.well.ox.ac.uk/foster/circadian.shtml)