Sync Map Description of Coupled Oscillator Systems

TREND 2008 FAIR

Institute for Research in Electronics & Applied Physics

Gilad Barlev

Advisors: E. Ott, M. Girvan

Abstract

Synchronization of many coupled oscillators is a generic issue in a wide variety of natural situations (e.g. pacemaker cells in the heart). We introduce a map model for the study of such problems. Issues addressed include the effects of oscillation frequency diversity, noise, and network topology, especially community structure.

The Model

-Discrete time, t

-Phase of oscllator i at time $t = \theta^i_t$

$$\theta^i_{t+1} = \omega^i + \theta^i_t + K \sum_{j=1}^{N} A_{ij} \sin(\theta^j_t - \theta^i_t) + \eta^i_t$$

ω^i = natural freq. of i^{th} osc., PDF $g(\omega)$

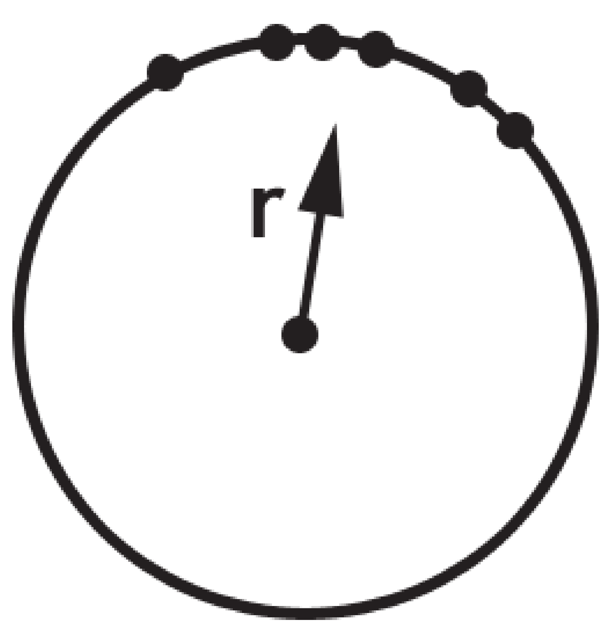
K = coupling factor

A = connectivity matrix

N = number of oscillators >> 1

η^i_t = random noise, PDF $P(\eta)$

-Order Parameter r:

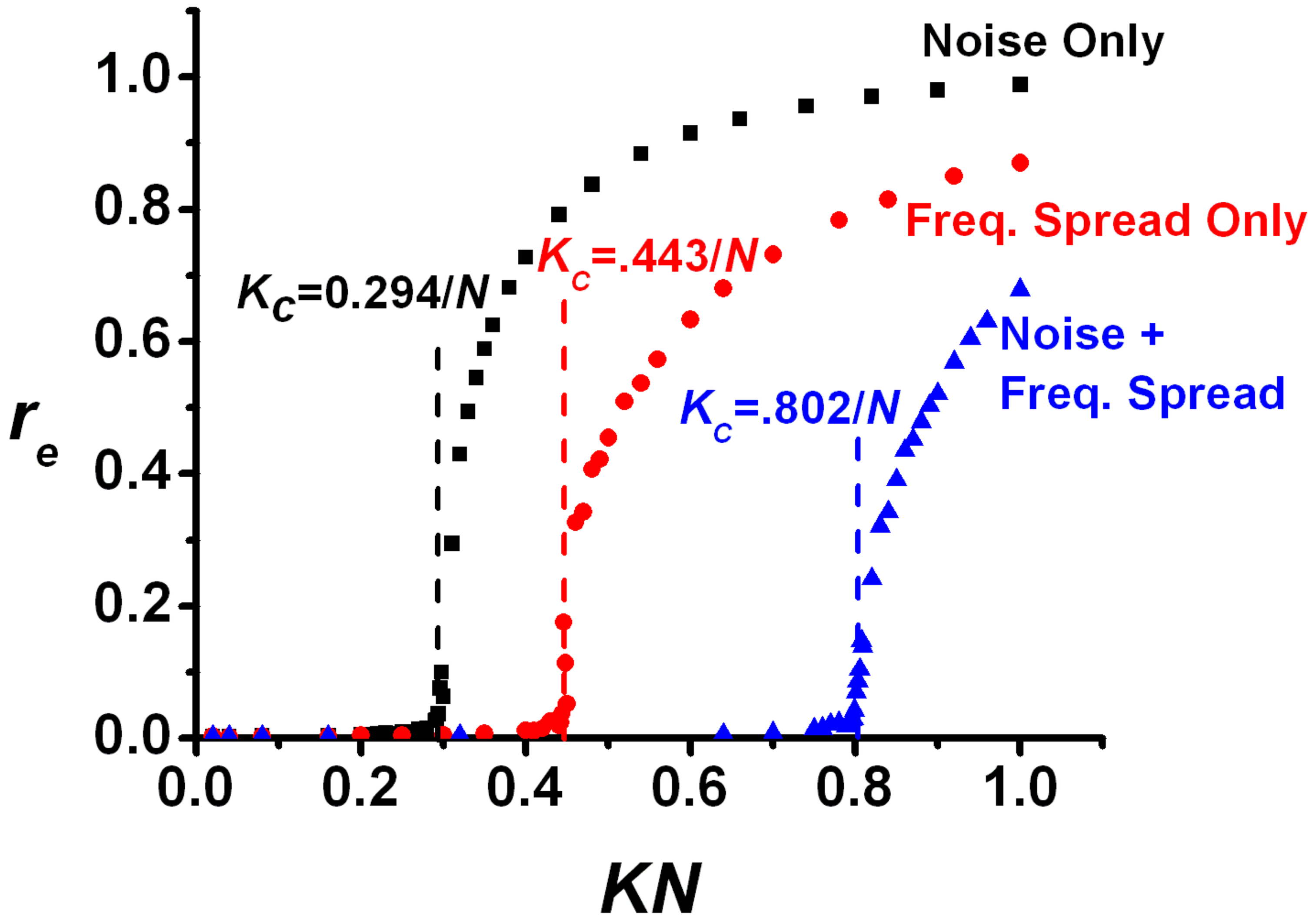


$$r_t = \frac{1}{N} \left| \sum_{j=1}^{N} e^{i\theta^j_t} \right|$$

Equilibrium r vs. K

-Global coupling: $A_{ij}=1$ for all i, j

-Equilibrium r, $r_e = r_t$ as $t \to \infty$

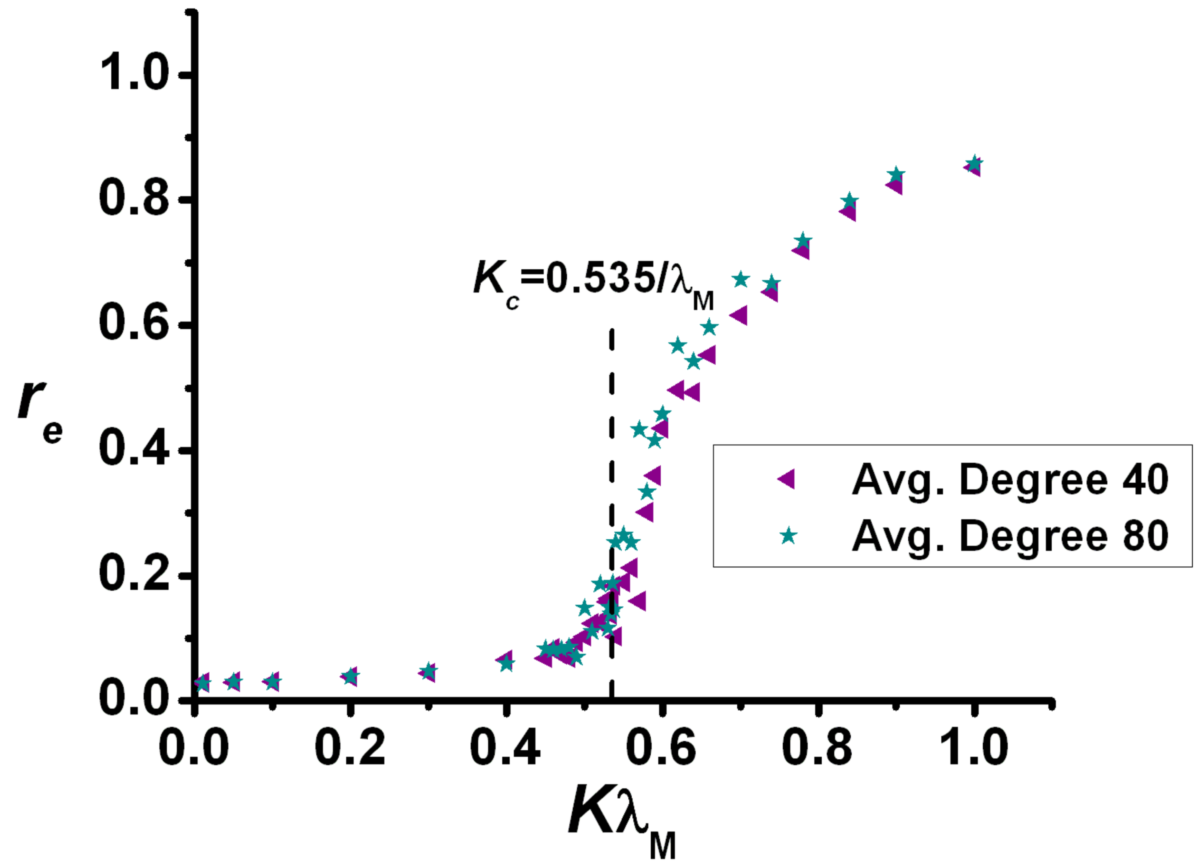


Vertical dashed lines = theoretical K_c

Gaussian $P(\eta)$ & Lorentzian $g(\omega)$

$\eta_{rms}=\pi/6$; $(\Delta\omega)_{rms}=0.2$; $N=10^5$

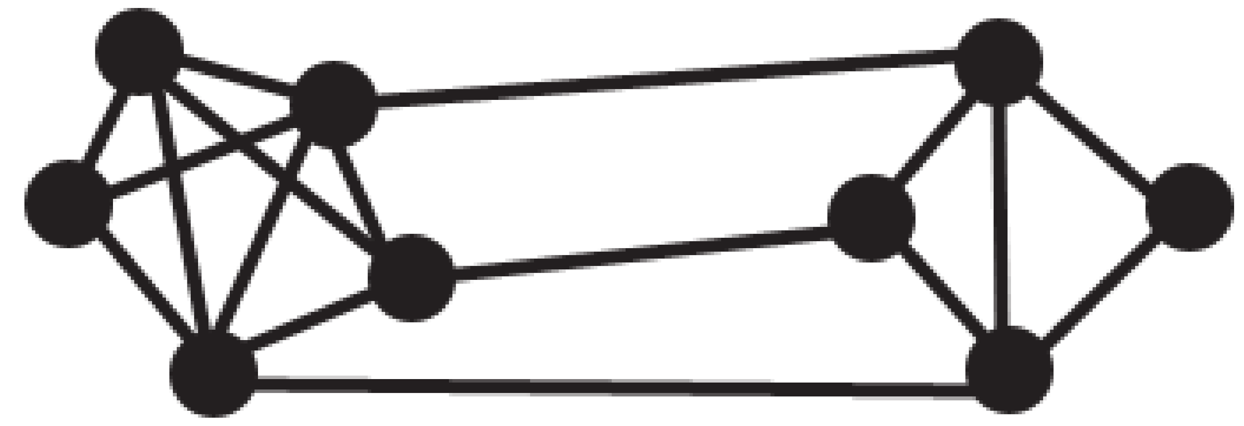
-Non-globally connected networks: K_C scales as $1/\lambda_M$, the largest eigenvalue of the network connectivity matrix A, rather than as $1/N$.



$<\eta^2>=\pi/6$; $<(\Delta\omega)^2>=0.1$; $N=10^3$

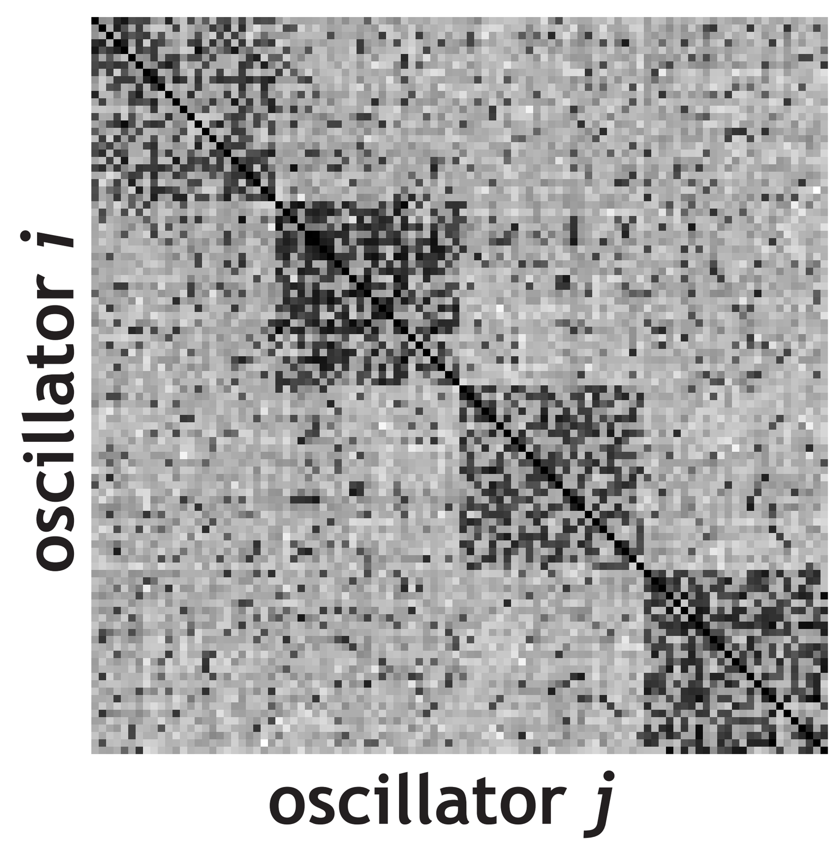
Community Structure

-More connections between members of the same community than members of different communties.



-We observe that members of the same community are more synchronized to each other than to members of different communities.

-Example: four-communtity network, with N=100. Matrix of the time-averaged distances between oscillator phases:



darker color means smaller distance

Avg. intra-community degree: 12

Avg. inter-community degree: 6

-We have investigated how the distance matrix can be used to discover communities when they are unknown.