The Emergence of Coherence in Networks of Coupled Oscillators: From Local to Global Synchronization

John Stout, Matthew Whiteway

Advisors: Ed Ott, Michelle Girvan, Tom Antonsen

Graduate Student: Gilad Barlev

NC STATE UNIVERSITY

TREND
FAIR 2010

INTRODUCTION

Motivation

Networks of coupled oscillators can describe interesting physical phenomena:

- Pacemaker cells in the heart
- Synchronous flashing of fireflies
- Neuronal activity and circadian rhythms

Question

As the coupling between oscillators increases from zero, how do these systems evolve toward global synchronization?

THE MODEL

$$\frac{d\theta_i}{dt} = \omega_i + K\sum_{j=1}^{N} a_{ij} sin(\theta_j - \theta_i)$$

θ_i = Phase of the ith Oscillator

ω_i = Natural Frequency of the ith Oscillator

K = Coupling Constant

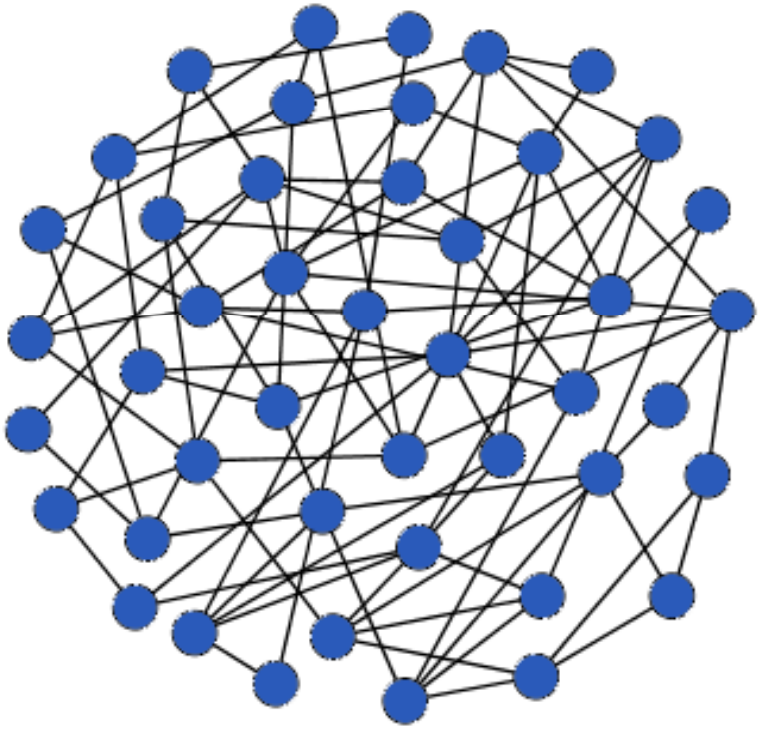
N = Number of Oscillators

a_{ij} = Network Matrix

THE NETWORK

ERDÖS-RÉNYI

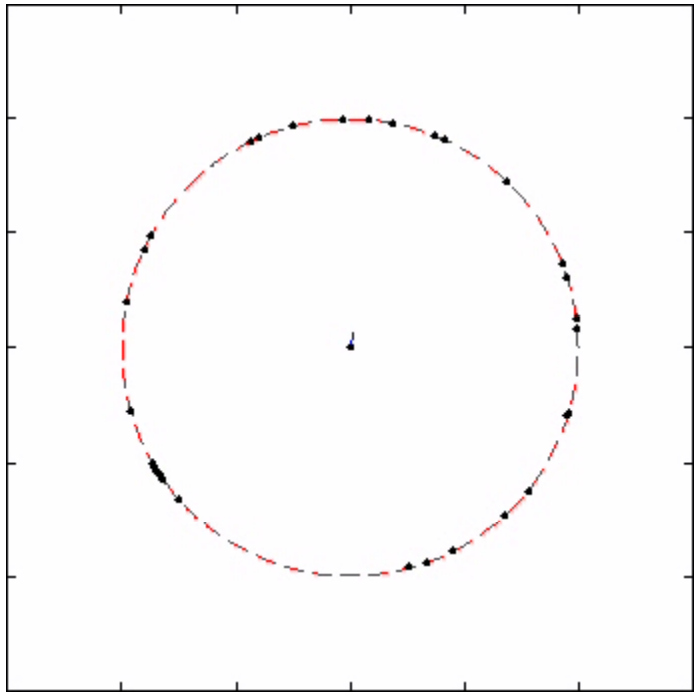
- Every pair of nodes has the same probability of being connected.
- As a result, every node has about the same number of links.



$$a_{ij} = a_{ji} = \begin{cases} 1 & \text{if the nodes i and j are linked} \\ 0 & \text{if nodes i and j are not linked} \end{cases}$$

MEASURES OF SYNCHRONIZATION

$$r_\infty = \left| \frac{1}{N} \sum_{i=1}^{N} e^{i\theta_i} \right|$$

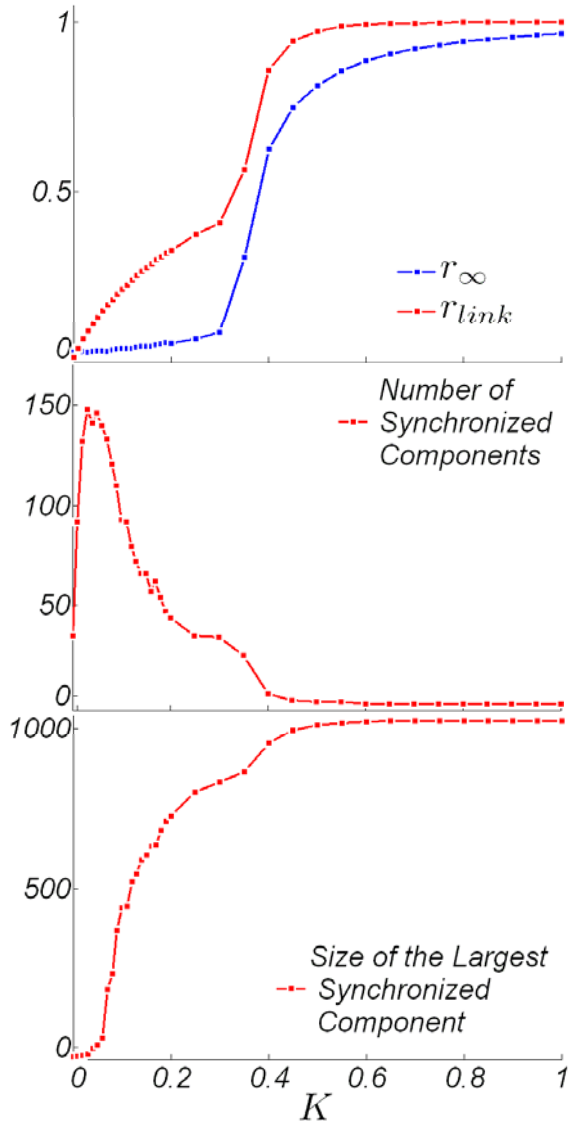


r_∞ **measures the degree of global synchronization of the system**

$$r_{link} = \frac{1}{2N_{links}} \sum_{i,j} a_{ij} \left| \left\langle \exp(i(\theta_j - \theta_i)) \right\rangle \right|$$

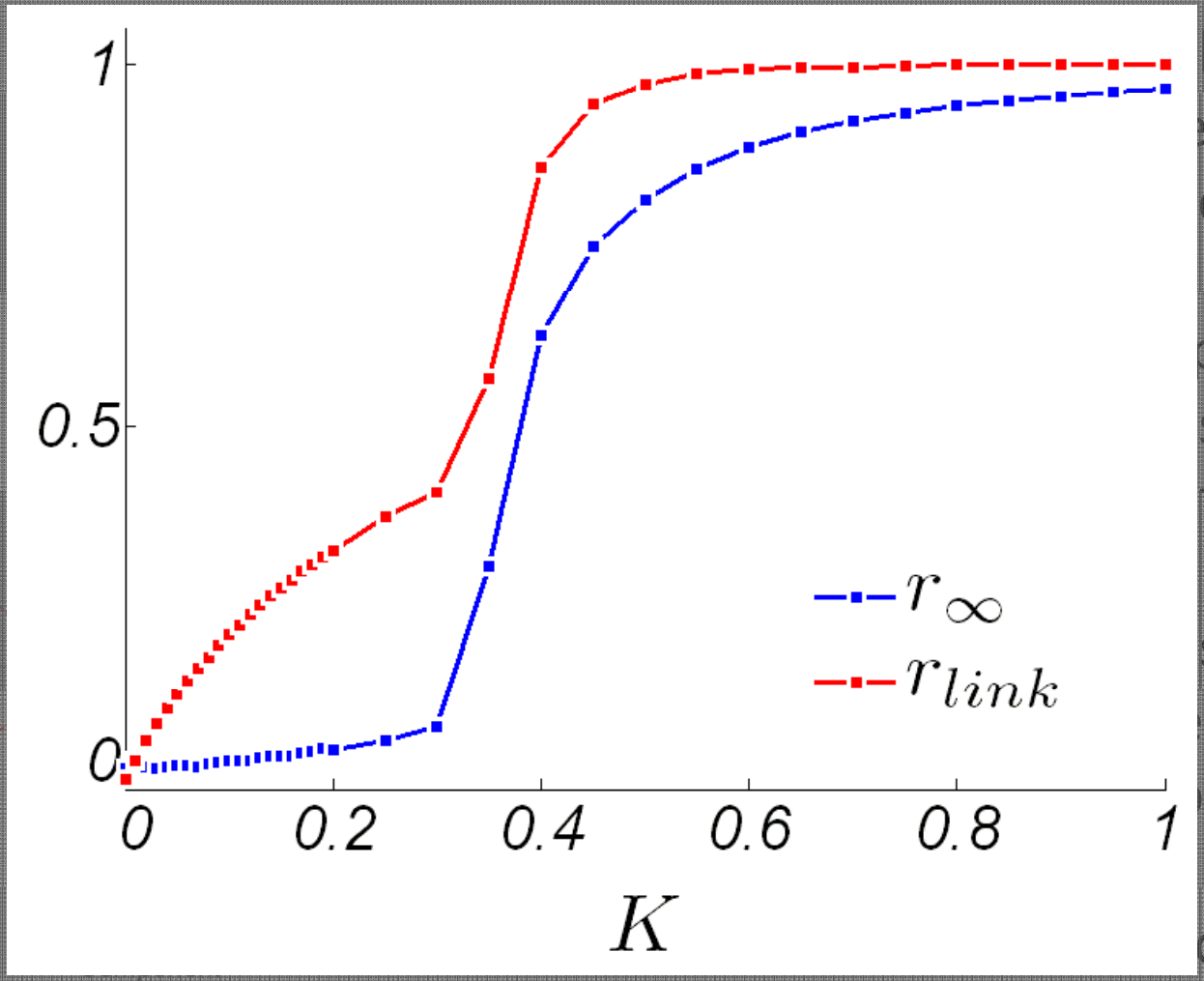
r_{link} **measures the degree of local synchronization of the system.**

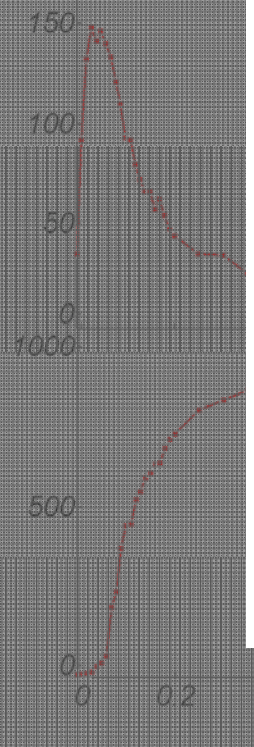
FROM LOCAL TO GLOBAL SYNCHRONIZATION



- Before the onset of global synchronization, the system displays a significant amount of local synchronization
- For low values of the coupling strength, there are many small connected components of nodes that are locally synchronized
- As the coupling strength is increased, these groups join together to form larger synchronized components, until the network is global synchronized in a single connected component

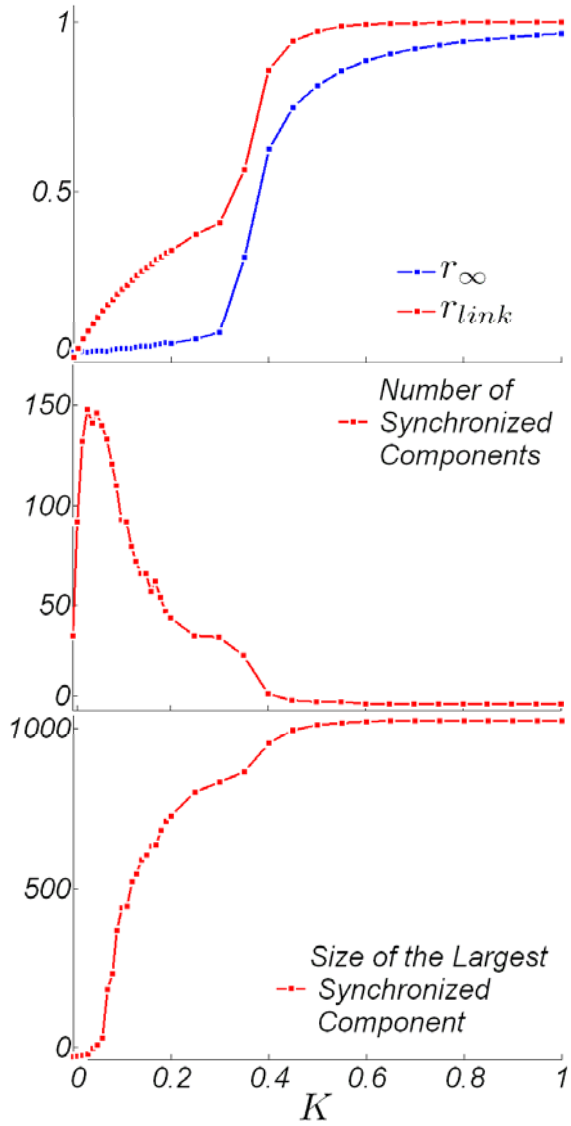
FROM LOCAL TO GLOBAL SYNCHRONIZATION





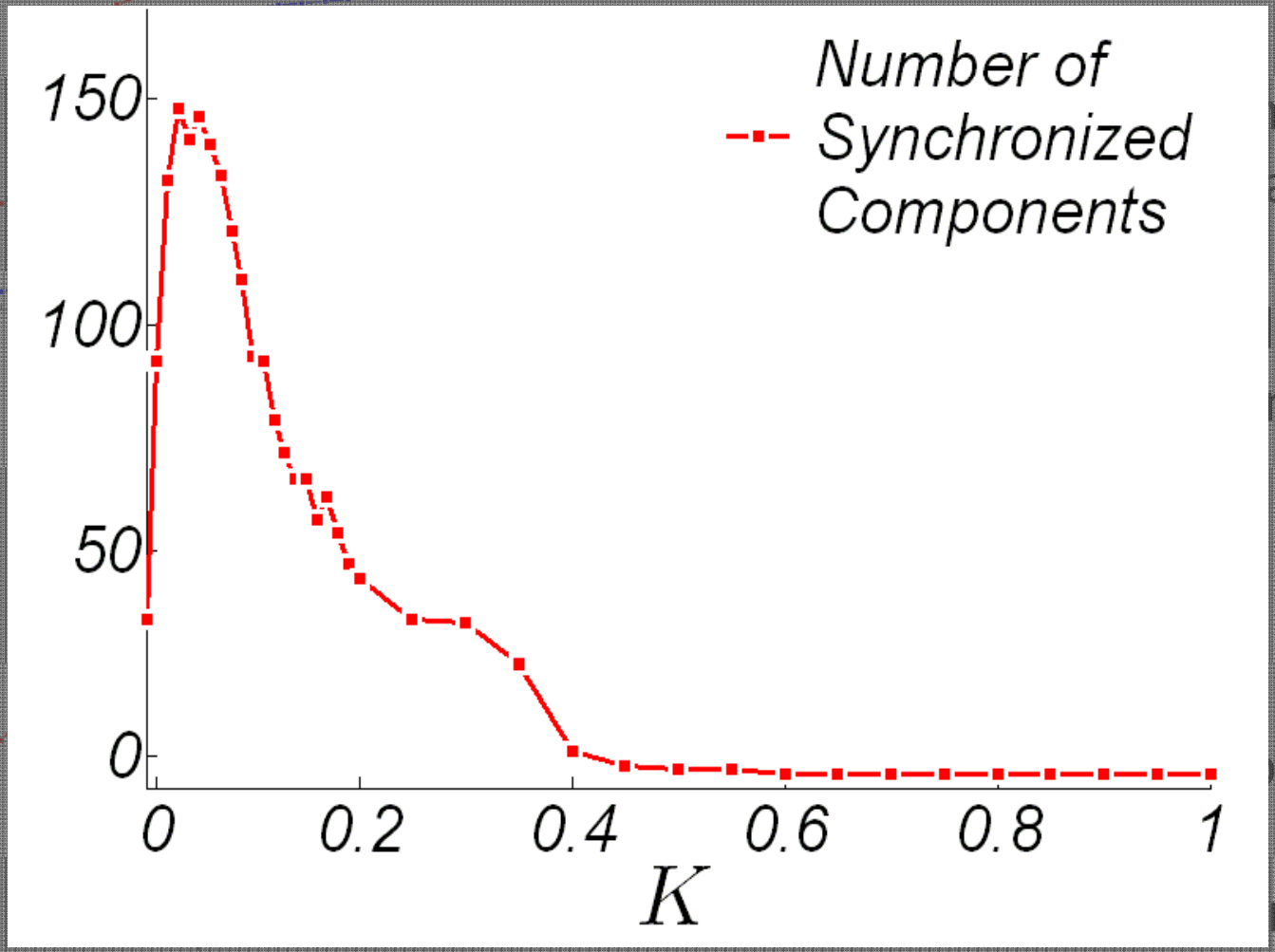
l
em displays
cal
pling
small
f nodes that
s increased,
r to form
onents,
l
onnected
component

FROM LOCAL TO GLOBAL SYNCHRONIZATION



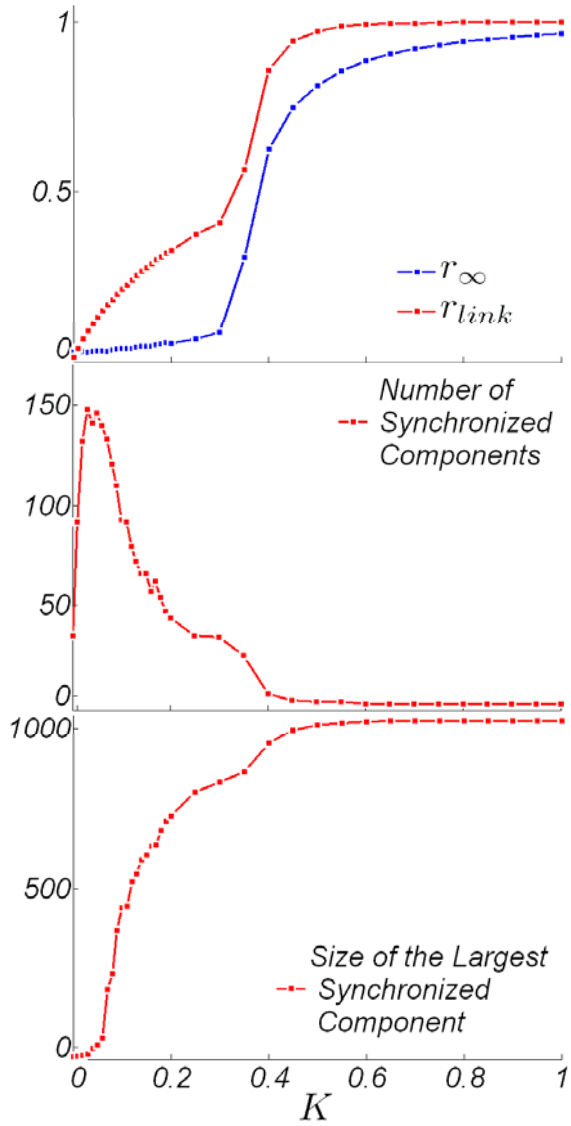
- Before the onset of global synchronization, the system displays a significant amount of local synchronization
- For low values of the coupling strength, there are many small connected components of nodes that are locally synchronized
- As the coupling strength is increased, these groups join together to form larger synchronized components, until the network is global synchronized in a single connected component

FROM LOCAL TO GLOBAL SYNCHRONIZATION



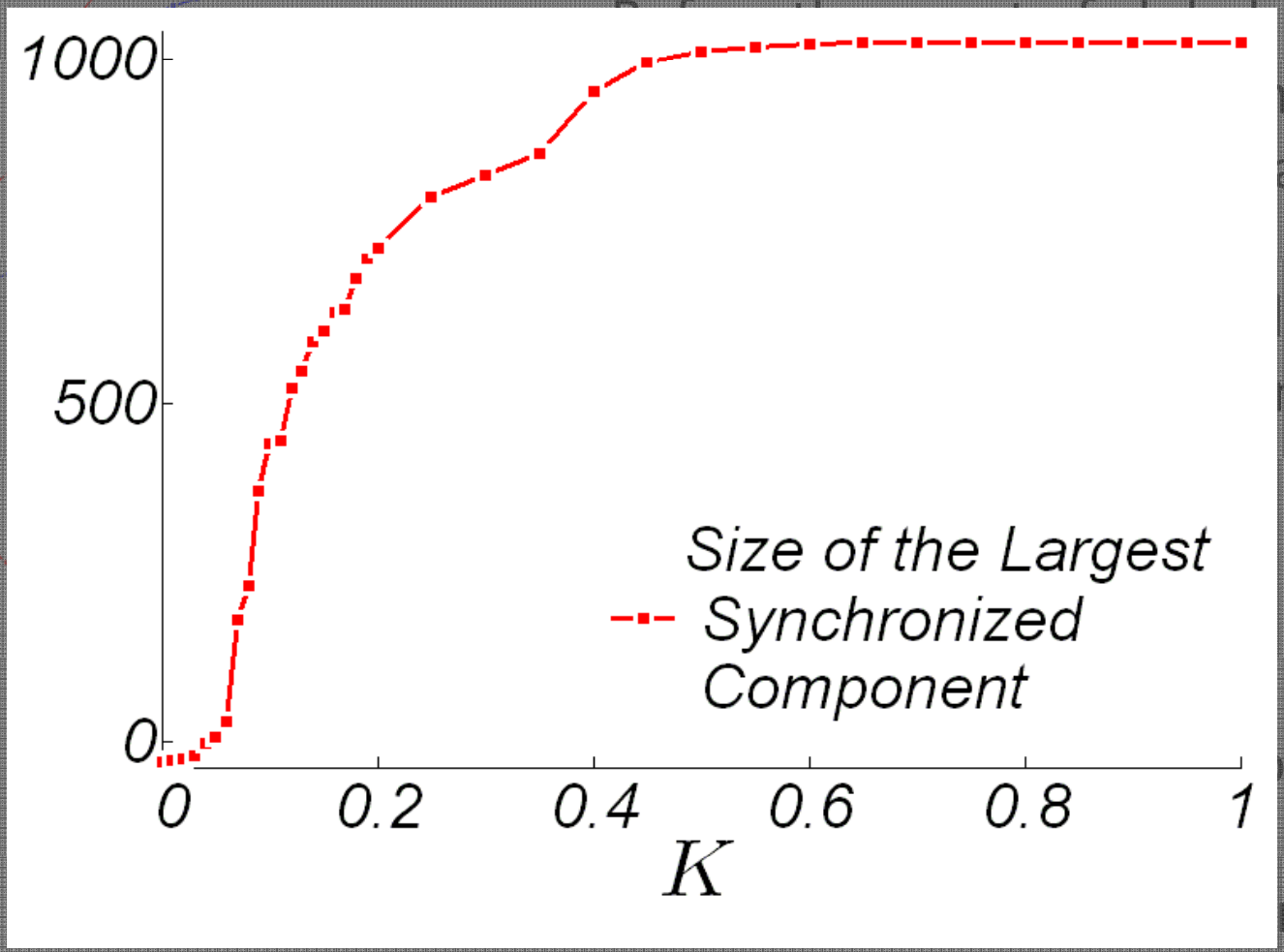
n displays
al
ling
mall
nodes that
increased,
to form
nents,
nnected
component

FROM LOCAL TO GLOBAL SYNCHRONIZATION



- Before the onset of global synchronization, the system displays a significant amount of local synchronization
- For low values of the coupling strength, there are many small connected components of nodes that are locally synchronized
- As the coupling strength is increased, these groups join together to form larger synchronized components, until the network is global synchronized in a single connected component

FROM LOCAL TO GLOBAL SYNCHRONIZATION



m displays
al
ling
mall
nodes that
increased,
to form
nents,
nnected
component

CONCLUSION

- Before the onset of global synchronization, nodes form locally synchronized components. These components ultimately coalesce into a single synchronized component
- Other questions explored:
  - How does the path to synchronization depend on the network structure?
  - How does the path to synchronization depend on how the natural frequencies of the oscillators are distributed?