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Abstract

Title of Dissertation: HIGHER-DIMENSIONAL NONLINEAR DYNAMICAL SYSTEMS: BURSTING AND SCATTERING

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We investigate two classes of nonlinear dynamical systems which have more than two state-space dimensions. In the first part, the dynamics of the system under study is given by a partial differential equation – an infinite dimensional system – but is modeled as a large set of ordinary differential equations, i.e., a system with many (thousands) of dimensions, but not an infinite number. In the second part, the systems studied have four state space dimensions.

In Part I, "The Onset of Magnetic Dynamo Action", we look at a converged simulation of the magnetohydrodynamic equations in three dimensions with periodic boundary conditions in a parameter range where a forced fluid flow is chaotic. In particular, we study how the magnetic field energy evolves over time near the transition to *dynamo action*, a situation where magnetic energy grows to some saturation value determined by details of the nonlinearity in the system. The magnetic energy exhibits a bursting behavior, or on-off intermittency,

characterized by several scaling laws. These scaling laws define this transition as a blowout bifurcation. By showing that these scaling laws hold for the simulation, we show that the transition is a blowout bifurcation. These results imply specific, testable, predictions for experimental dynamos.

The first chapter of Part II, "Fractal Dimension of Higher-Dimensional Chaotic Saddles", tests, using a physical system as an example, formulae previously conjectured to give the information dimension, in "typical" systems, of chaotic saddles and their stable and unstable manifolds in terms of the Lyapunov exponents and the characteristic escape time of the saddle. The example system is shown to possess a novel structure for its invariant manifolds and to be an atypical system. The typicality of the system is restored after a small perturbation, consistent with the conjecture.

Building on the example presented in Part II, Chapter 1, Part II, Chapter 2, "Complex Topology in Chaotic Scattering: A Laboratory Observation" presents an optical billiard, a chaotic light-scattering system made from reflective surfaces (spheres in this case). The physical manifestation of the *Wada Property* of basins (sets of initial conditions in state space which all asymptote to the same motion) and their boundaries is discussed, as is a method of demonstrating this property using the billiard. Three or more basins are said to possess the Wada Property when they share a single boundary. This property has been discussed previously in nonlinear dynamical systems theory, but had not been seen in a physical system.

Part II, Chapter 3, "Three Dimensional Optical Billiard Chaotic Scattering", looks more closely at optical billiards. We photograph reflective spheres in five configurations and measure the fractal dimension of the boundaries of the basin.

Four of the configurations contain two basins which are divided by a boundary which is a continuous, nowhere-differentiable surface. The fifth configuration (which is the same optical billiard as was discussed in Part II, Chapter 2) contains four basins which are divided by a boundary which is a higher-dimension generalization of a Cantor set.