Physics 270, Assignment 3

33.2

We are given that the earth’s magnetic field is $B = 5 \times 10^{-5}$ T. How fast should we drive a care to create a 1.0 V motional emf along a 1.0 m antenna if the antenna is perpendicular to the magnetic field? We know that the motional emf is given by equation 33.2

$$\Delta V = vlB$$

where $l$ is the length of the antenna and $v$ is the velocity. Solving for $v$, we have

$$v = \frac{\Delta V}{lB} = \frac{1.0 \text{ V}}{(1.0 \text{ m}) (5 \times 10^{-5} \text{ T})} = 20000 \text{ m/s}$$

33.4

The magnetic flux through the loop shall be

$$\Phi = \oint \vec{B} \cdot d\vec{A} = \int_1 \vec{B} \cdot d\vec{A} + \int_2 \vec{B} \cdot d\vec{A}$$

where integral 1 is over the area with a field into the page and integral 2 is over the area with the field out of the page. Within these two areas, the $\vec{B}$ field is constant. So we have

$$\Phi = B_1 \int_1 dA - B_2 \int_2 dA$$

where I have chosen the area vector to point into the page (accounting for the minus sign on the second integral.)

$$\Phi = B_1 \int_1 dA - B_2 \int_2 dA = B_1 A_1 - B_2 A_2 = (2.0 \text{ T})(0.2 \text{ m} \times 0.2 \text{ m}) - (1.0 \text{ T})(0.2 \text{ m} \times 0.2 \text{ m}) = 0.04 \text{ Tm}^2$$

33.6

We know that Lenz’s law requires that the induced flux oppose the change in magnetic flux. The induced current here results in an induced magnetic field out of the page (as given by the right hand rule). Because it must be in opposition to the change in flux, I know that the flux is changing such that more magnetic field lines are penetrating the surface and going into the page. I may thus be certain that the magnetic field inside the loop is increasing in strength.

33.12

Here, we have a loop being pushed into a magnetic field coming out of the page. As such, we know that magnetic flux is increasing out of the page. By Lenz’s law, the induced current must cause a magnetic field that opposes this change. Hence, the current causes a magnetic field going into the page. This is how we know that the current must be going clockwise through the loop. To find it’s magnitude, we know that the $I = \frac{\varepsilon}{R}$ where $\varepsilon$ is the induced emf and $R$ is the resistance of the loop. Using Faraday’s law and remembering that the field is constant in space, we have

$$I = \frac{1}{R} \varepsilon = \frac{1}{R} \left| \frac{d\Phi}{dt} \right| = \frac{1}{R} \left| \frac{d(BA)}{dt} \right| = \frac{1}{R} \left| B \frac{dA}{dt} \right|$$
Now we know that \( A = lx \) where \( l = 0.05 \) m and \( x \) is the depth to which the loop is inserted into the magnetic field. Therefore, \[
\frac{dA}{dt} = \frac{d}{dt}(lx) = l \frac{dx}{dt} = lv
\]
Giving us
\[
I = \frac{1}{R} |Blv| = \frac{1}{0.10 \, \Omega} |(0.20 \, \text{T})(0.05 \, \text{m})(50 \, \text{m/s})| = 5 \, \text{A}
\]

33.36

Now we are given a loop antenna that is 0.25 m in diameter. The plane of the loop is perpendicular to the oscillating magnetic field of a 150 MHz (an angular frequency of \( 2\pi \left( 150 \times 10^6 \right) \, \text{s}^{-1} \)) electromagnetic wave. The field through the loop is
\[
B = (20 \, \text{nT}) \sin \omega t = (20 \times 10^{-9} \, \text{T}) \sin \left( 2\pi \left( 150 \times 10^6 \right) t \right)
\]
The emf induced by this changing field will be given by Faraday’s law,
\[
\varepsilon = \left| \frac{d\Phi}{dt} \right| = \left| \frac{d}{dt}(BA) \right| = \left| A \frac{dB}{dt} \right| = \left| \pi r^2 \frac{dB}{dt} \right|
\]
where \( r = 0.125 \) m is the radius of the loop and \( \frac{dB}{dt} \) is given by
\[
\frac{dB}{dt} = \frac{d}{dt}[(20 \, \text{nT}) \sin \omega t] = (20 \, \text{nT}) \omega \cos \omega t = (20 \, \text{nT}) 2\pi \left( 150 \times 10^6 \right) \cos \left( 2\pi \left( 150 \times 10^6 \right) t \right)
\]
The maximum \( \frac{dB}{dt} \) will be when the cosine term is 1. Thus, the maximum \( \frac{dB}{dt} \) is
\[
\frac{dB}{dt}_{\text{max}} = (20 \, \text{nT}) 2\pi \left( 150 \times 10^6 \, \text{s}^{-1} \right)
\]
Therefore, the maximum emf induced is
\[
\varepsilon_{\text{max}} = \left| \pi r^2 \frac{dB}{dt}_{\text{max}} \right| = \left| \pi (0.125 \, \text{m})^2 (20 \times 10^{-9} \, \text{T}) 2\pi \left( 150 \times 10^6 \, \text{s}^{-1} \right) \right| = 0.925 \, \text{V}
\]

part b

If the loop is now turned 90° to to be perpendicular with to the oscillating field, then we have
\[
\varepsilon = \left| \frac{d\Phi}{dt} \right| = \left| \frac{d}{dt}(BA \cos (90°)) \right| = \cos (90°) \left| \frac{d}{dt}(BA) \right| = (0) \left| \frac{d}{dt}(BA) \right| = 0
\]
There is no emf generated when the magnetic field lines do not actually penetrate the surface.

33.37

We are now given a 50-turn, 0.04 m diameter coil with \( R = 0.50 \, \Omega \) surrounding a 0.02 m diameter solenoid. The solenoid is 0.2 m long and has 200 turns. The 60 Hz current through the solenoid is
\[
I_{\text{sol}} = (0.50) \sin (2\pi ft) = (0.50 \, \text{A}) \sin (2\pi (60 \, \text{Hz}) t)
\]
The magnetic field generated by that current will be
\[
B = \frac{\mu_0 N_{\text{sol}} I_{\text{sol}}}{L} = \frac{\mu_0 N_{\text{sol}}}{(0.2 \, \text{m})} \left(0.50 \, \text{sin} \left(2\pi ft \right) \right) = \frac{(1.26 \times 10^{-6} \, \text{Tm/A})(200 \, \text{turns})}{(0.2 \, \text{m})} (0.50 \, \text{A}) \sin (2\pi (60 \, \text{Hz}) t)
\]
The current induced in the coil $I_{\text{coil}}$ will be

$$I_{\text{coil}} = \frac{\varepsilon}{R} = \frac{1}{R} \left| \frac{d\Phi}{dt} \right| = \frac{1}{R} \left| \frac{d(NA)}{dt} \right| = \frac{A}{R} \left| \frac{dI_{\text{sol}}}{dt} \right| = \frac{A}{R} \left| \frac{\mu_0 N_{\text{sol}}}{L} \right| \left| \frac{dI_{\text{sol}}}{dt} \right| = \frac{A}{R} \left| \frac{\mu_0 N_{\text{sol}}}{L} \right| \left| (0.50) (2\pi f) \cos (2\pi ft) \right|$$

by Faraday’s law. Notice that the $A$ refers to the area of the solenoid (the only area with magnetic field lines through it) and not the area of the coil. Also, the flux through the coil is magnified by the number of turns $N$ of the coil. With $A = \pi (0.02 \text{ m})^2$, this gives us

$$I_{\text{coil}} = \frac{\pi (0.01 \text{ m})^2 (1.26 \times 10^{-6} \text{ Tm}/\text{A}) (50) (200) \left| (0.50 \text{ A}) (2\pi (60 \text{ s}^{-1})) \cos (2\pi (60 \text{ s}^{-1}) t) \right|}{0.2 \text{ m}}$$

$$= (7.46 \times 10^{-5} \text{ A}) \cos (2\pi (60 \text{ s}^{-1}) t)$$

33.46

We are now given a rail gun with magnetic field $B$. The switch closes at time $t = 0$ while the bar is at rest. What will the terminal velocity of the bar be? The bar will continue to move faster until no more current is flowing through it. That will happen when the induced emf (in opposition to the emf $\varepsilon_{\text{bat}}$ of the battery) is equal to $\varepsilon_{\text{bat}}$. Hence, we have

$$\varepsilon_{\text{bat}} = \varepsilon = \left| \frac{d\Phi}{dt} \right| = \left| \frac{d(NA)}{dt} \right| = B \left| \frac{dA}{dt} \right|$$

As before, we have $A = lx$ where $x$ is the $x$ coordinate of the bar. Therefore

$$\varepsilon_{\text{bat}} = B \left| \frac{dA}{dt} \right| = B \left| \frac{d(lx)}{dt} \right| = Bl \frac{dx}{dt} = Bl |v_{\text{term}}| = Bl v_{\text{term}}$$

or

$$v_{\text{term}} = \frac{\varepsilon_{\text{bat}}}{Bl}$$

part b

Putting numbers into this formula for the terminal velocity, we have

$$v_{\text{term}} = \frac{\varepsilon_{\text{bat}}}{Bl} = \frac{(1.0 \text{ V})}{(0.50 \text{ T}) (0.06 \text{ m})} = 33.3 \text{ m/s}$$

33.28

Now we have a 100 turn, 0.02 m diameter coil at rest in a horizontal plane. The $B$ field is inclined 60° away from the vertical axis and it increases in magnitude from 0.50 T to 1.50 T in 0.60 s. We want the induced emf of the coil assuming the increase is constant in time ($\frac{d\Phi}{dt} = \frac{\Delta \Phi}{\Delta t}$). Using Faraday’s law, we have

$$\varepsilon = \left| \frac{d\Phi}{dt} \right| = \left| \frac{\Delta \Phi}{\Delta t} \right| = \frac{NB_f A \cos (60) - NB_i A \cos (60)}{0.06 \text{ s}} = \frac{NA \cos (60)}{0.06 \text{ s}} |B_f - B_i|$$

$$= \frac{(100 \text{ turns}) (\pi (0.01 \text{ m})^2) \cos (60)}{0.06 \text{ s}} |(1.50 \text{ T}) - (0.50 \text{ T})| = 0.26 \text{ V}$$
Written Problem

Here we are given a wire loop in the xy plane. It has width $w$ in the x direction and height $h$ in the y direction with a magnetic field in the z direction of the form

$$B = \frac{B_0}{w}x$$

where $B_0$ is a constant. We would like the total magnetic flux through this loop. It is given as follows

$$\Phi = \oint \vec{B} \cdot d\vec{a} = \oint B da$$

because $\vec{B}$ is parallel to the area vector $d\vec{a}$. So

$$\Phi = \oint (B) da = \int_0^h \int_0^w \left(\frac{B_0}{w}x\right) dxdy = \int_0^h dy \int_0^w dx \left(\frac{B_0}{w}x\right) = \frac{B_0hw}{2}$$