

# Chaos Experiments at Microwave Frequencies

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# Two Types of Experiments

## “Quantum Chaos”

Basic study of wave dynamics in enclosed boxes

Relevant to EM interference and damage mechanisms

## “Classical Chaos”, addressing the question:

Does chaos enhance the susceptibility of electronic circuits to damage at low power levels?

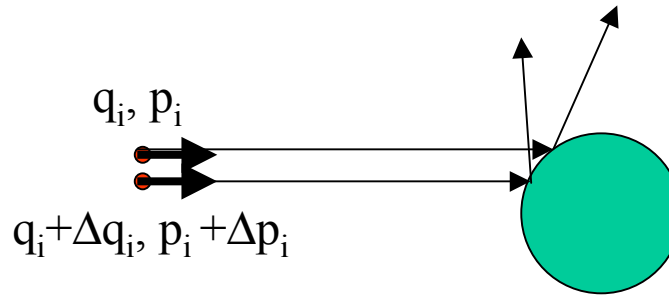
Continuation of work funded by STEP/STIC project

# Chaos

Classical: Extreme sensitivity to initial conditions

$$\dot{q}_i = \partial H / \partial p_i$$

$$\dot{p}_i = -\partial H / \partial q_i$$



Manifestations of classical chaos:

Chaotic oscillations, difficulty in making long-term predictions, sensitivity to noise, etc.

Quantum: ???

Heisenberg Uncertainty principle limits knowledge of initial conditions

$$\Delta p \Delta q > \hbar/2$$

$$\frac{1}{2m}(-i\hbar\nabla - qA)^2\Psi + V\Psi = i\hbar\frac{\partial\Psi}{\partial t}$$

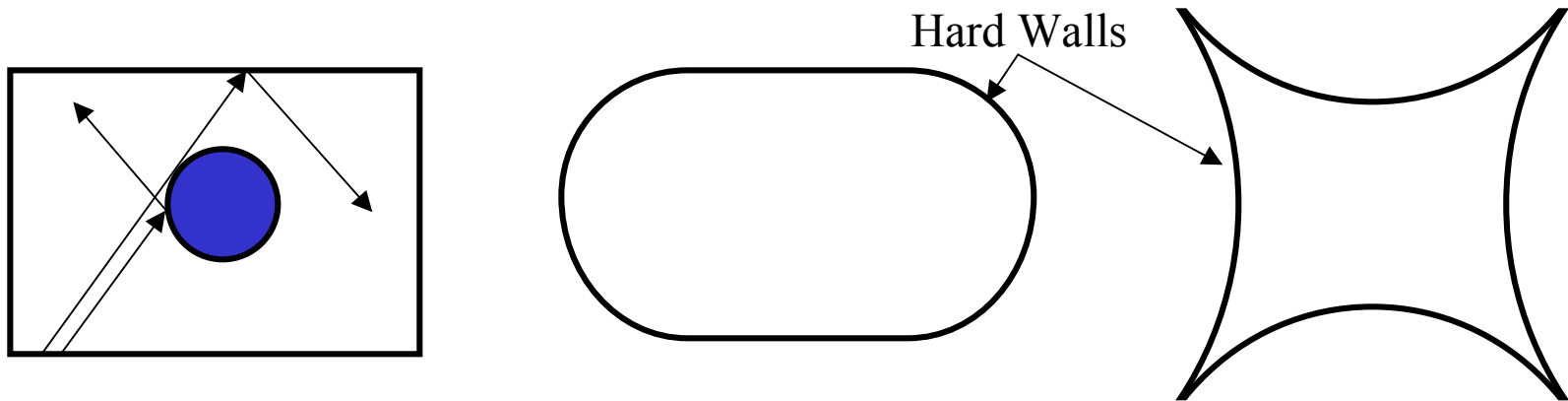
Manifestations of quantum chaos:

Breaking of degeneracy, Scars, Strong eigenfunction fluctuations



## Wave Chaos in Bounded Regions

Consider a two-dimensional infinite square-well potential (i.e. a box) which shows chaos in the classical limit:



Now solve the electromagnetic wave equation (or Schrodinger equation) in the same potential well

Examine the solutions in the semiclassical regime:  $\lambda \ll L$

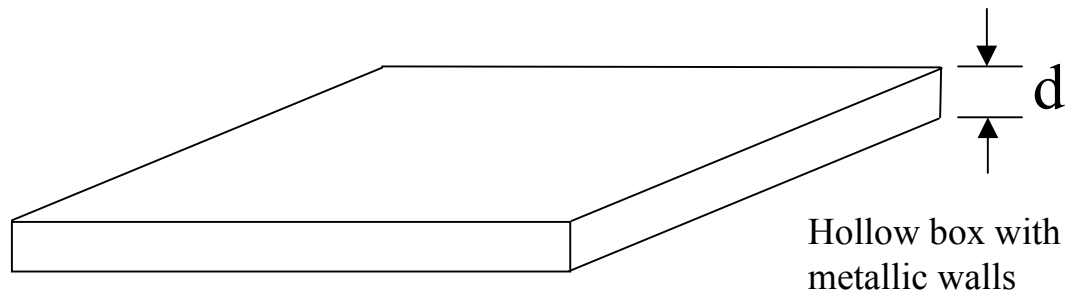
What will happen?



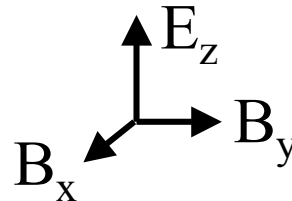
# Principle of Measurement



- Exploit the Helmholtz - Schrodinger Analogy in a “two-dimensional” electromagnetic resonator



Only transverse magnetic (TM) propagate for  $f < c/2d \sim 19$  GHz, in our case



$$\begin{aligned} \nabla^2 \psi_n + 2m(\epsilon_n - V) \psi_n &= 0 \\ \psi_n &= 0 \text{ at boundaries} \end{aligned}$$

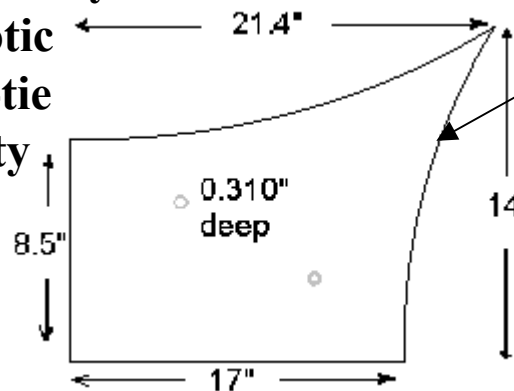
$$\begin{aligned} \nabla^2 E_z + k_i^2 E_z &= 0 \\ E_z &= 0 \text{ at boundaries} \end{aligned}$$



# Experiment



Classically  
Chaotic  
Bow-tie  
Cavity



“Hard Walls”

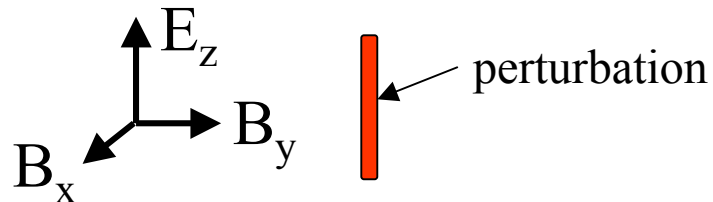
$$\nabla^2 \psi_n + 2m(\epsilon_n - V)/\hbar^2 \psi_n = 0$$

$\psi_n = 0$  at boundaries

$$\nabla^2 E_z + k_i^2 E_z = 0$$

$E_z = 0$  at boundaries

- Excite an eigenmode of the cavity with frequency  $\omega_0$
- Scan Perturbation through the cavity
- Measure  $\omega$  at each point
- Use :  $\omega^2 = \omega_0^2 (1 + \int (|B|^2 - |E|^2) dV_p)$  , to get  $|E|^2$  (Slater)





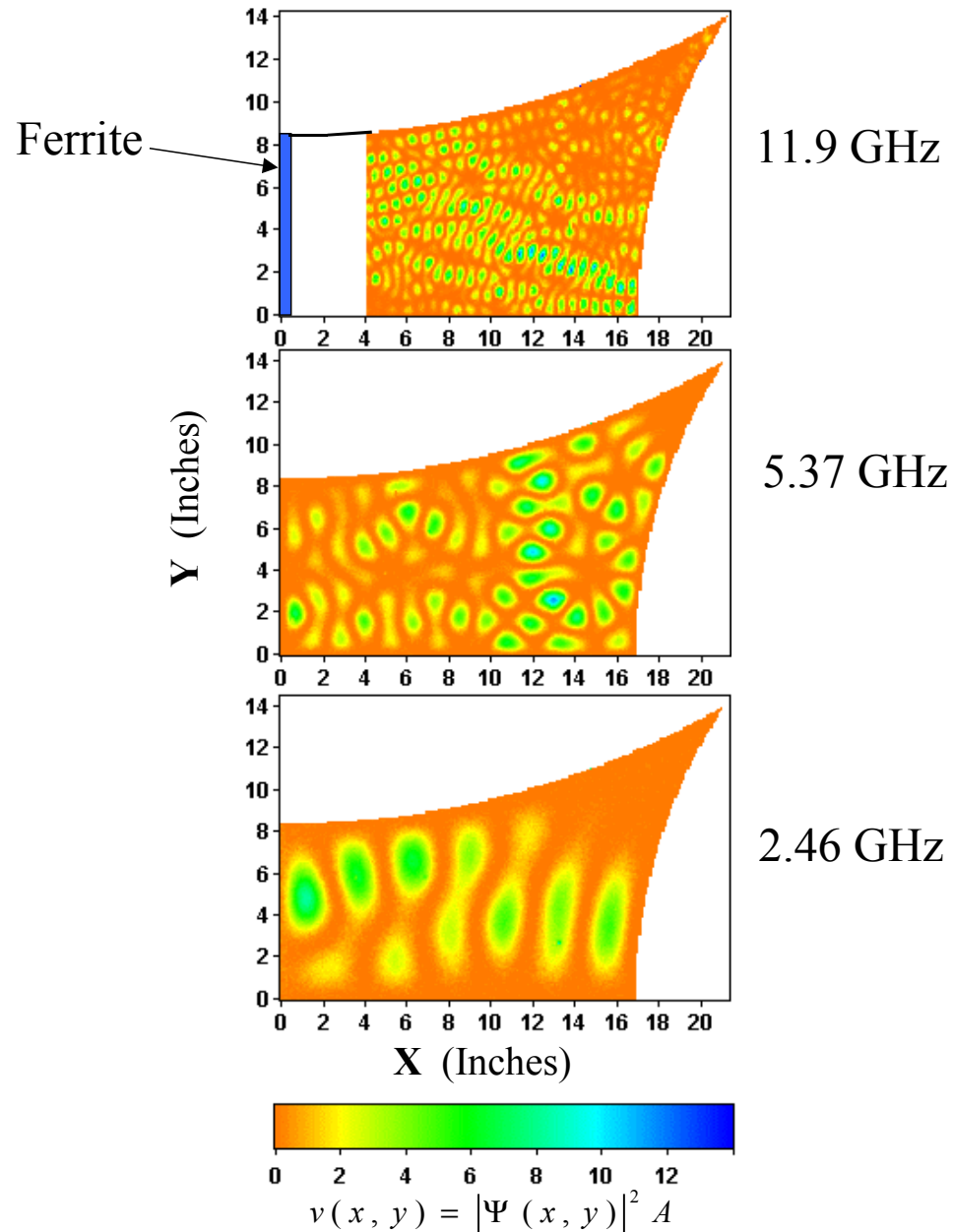
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## Eigenfunctions

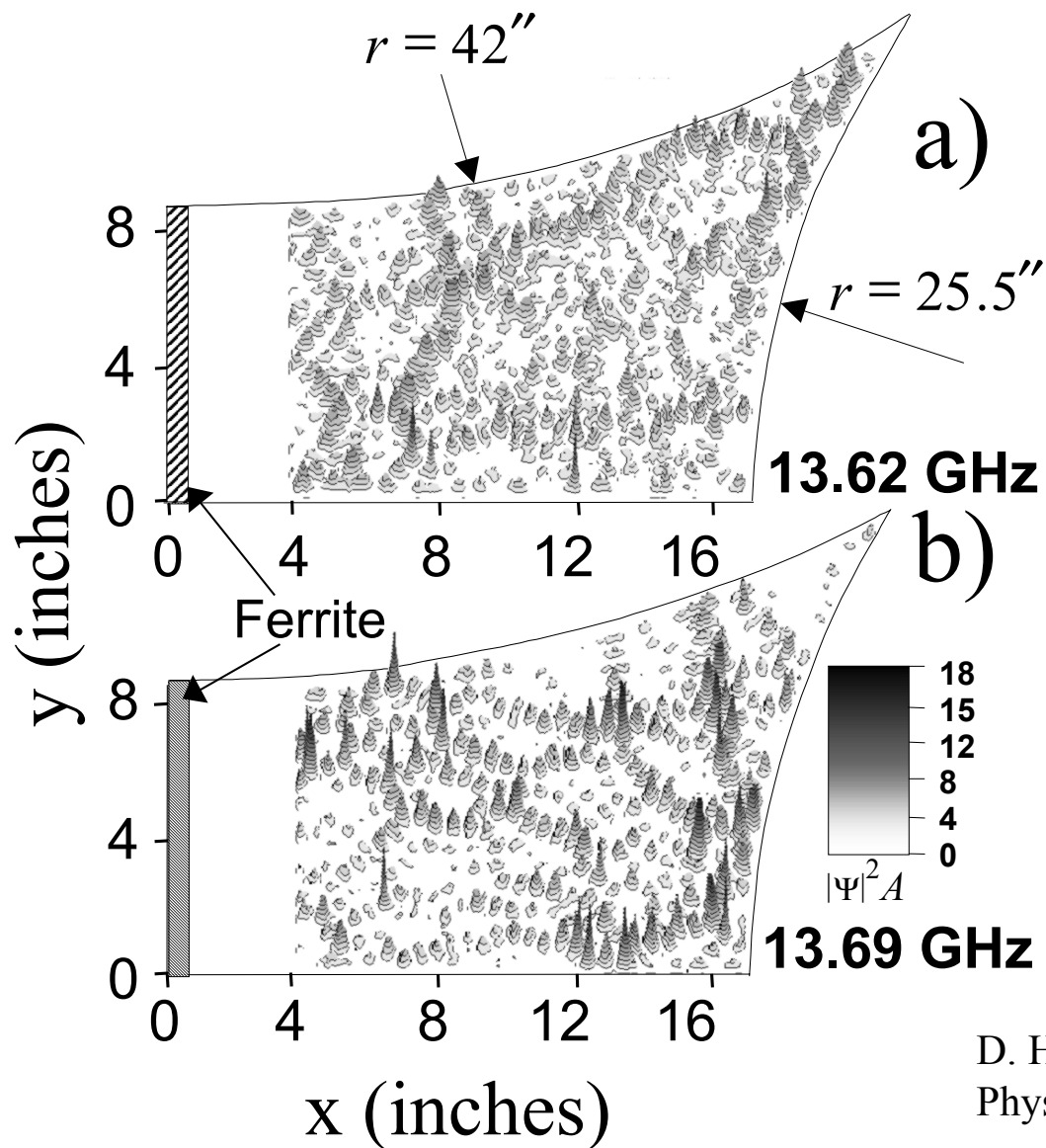
Quarter bow-tie cavity

A magnetized ferrite (top Fig.)  
breaks time-reversal symmetry  
for the microwaves





# Wave Chaotic Eigenfunctions with and without Time Reversal Symmetry



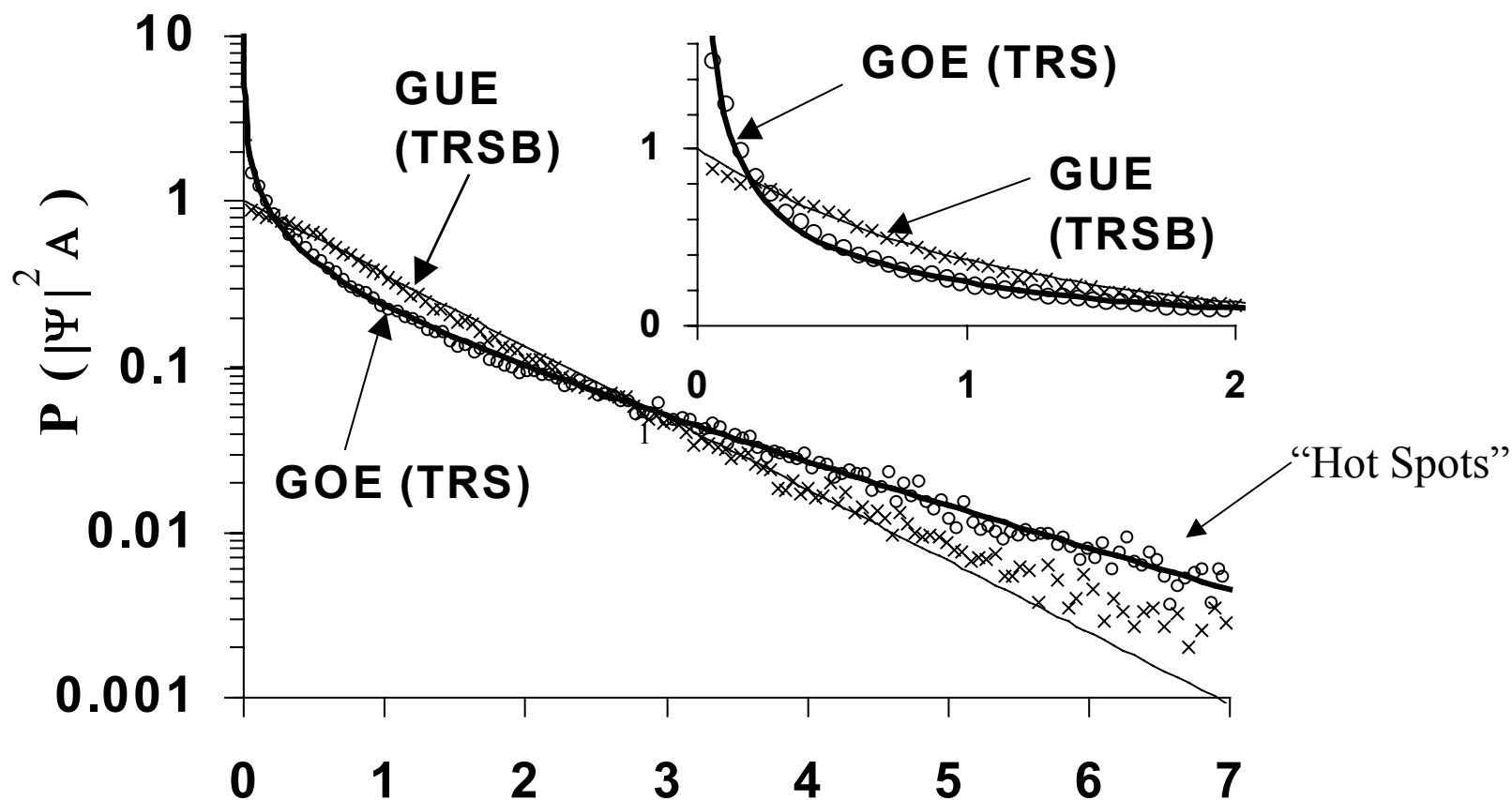
TRS Broken  
(GUE)

TRS  
(GOE)

D. H. Wu and S. M. Anlage,  
Phys. Rev. Lett. 81, 2890 (1998).



# Probability Amplitude Distribution with and without Time Reversal Symmetry



$$P(v) = \begin{cases} (2\pi v)^{-1/2} e^{-v/2} & \text{TRS (GOE)} \\ e^{-v} & \text{TRSB (GUE)} \end{cases} \quad |\Psi|^2 A$$

D. H. Wu and S. M. Anlage,  
Phys. Rev. Lett. 81, 2890 (1998).



# Wave Chaos Experiments



## Cavity with dielectric slab

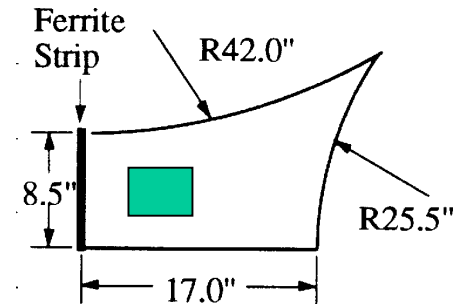
Investigate wave chaos at the length scale  
of a pc board

Ray splitting

Fields confined mainly in slab

Image  $|E_z|^2$  under and around the slab

Add loss to the slab: when does the eigenfunction picture break down?



Make a connection to device-level studies:

start with low-loss dielectric

- blank pc board
- pc board with Cu ground plane
- pc board with interconnects, passive circuit elements
- pc board with active elements (nonlinear circuits!)



# Other Wave Chaos Experiments

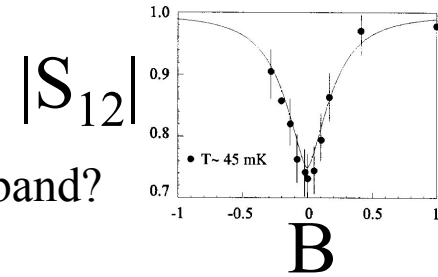


## Magnetic Field Dependence of Standing Wave Characteristics

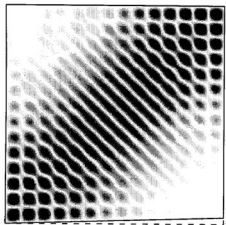
How are the  $|E|^2$  maxima decreased with applied  $B$ ?

“Weak Localization”

Can the standing wave suppression be made broad-band?



## Localized modes in trapezoidal cavities

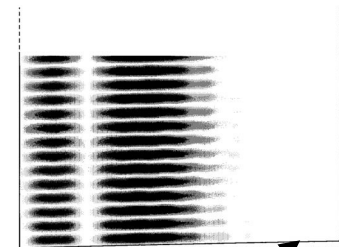


Investigate the effects of slight irregularities in the shape

Square/rectangular cavity + wedge

Enhanced  $|E_z|^2$  mode near wedge - calculated by Prange

wedge

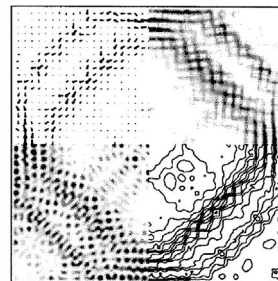
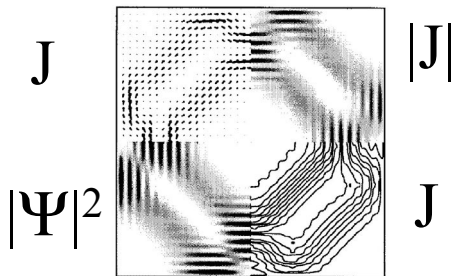


wedge

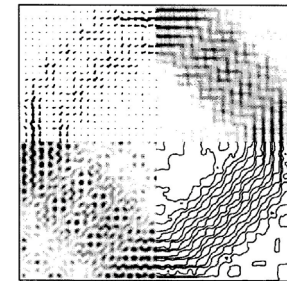
Calculated by  
Zaitsev, *et al.*

## Circulating modes in cavities with magnetized ferrite

Square cavity + magnetized ferrite show circulating currents



Diamagnetic



Diamagnetic

Calculated by  
Narevich.  
PRE 2000



# Objectives of Classical Chaos Experiments



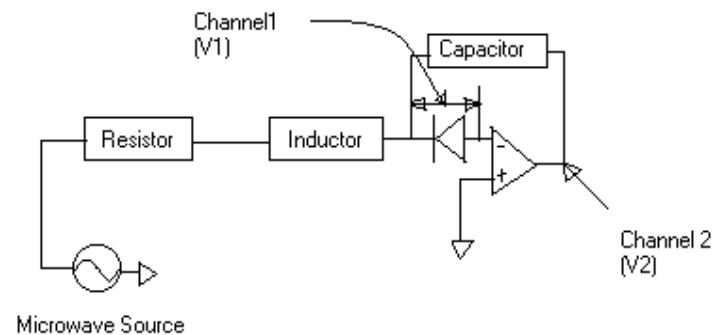
- Investigate the idea that chaos suppresses the threshold for damage in nonlinear circuits
  - Continue to examine the series R-L-Diode-Op-Amp circuit
    - Does period doubling lead to damage of components?
- Investigate the effects of high power rf signals on nonlinear circuits
  - RF-induced chaos may bring about a lowering of damage thresholds of electronic devices.
- Simulate the behavior of nonlinear circuits under these conditions



# Brief History of the Problem



- C. Wallace (TRW) claims that period-doubling transition in this circuit leads to op-amp failure at absorbed power levels far below thermal failure limits:



900 MHz  
100 W input  
70 mW absorbed

Wallace's observation: "violent chaotic oscillation state and failure of the op-amp, in which the output voltage went to zero and stayed there while the diode remained perfectly functional."

- Our experiments on a similar circuit at 10 - 60 MHz, 0.1 W input showed period-doubling and chaos, but no component failure.
- Jaycor/MRC examined a similar circuit up to 3 W and found no period-doubling or chaos, but did observe op-amp and diode burnout due to thermal effects



# Classical Chaos Experiments in Progress

Understand why the results are so sensitive to measurement conditions, power levels, etc. Arrive at a consensus

Employ high-impedance voltage probes to minimize perturbation of circuit

Establish 100 W, 900 MHz measurement setup

Monitor op-amp supply voltage and current. Preliminary results show significant changes at the onset of period-doubling

Identify a simpler circuit which shows the essential behavior



# Conclusions



Wave Chaos offers insights into the effects of microwaves in enclosures

Statistical properties of eigenvalues and eigenfunctions

Most results independent of box shape

Benefits of breaking Time-Reversal Symmetry

Ray Splitting

Does chaos lower the threshold for damage to electronic components?

Develop a consensus on experimental results

Is there a simpler circuit which shows the effect unambiguously?

Generalize the results

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