Wave Chaos and Coupling to EM Structures

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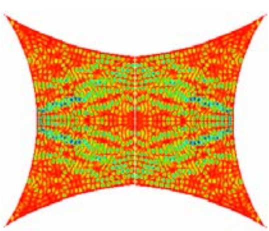

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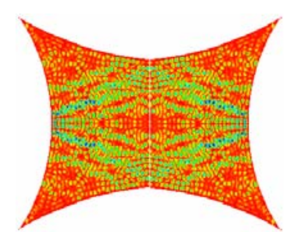
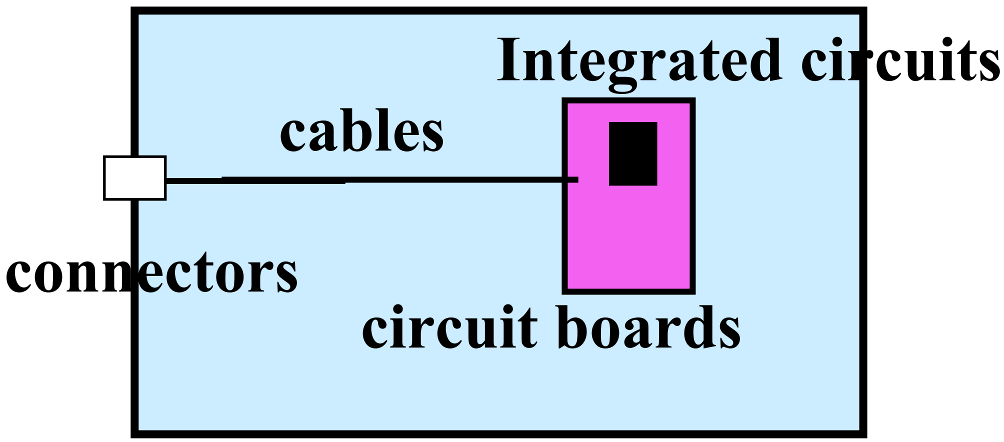


Electromagnetic Coupling in Computer Circuits

Schematic



• Coupling of external radiation to computer circuits is a complex process:

apertures
resonant cavities
transmission lines
circuit elements

• What can be said about coupling without solving in detail the complicated EM problem ?

• Statistical description !

Outline

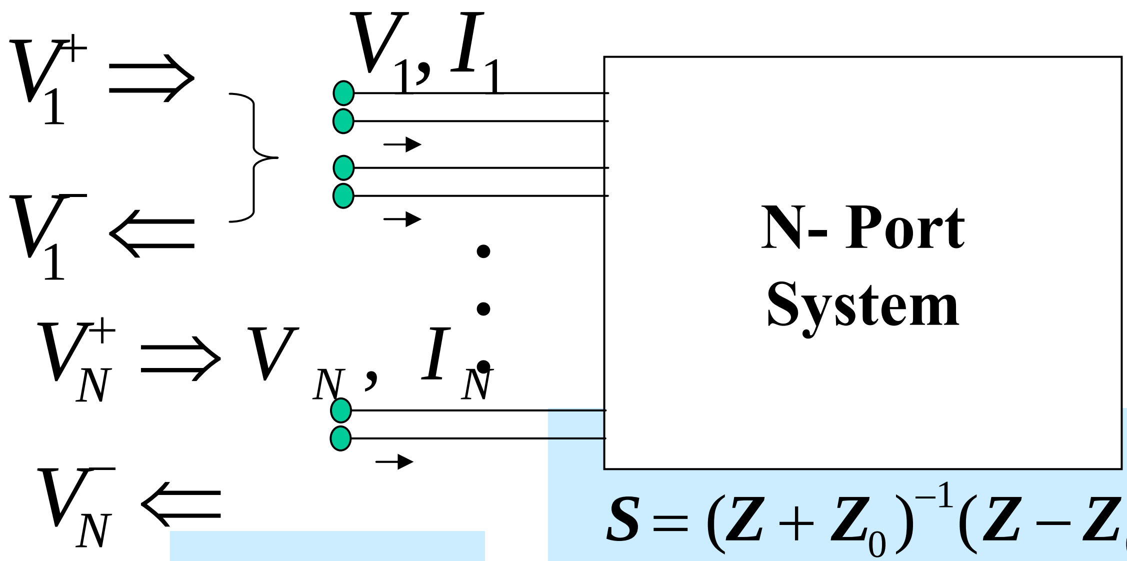
- **Part I: Frequency Domain**
  - **Extracting the universal impedance and scattering statistics**
  - **Predictions and tests**
- **Part II: Time Domain**
  - **Model**
  - **Predictions**

Part I:

Frequency Domain

Z and S-Matrices
What is S_{ij} ?

N ports
- voltages and currents,
- incoming and outgoing waves



$$S = (Z + Z_0)^{-1}(Z - Z_0)$$

Z matrix

$$\begin{pmatrix} V_1 \\ V_2 \\ \vdots \\ V_N \end{pmatrix} = Z \begin{pmatrix} I_1 \\ I_2 \\ \vdots \\ I_2 \end{pmatrix}$$

voltage **current**

S matrix

$$\begin{pmatrix} V_1^- \\ V_2^- \\ \vdots \\ V_{N1}^- \end{pmatrix} = S \begin{pmatrix} V_1^+ \\ V_2^+ \\ \vdots \\ V_{N1}^+ \end{pmatrix}$$

refl. **inc.**

$Z(\omega)$ $S(\omega)$

- Complicated function of frequency
- Details depend very sensitively on unknown parameters.

Statistical Model of Z Matrix

$$\underline{\underline{Z}}(\omega) = -\frac{j}{\pi}\sum_n \underline{\underline{R}}_R^{1/2}(\omega_n)\frac{\underline{w}_n \underline{w}_n^T \Delta\omega_n^2}{\omega^2(1-jQ^{-1})-\omega_n^2}\underline{\underline{R}}_R^{1/2}(\omega_n)$$

$\underline{\underline{Z}}$ = **MxM matrix**

$\underline{\underline{R}}_R$ = **MxM radiation resistance matrix**

$\Delta\omega_n^2$ = **Mean spectral spacing**

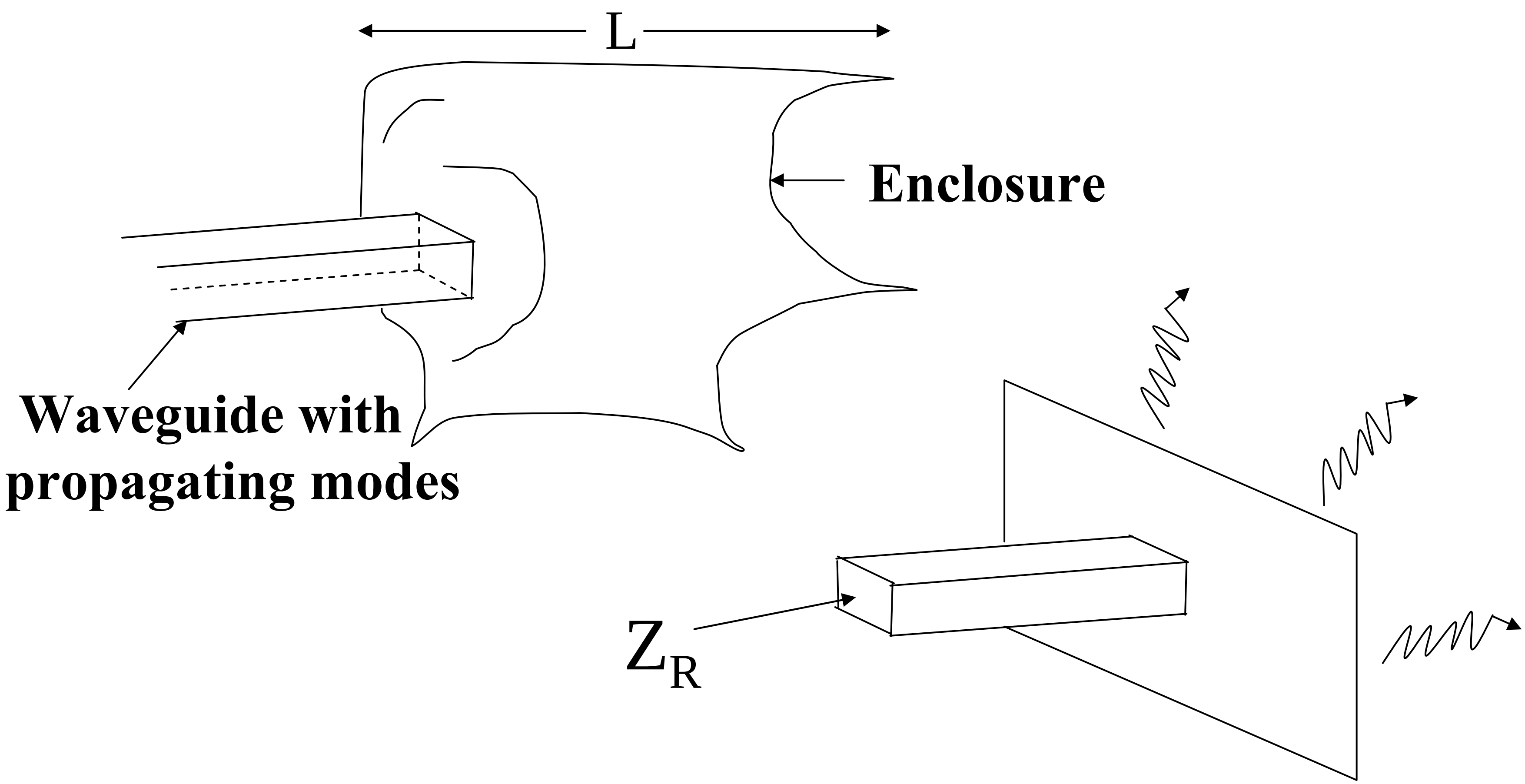
Q = **Quality factor**

ω_n^2 = **Random Spectrum from RMT**

$\underline{w}_n$ = **M vector of Gaussian random variables**

$$\left\langle \underline{w}_n \underline{w}_n^T \right\rangle = \underline{\underline{1}}$$

Radiation Impedance



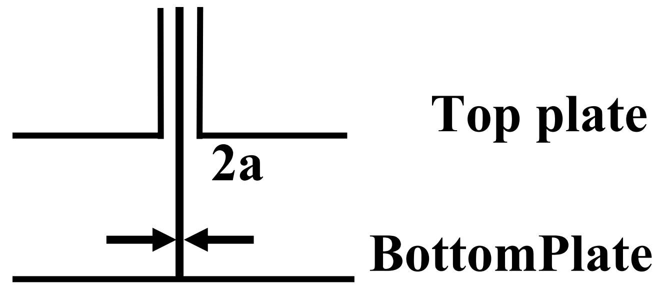
$$\underline{\underline{Z}}_R = \underline{\underline{R}}_R + j\,\underline{\underline{X}}_R$$

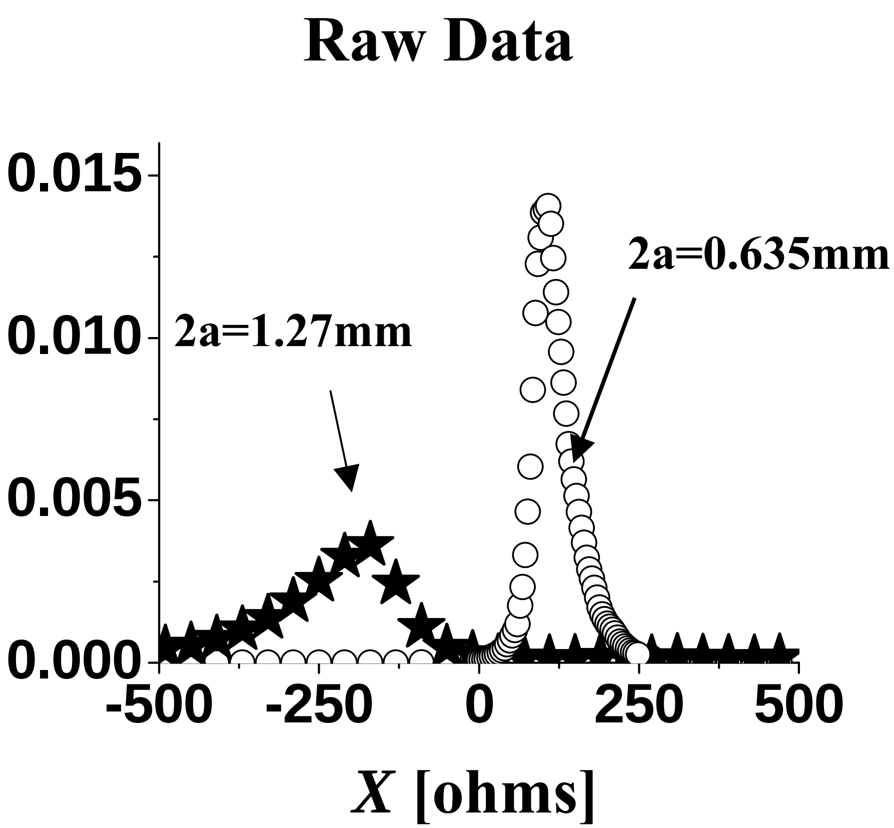
Universal Properties of Impedance

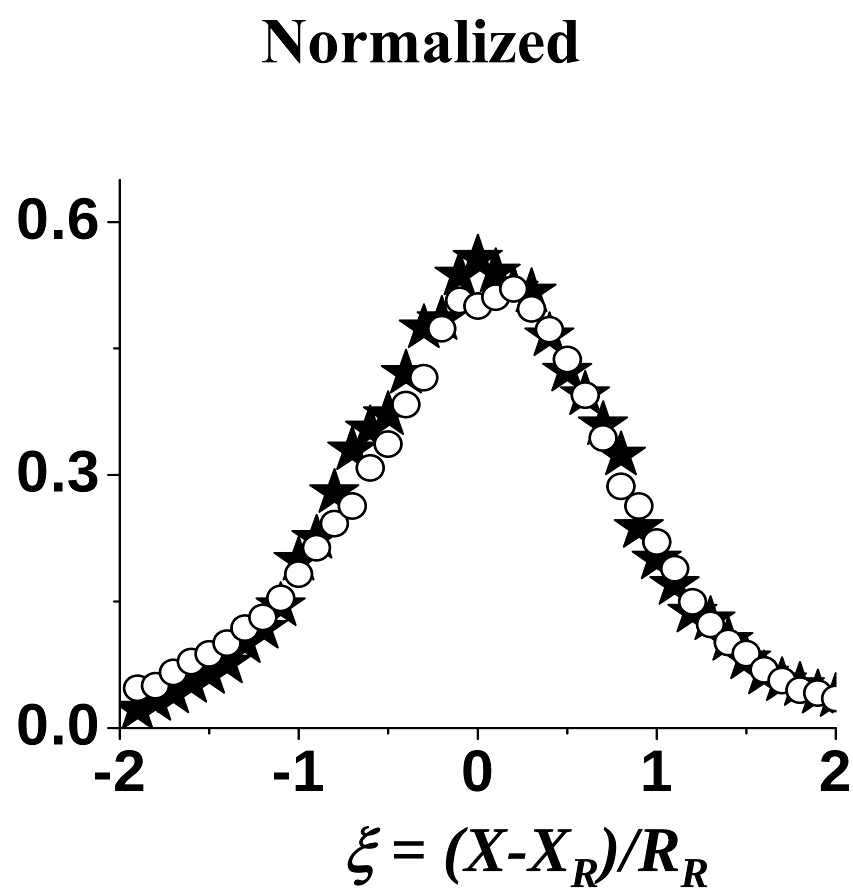
$$\underline{\underline{\xi}} = \underline{\underline{R}}_R^{-1/2}(\underline{\underline{Z}} - j\underline{\underline{X}}_R)\underline{\underline{R}}_R^{-1/2}$$

- **ξ is universal and obtainable from Random Matrix Theory.**
- **This applies for (λ/L) small.**

Importance of Normalization







Past Results

- Predictions for statistics of S and Z matrices.
- Tests of predictions against numerical solutions of Maxwell's equations.
- Tests of predictions against laboratory experiments.

Predictions Tested

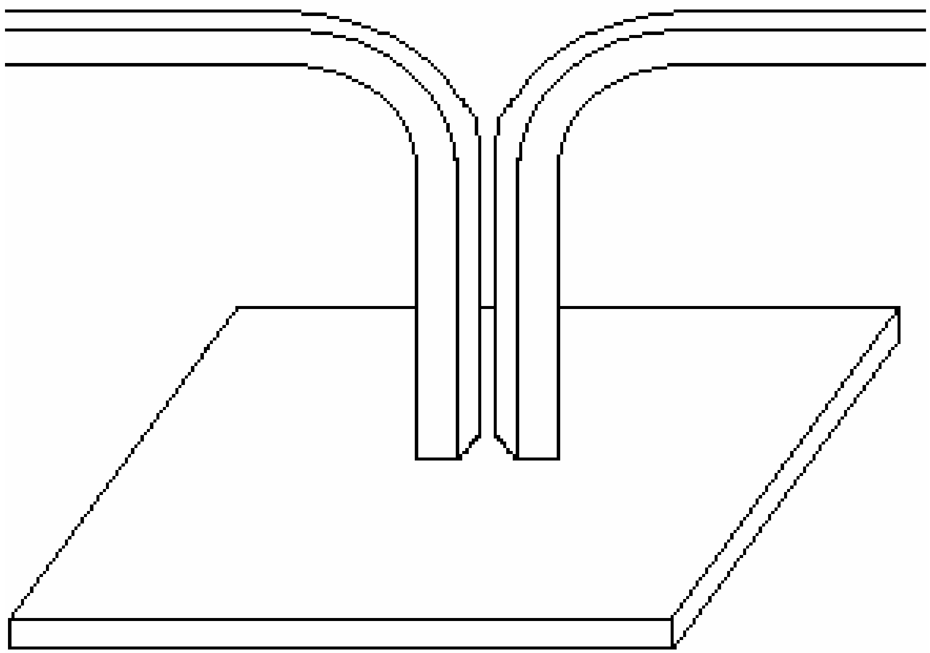
- **M=1**
  - **PDF's of normalized impedances and scattering coefficients as a funct. of loss.**
- **M=2**
  - **Statistics of normalized 2x2 impedance and scattering matrices as a funct. of loss.**
  - **Variance ratios as a funct. of loss [also Fiachetti and Michielsen, Elect. Lett. '03].**

$$VR_Z = \frac{Var[Z_{12}]}{\sqrt{Var[Z_{11}]Var[Z_{22}]}}$$

Some Predictions Not Yet Tested

- Situations where reciprocity does not apply
  - Magnetized ferrite → different statistics

- Situations where off-diagonal elements of Z_R are significant.



Some Predictions Not Yet Tested (continued)

- **M>2: E.g.,**

reciprocal

$$\left\langle \left| S_{ij} \right|^2 \right\rangle = \begin{cases} \dfrac{2}{M+1}, & i = j \\ \dfrac{1}{M+1}, & i \neq j \end{cases}$$

non-reciprocal

$$\left\langle \left| S_{ij} \right|^2 \right\rangle = \frac{1}{M}$$

Part II:

Time Domain

Time Domain Model

Frequency Domain

w_n- Guassian Random variables

$$Z(\omega) = -\frac{j\omega}{\pi}\sum_n \frac{R_R(\omega_n)}{\omega_n}\frac{\Delta\omega_n^2 w_n^2}{\omega^2(1-jQ^{-1})-\omega_n^2}$$

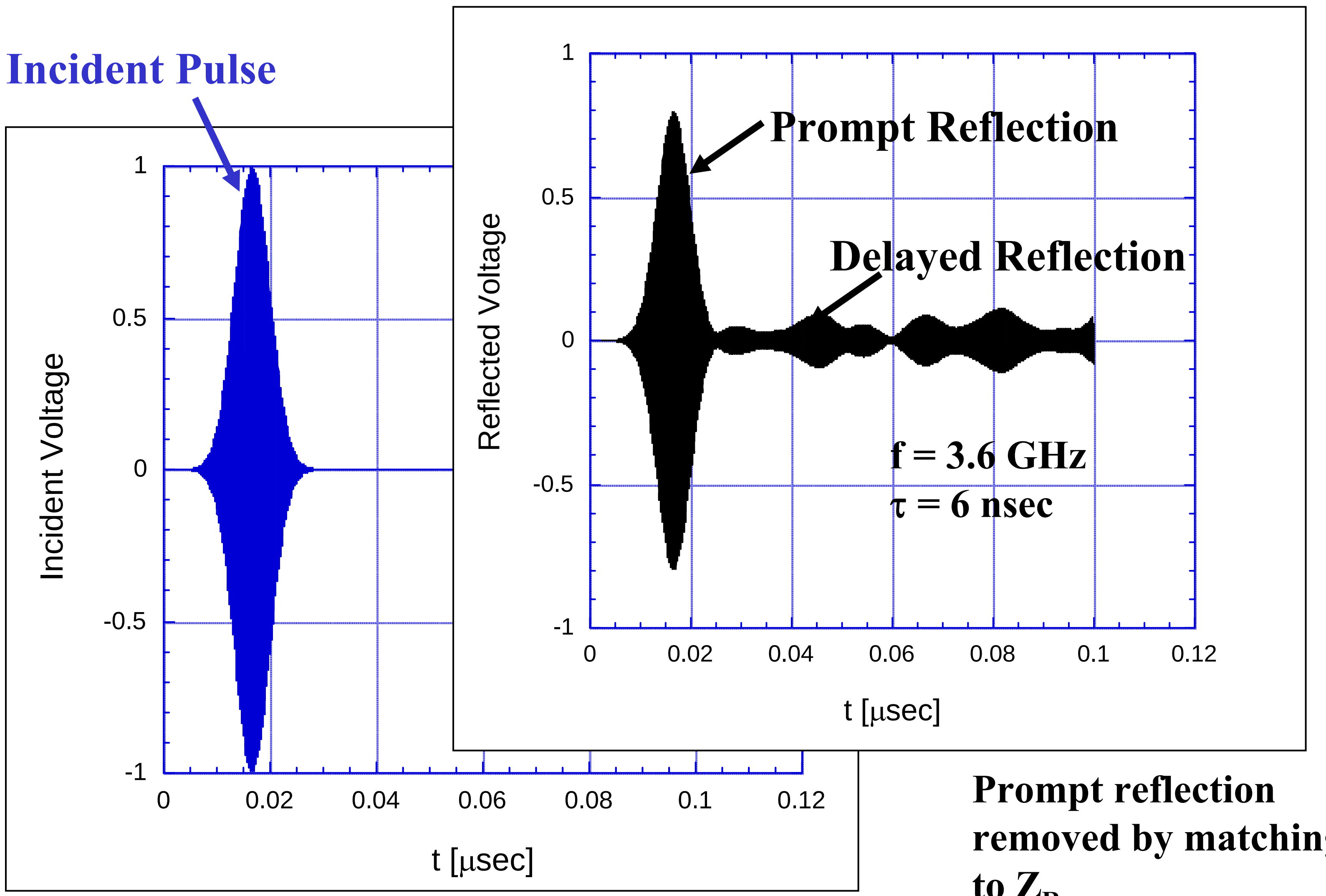
ω_n - random spectrum

Time Domain

$$\left(\frac{d^2}{dt^2} + 2\nu_n\frac{d}{dt} + \omega_n^2\right)V_n(t) = -\frac{1}{\pi}\frac{R_R(\omega_n)\Delta\omega_n^2 w_n^2}{\omega_n}\frac{d}{dt}I(t)$$

$$V(t) = \sum_n V_n(t) \qquad \nu_n = \frac{\omega_n}{Q}$$

Incident and Reflected Pulses for One Realization



Prompt reflection removed by matching Z_0 to Z_R

Decay of Port Voltage - Lossless Case

- **One Port with an Incident Pulse:** $\left\langle V^2(t) \right\rangle \approx 1/t^{5/2}$

- **Two Ports Excited Through Port 1,**

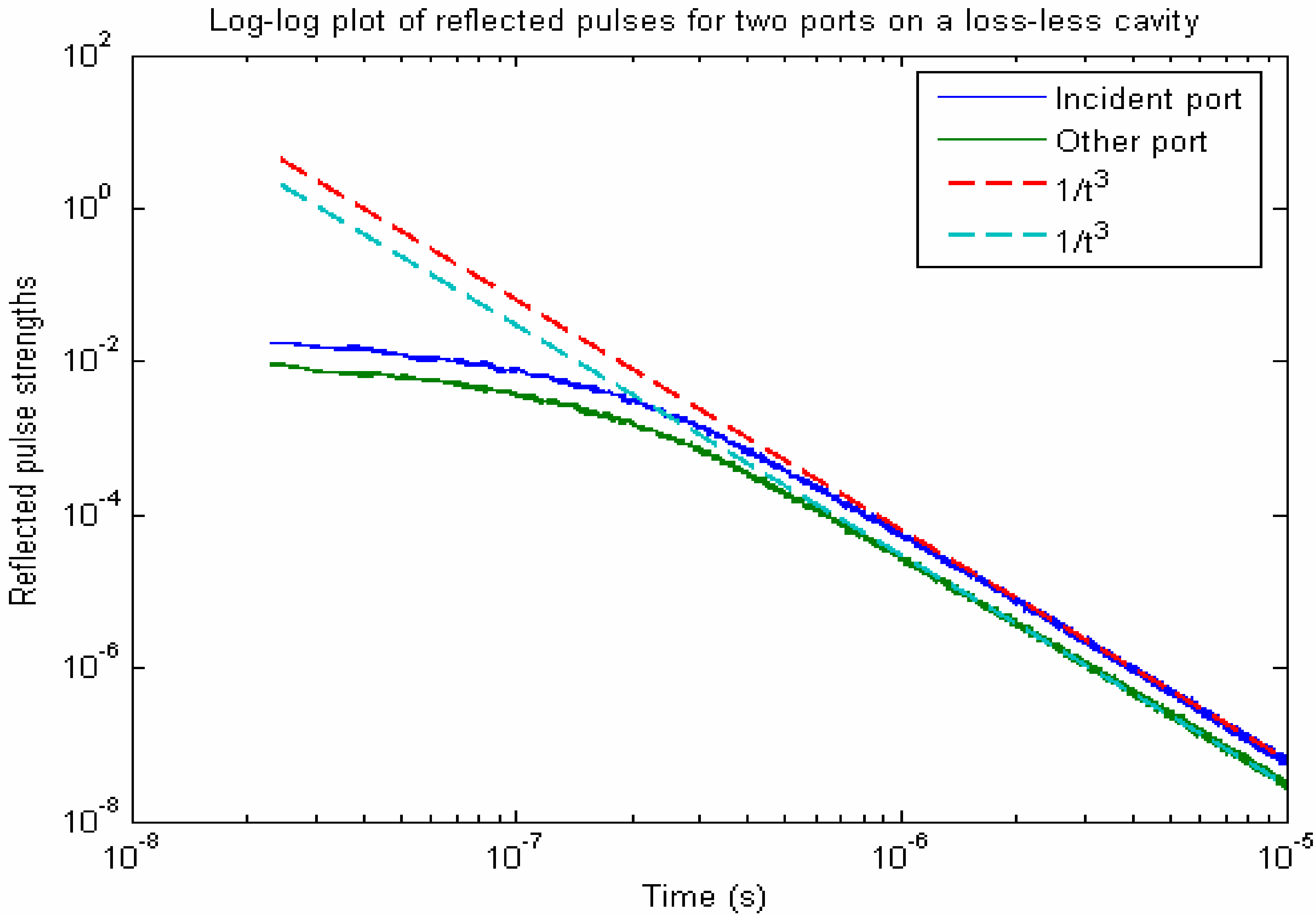
  a) all ports matched: $\left\langle V_1^2(t) \right\rangle = 2\left\langle V_2^2(t) \right\rangle \approx 1/t^3$

  b) Port 1 matched Port 2 strongly mismatched $\left\langle V_1^2(t) \right\rangle \approx 1/t^{5/2}$

  $\left\langle V_2^2(t) \right\rangle \approx 1/t^{3/2}$

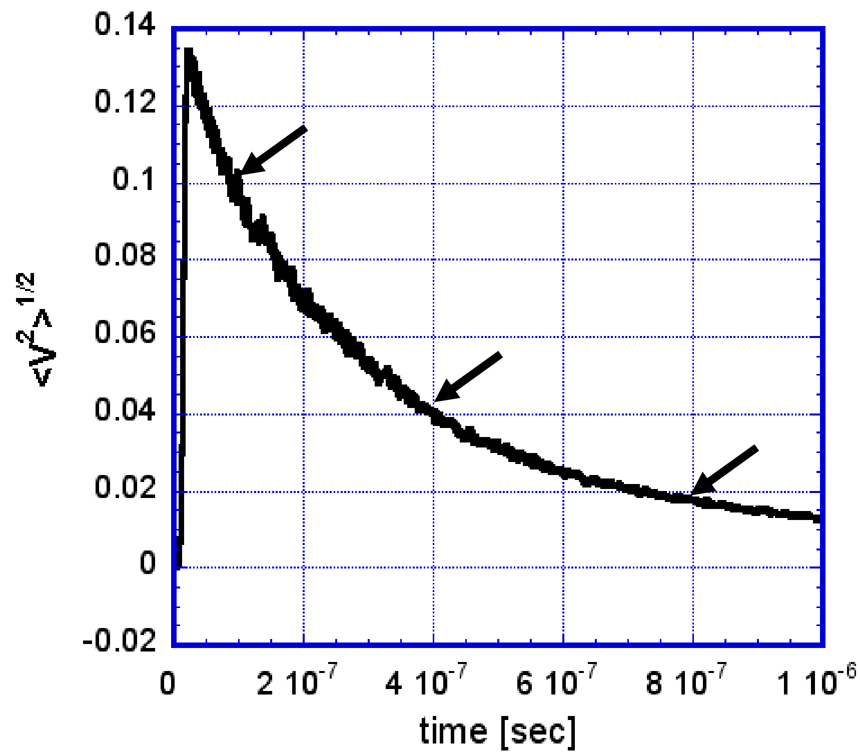
- **N Ports Excited Through Port 1, all ports matched:** $\left\langle V_1^2(t) \right\rangle = 2\left\langle V_{i\neq 1}^2(t) \right\rangle \approx 1/t^{(4+N)/2}$

Simulations of Average Decay

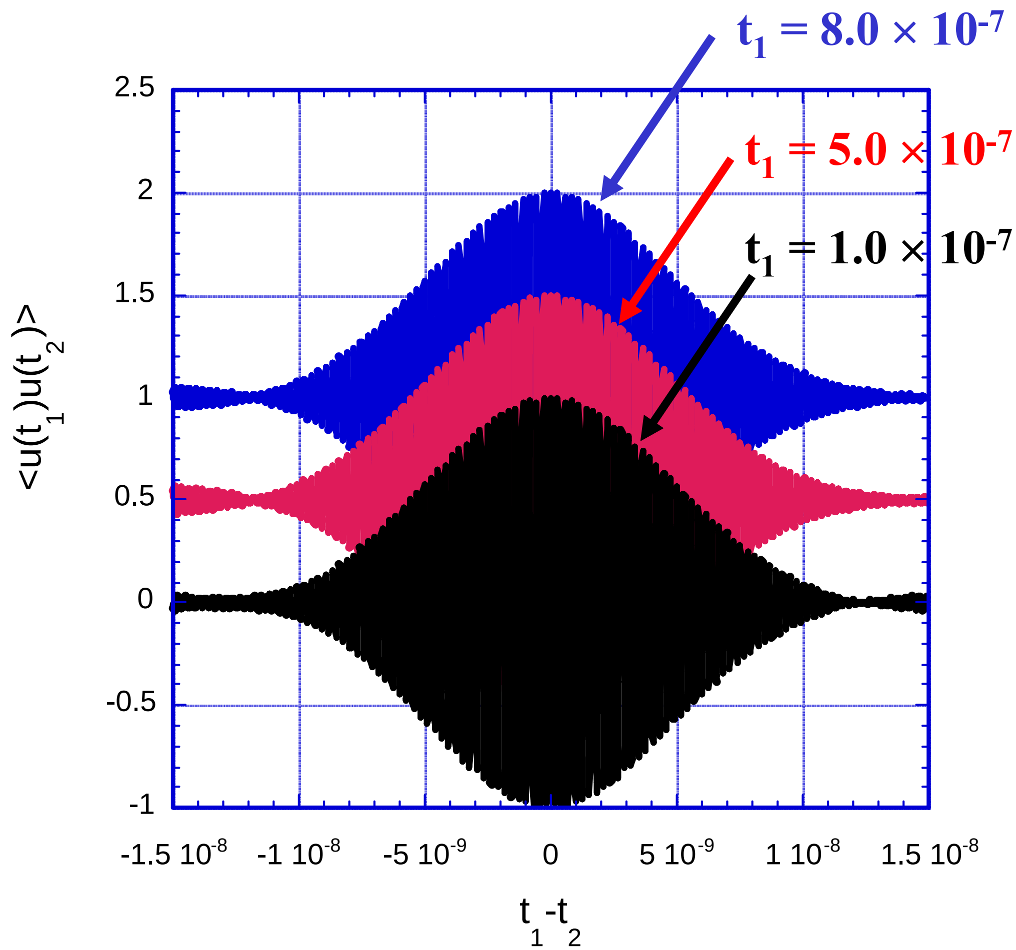


Quasi-Stationary Process

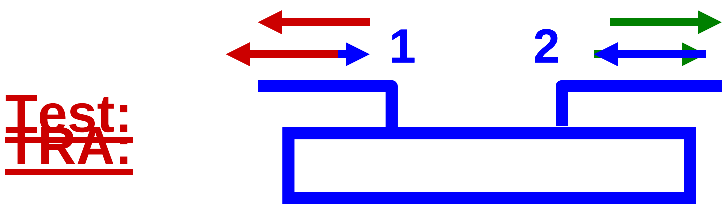
Normalized Voltage
$u(t)=V(t) / \langle V^2(t) \rangle^{1/2}$

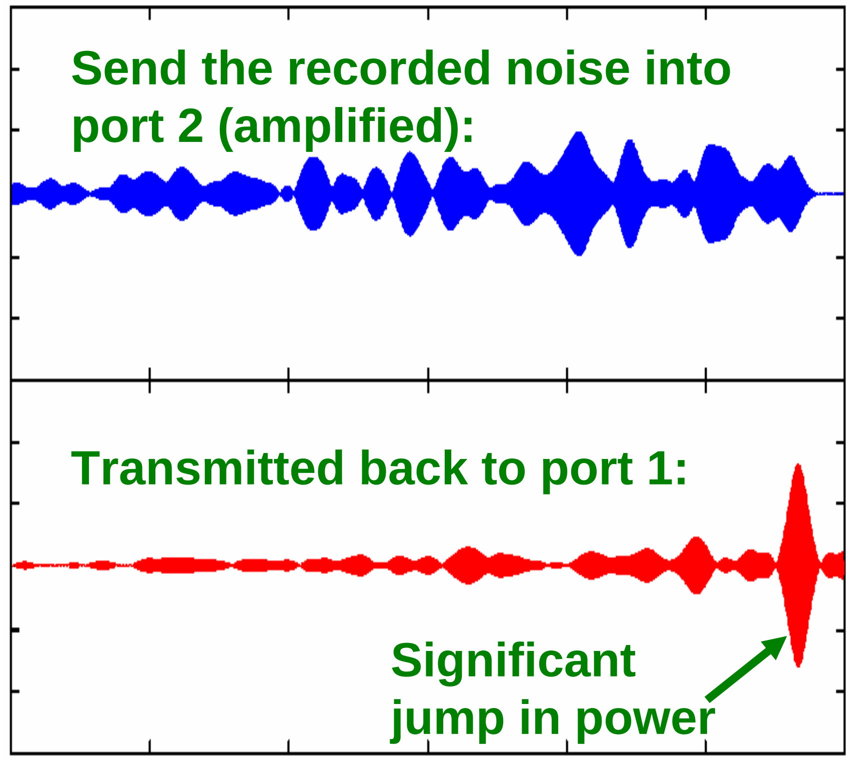


2-time Correlation Function (Matches initial pulse shape)



Time Reversal Attack (TRA)





Issues:
- **‘Fidelity’ under study**
- **Magnetized ferrite: breaks time reversal symmetry**

Future TheoreticalWork:

- Corrections for deviations from RMT that occur when $(\lambda/L) \ll 1$ is not well satisfied
- Scars - “Anomalous” hot spots
- Networks formed by transmission line links
- Statistical aspects of coupling of pulsed signals

Publications

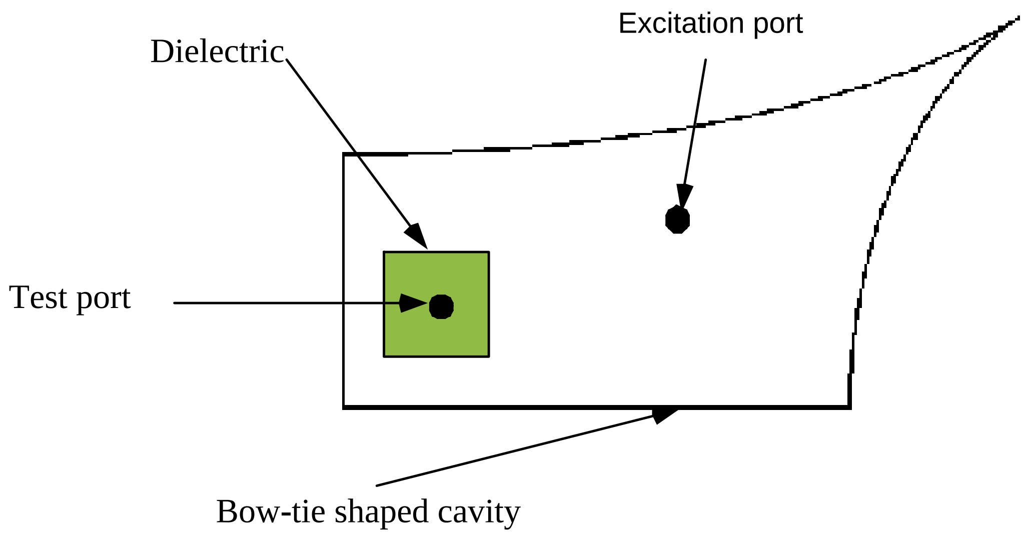
1. *S. Hemmady, X. Zheng, E. Ott, T. Antonsen, and S. Anlage, Universal Impedance Fluctuations in Wave Chaotic Systems, Phys. Rev. Lett. 94, 014102 (2005).*
2. *S. Hemmady, X. Zheng, T. Antonsen, E. Ott, and S. Anlage, Universal Statistics of the Scattering Coefficient of Chaotic Microwave Cavities, Phys. Rev. E 71*
3. *X. Zheng, T. Antonsen, E. Ott, Statistics of Impedance and Scattering Matrices in Chaotic Microwave Cavities: Single Channel Case, Electromagnetics 26, 3 (2006).*
4. *X. Zheng, T. Antonsen, E. Ott, Statistics of Impedance and Scattering Matrices of Chaotic Microwave Cavities with Multiple Ports, Electromagnetics 26, 37 (2006).*
5. *X. Zheng, S. Hemmady, T. Antonsen, S. Anlage, and E. Ott, Characterization of Fluctuations of Impedance and Scattering Matrices in Wave Chaotic Scattering, Phys. Rev. E 73 , 046208 (2006).*
6. *S. Hemmady, X. Zheng, T. Antonsen, E. Ott, S. Anlage, Universal Properties of 2-Port Scattering, Impedance and Admittance Matrices of Wave Chaotic Systems, Phys. Rev. E. submitted.*
7. *S. Hemmady, X. Zheng, T. Antonsen, E. Ott and S. Anlage, Aspects of the Scattering and Impedance Properties of Chaotic Microwave Cavities, Acta Physica Polonica A 109, 65 (2006).*

Photo by Tom Antonsen

Part III:

Open Problems

More Complexity in the Scatterer

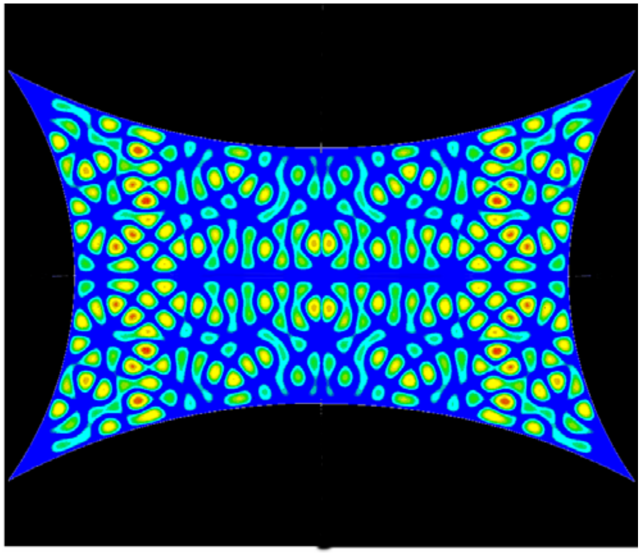


• **Can be addressed**

-theoretically

-numerically

-experimentally



Features:

Ray splitting

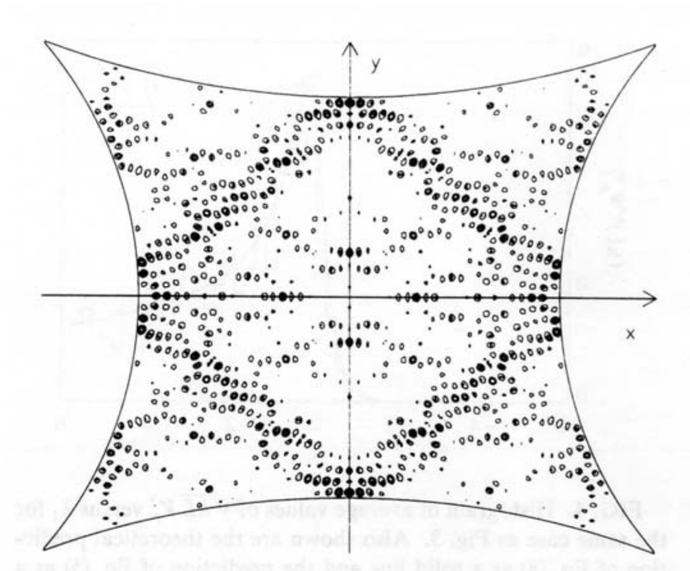
Losses

Additional complications can be added

Role of Scars?

- Eigenfunctions that do not satisfy the random plane wave assumption

- Scars are not treated by either random matrix or chaotic eigenfunction theory

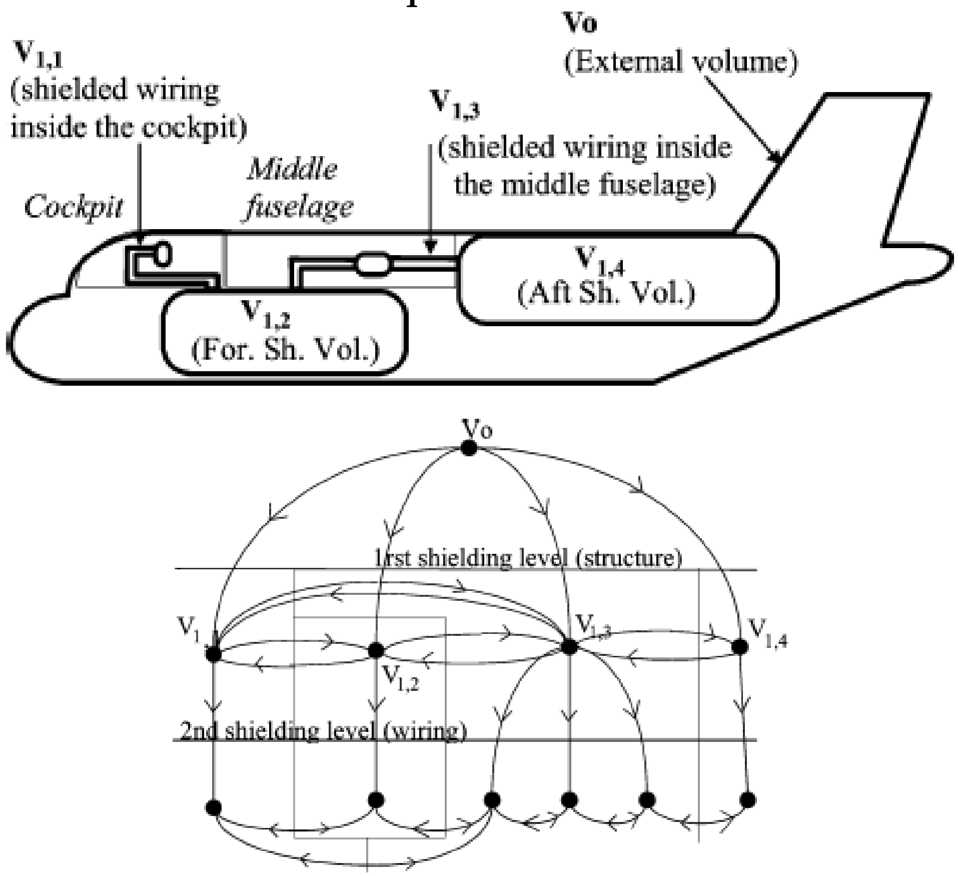


Bow-Tie with diamond scar

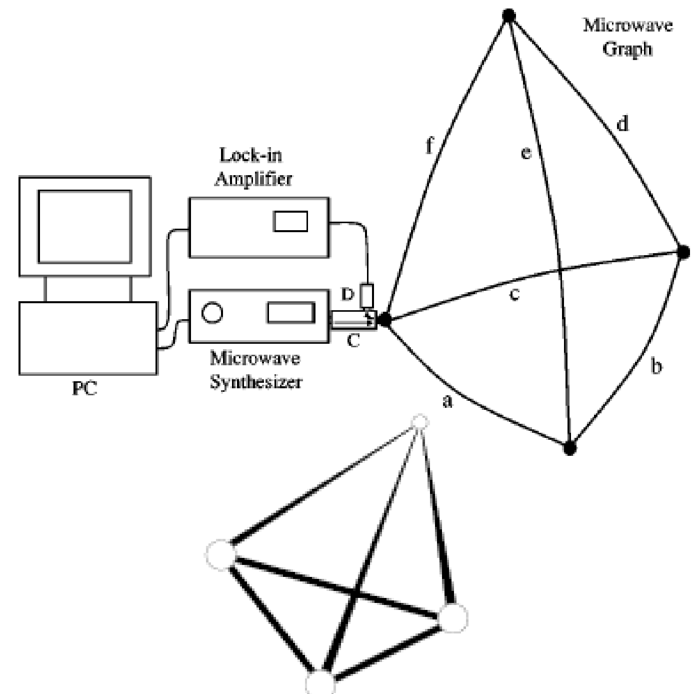
Ref: Antonsen et al., Phys. Rev E 51, 111 (1995).

Electromagnetic Topology andWave Graphs

Electromagnetic Topology
BLT Equations



J.-P. Parmantier, IEEE Trans. Electromag. Compat. 46 (3) 359-367 (2004). "Numerical coupling models for complex systems and results"



O. Hul, et al., Phys. Rev. E 69, 056205 (2004). "Experimental simulation of quantum graphs by microwave networks"