Chaos Experiments – Wave Chaos and Electromagnetic Interference in Enclosures

MURI Program Review

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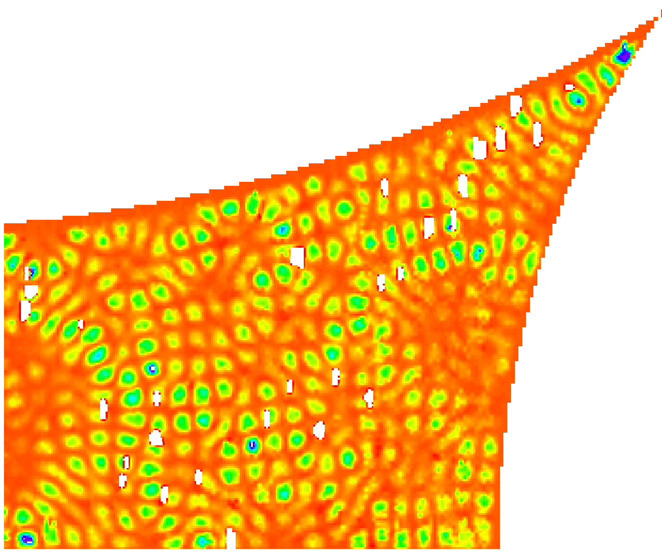
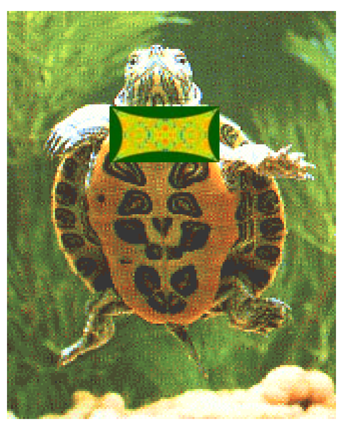
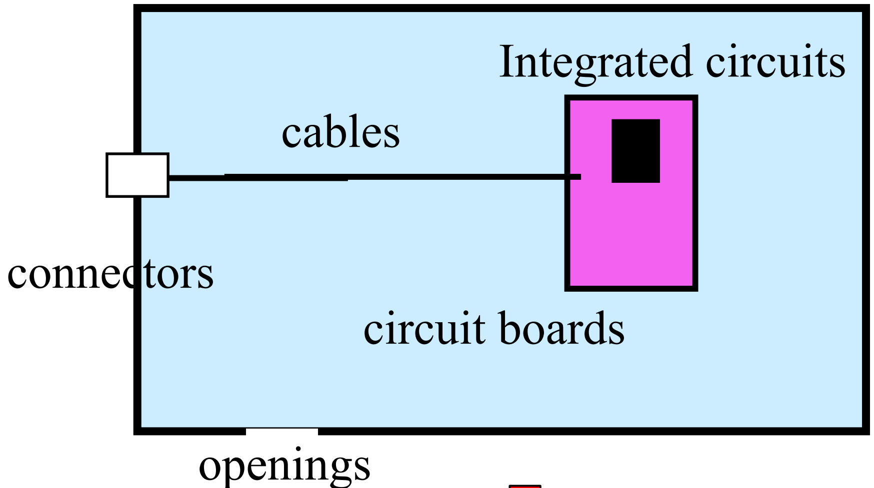
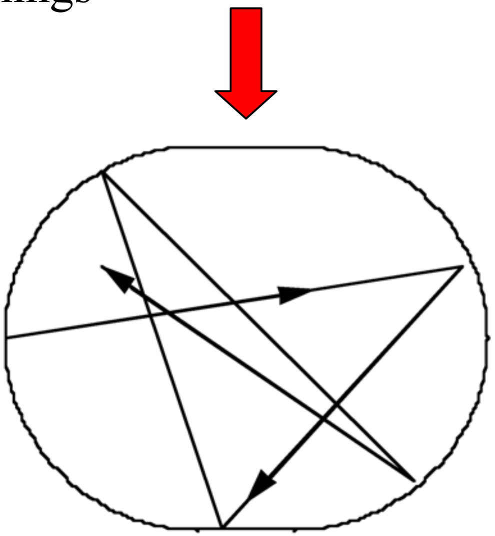




Electromagnetic Coupling in Computer Circuits

Schematic

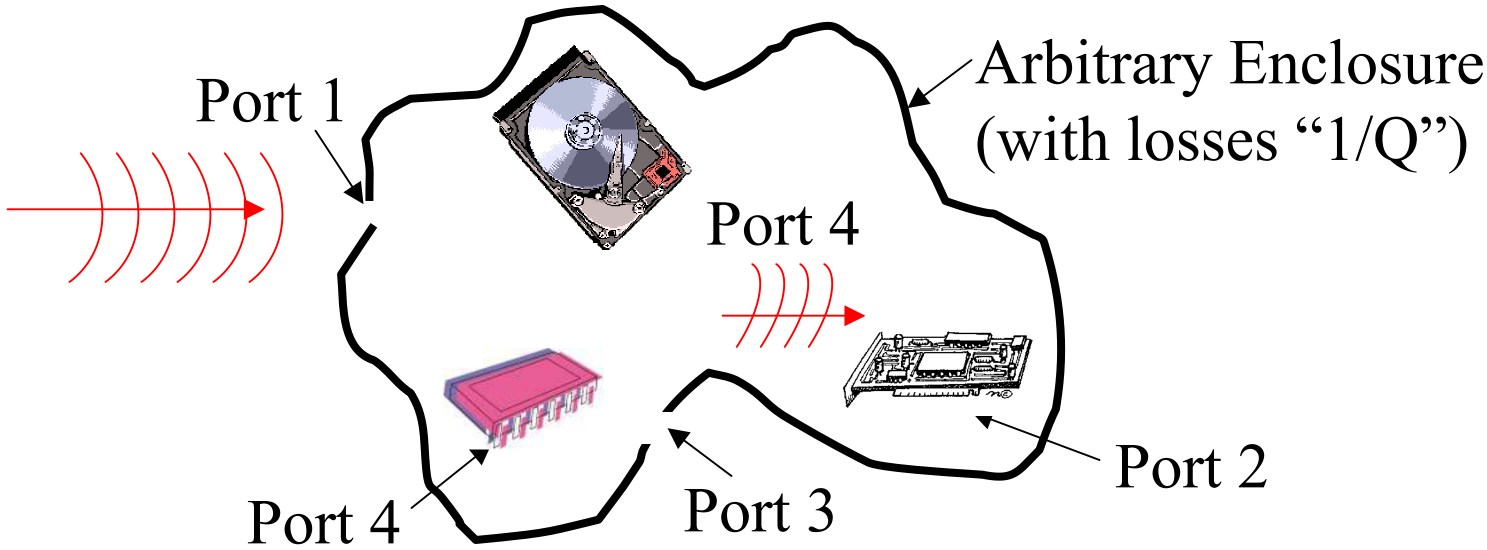




Chaotic Ray Trajectories

- Coupling of external radiation to computer circuits is a complex processes:
  - apertures
  - resonant cavities
  - transmission lines
  - circuit elements
- Intermediate frequency range involves many interacting resonances
- What can be said about coupling without solving in detail the complicated EM problem ?
- Wave Chaos

The Ultimate Goal: Scattering Matrix for an Arbitrary Enclosure



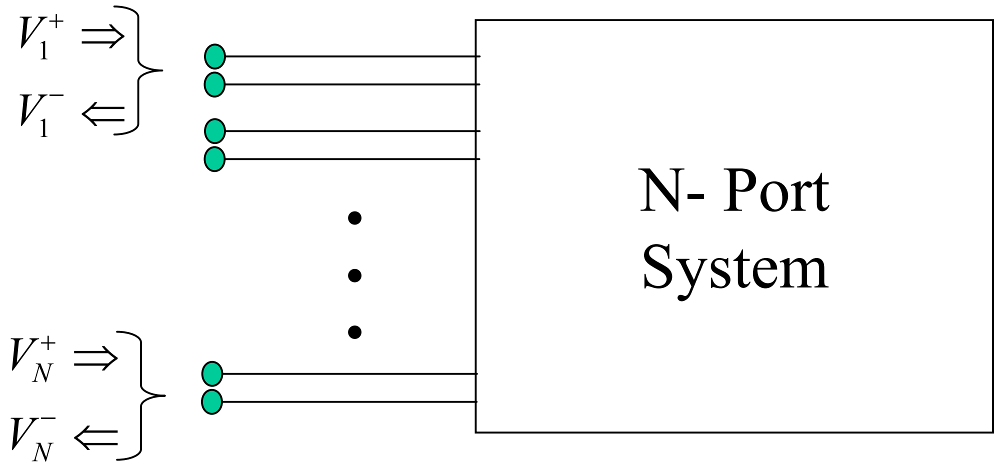
Random Coupling Model

Based on results and concepts from the field of Wave Chaos

The predictions are statistical in nature

S-Matrix
Characterizes Wave Coupling

N ports with incoming and outgoing waves



S matrix
$$\begin{pmatrix} V_1^- \\ V_2^- \\ \bullet \\ V_{N1}^- \end{pmatrix} = \boldsymbol{S} \begin{pmatrix} V_1^+ \\ V_2^+ \\ \bullet \\ V_{N1}^+ \end{pmatrix}$$
outgoing incoming

$\boldsymbol{S}(\omega)$

Complicated function of frequency
Details depend sensitively on unknown parameters

Z matrix $\hat{V} = Z\hat{I}$

$$S = Z_0^{1/2}\left(Z + Z_0\right)^{-1}\left(Z - Z_0\right)Z_0^{-1/2}$$

Some Predictions of the Random Coupling Model

(…those relevant to MURI objectives …)

- Z_{ij} is characterized by a pair of Probability Distribution Functions (PDFs)

  $Re[Z_{ij}]$ PDF
  $Im[Z_{ij}]$ PDF } Both depend on losses (Q) in the cavity

- RCM prescribes a normalization procedure for Z to eliminate / minimize dependence on geometry / details

  Z_{Norm} PDFs are a universal function of losses

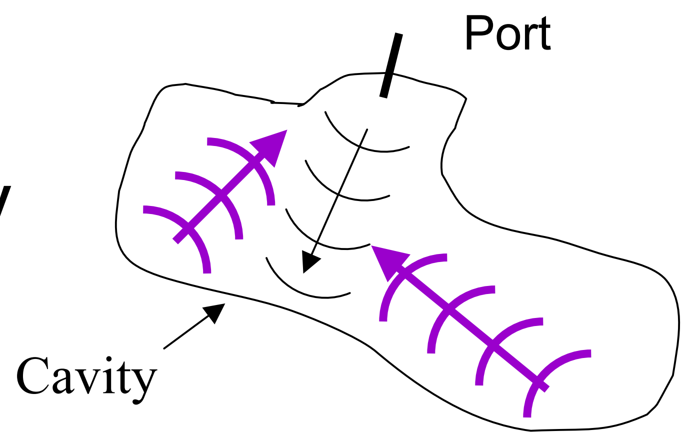
- Testable predictions of the RCM:

  Variance$\{Re[Z_{Norm}]\}$ = Variance$\{Im[Z_{Norm}]\}$
  PDFs smoothly evolve as loss changes
  Variances $\sim 1/f^{3/2}$ (when conductor losses dominate)
  Parametric dependencies:
  frequency, antenna details, cavity height, impedance of medium, etc.

Cavity and Radiation Impedance

It is convenient to think of these two types of impedances

Z_{Cavity}

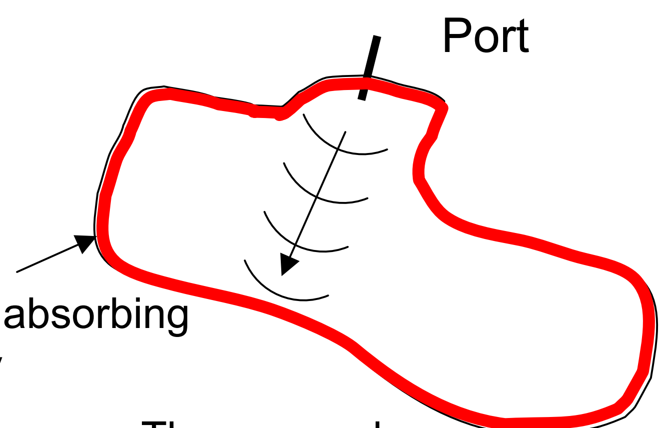


$$Z_{Cavity} = R_{Cavity} + j\,X_{Cavity}$$

Combine

$$Z_{Norm} = \frac{R_{Cavity}}{R_{Rad}} + j\,\frac{X_{Cavity} - X_{Rad}}{R_{Rad}}$$

Z_{Rad}



$$Z_{Rad} = R_{Rad} + j\,X_{Rad}$$

R_{Rad}: Radiation Losses

X_{Rad}: Reactive Impedance of Antenna

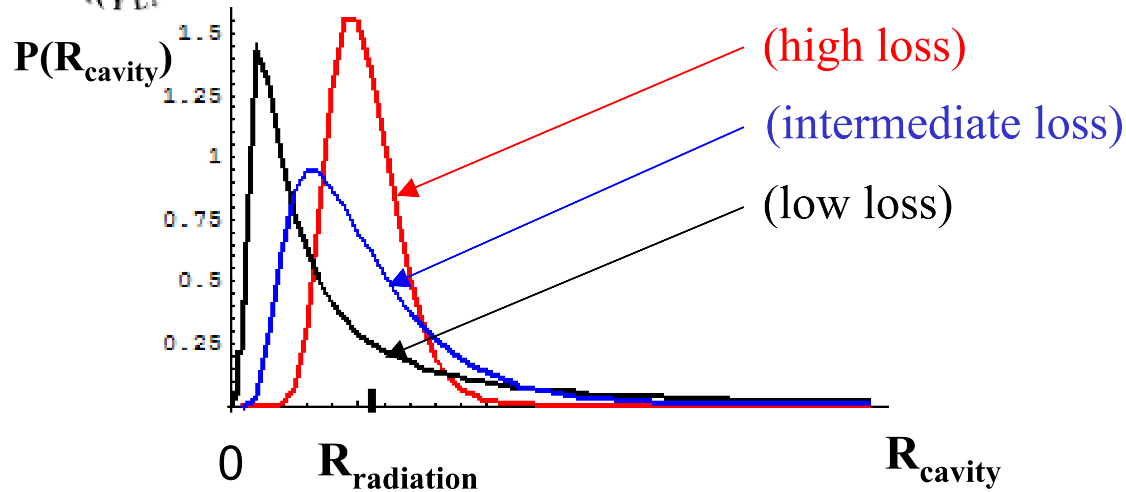
Predictions of the Random Coupling Model

Consider the 1-port case

PDFs	$Z_{Norm} = \frac{R_{Cavity}}{R_{Rad}}$	$+ j \frac{X_{Cavity} - X_{Rad}}{R_{Rad}}$
Zero Loss	$(R_{Cavity} = 0)$	Lorentzian of unit width and zero mean
"Small" Loss	Peaked between 0 and 1	Lorentzian ⇔ Gaussian
"Large" Loss	Gaussian: mean of 1, width $\sim\sqrt{Q}$	Gaussian: zero mean, width $\sim\sqrt{Q}$

Evolution of the Z_{Norm} PDFs with Losses

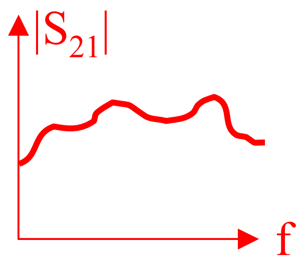
RCM Model Results

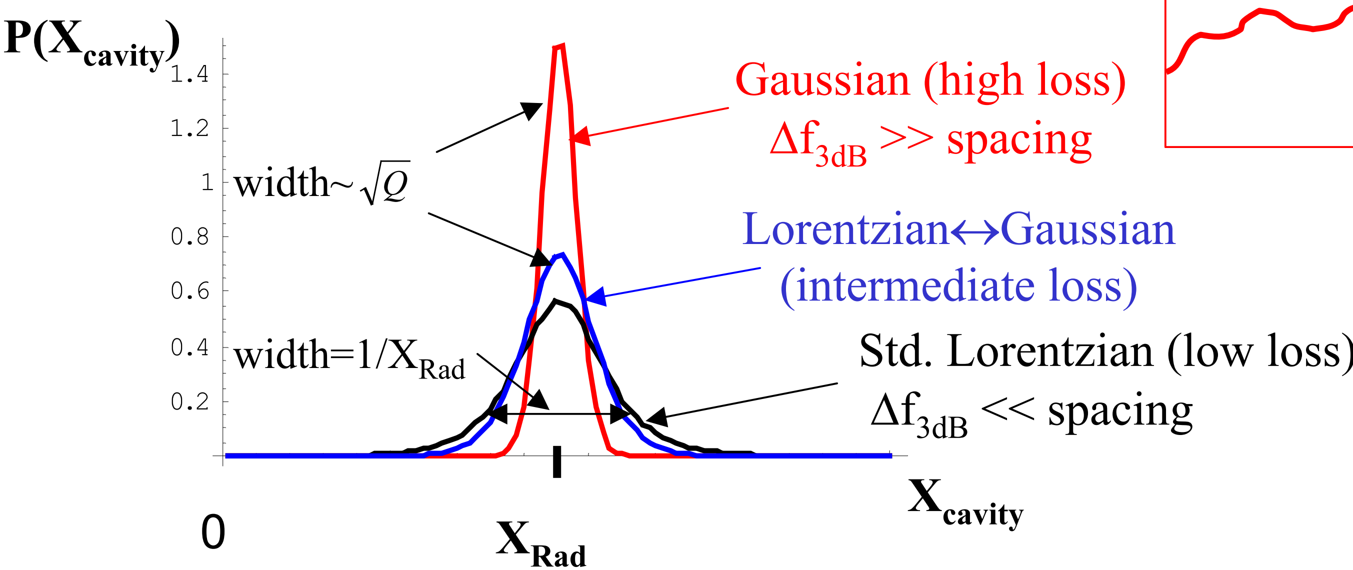


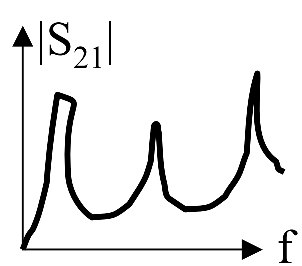
Lossy Case

$$\mathbf{Z_{cavity} = R_{cavity} + j\, X_{cavity}}$$

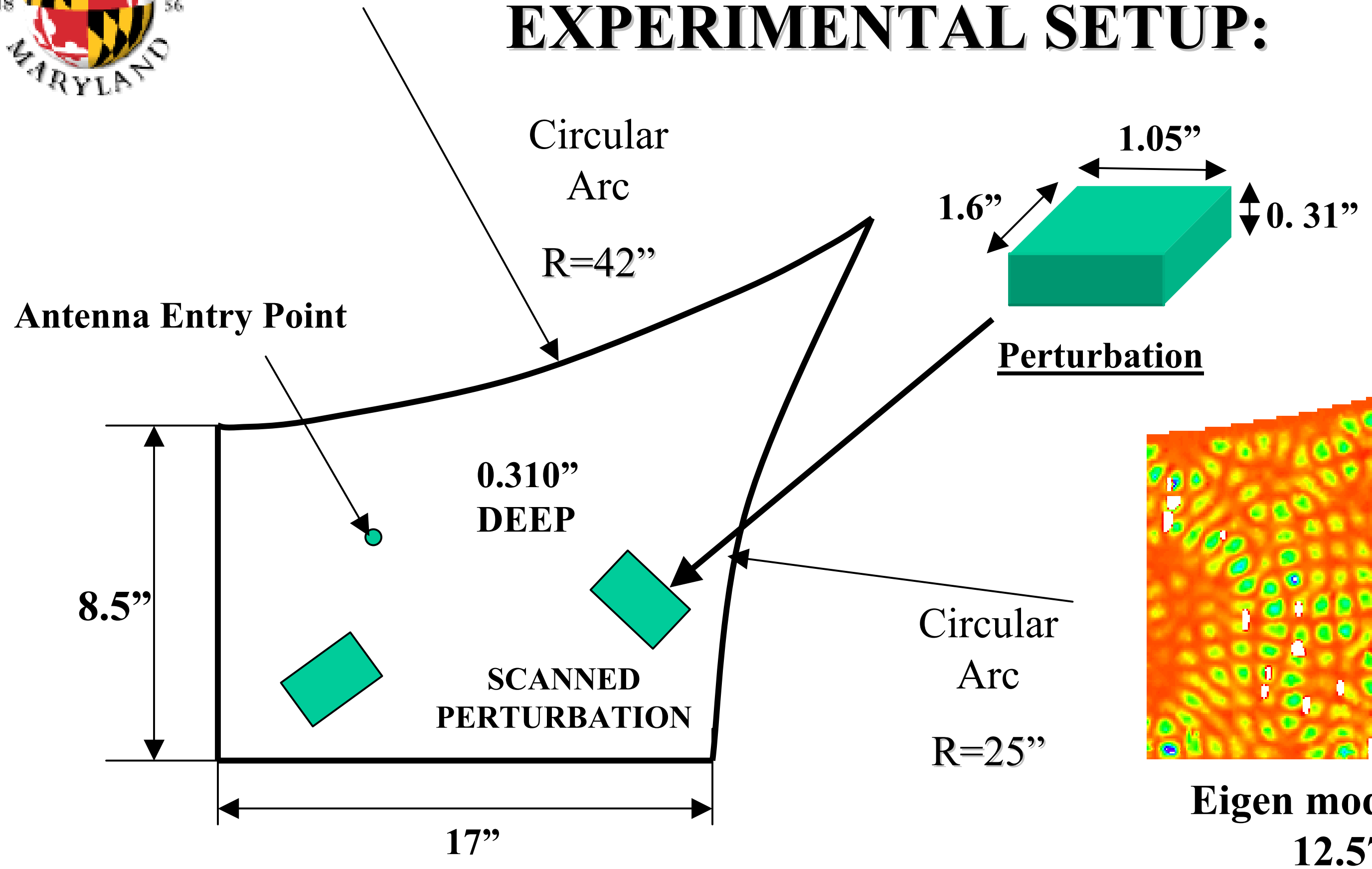
$$Z_{Norm} = \frac{R_{Cavity}}{R_{Rad}} + j\,\frac{X_{Cavity} - X_{Rad}}{R_{Rad}}$$

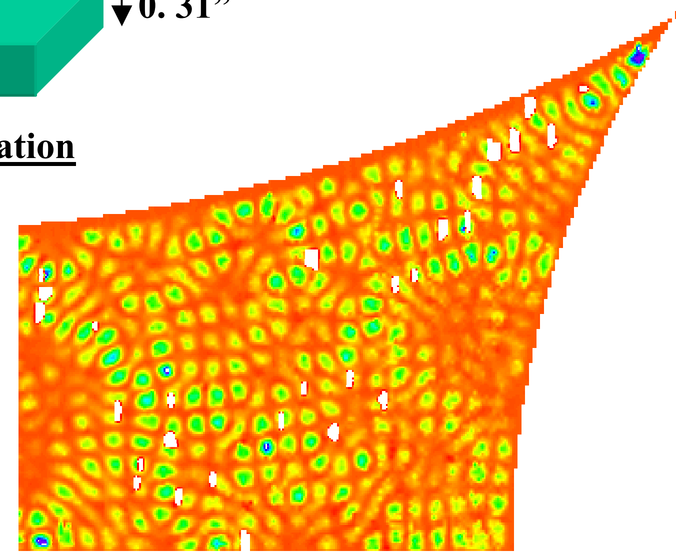






EXPERIMENTAL SETUP:



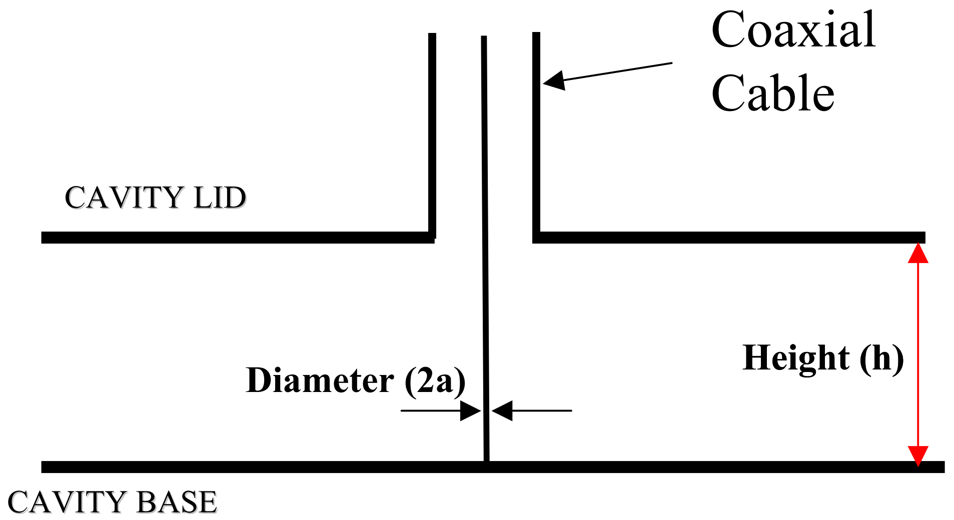


Eigen mode Image at 12.57GHz

- 2 Dimensional Quarter Bow Tie Wave Chaotic cavity
- Classical ray trajectories are chaotic - short wavelength - Quantum Chaos
- 1-port S and Z measurements in the 6 – 12 GHz range.
- Ensemble average through 100 locations of the perturbation (800,000 data points)

Antenna Detail

Cross Section View



For the particular case of a coax-driven quasi-2D cavity:

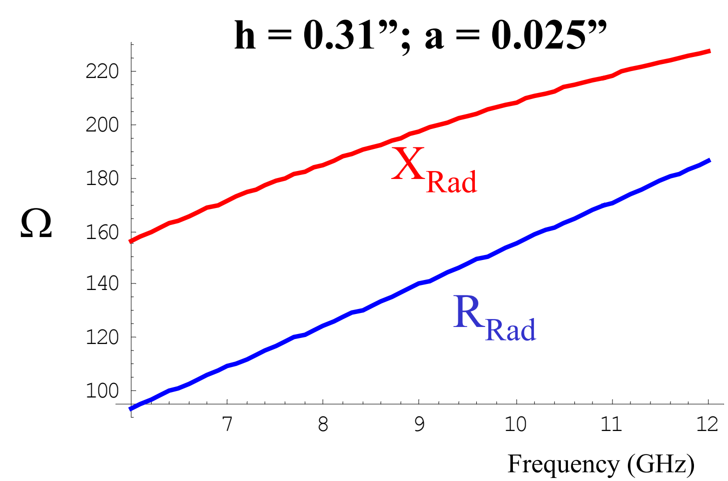
$$X_{Rad} = -\frac{kh\eta}{4} J_0(ka) Y_0(ka)$$

$$R_{Rad} \cong \frac{kh\eta}{4}$$

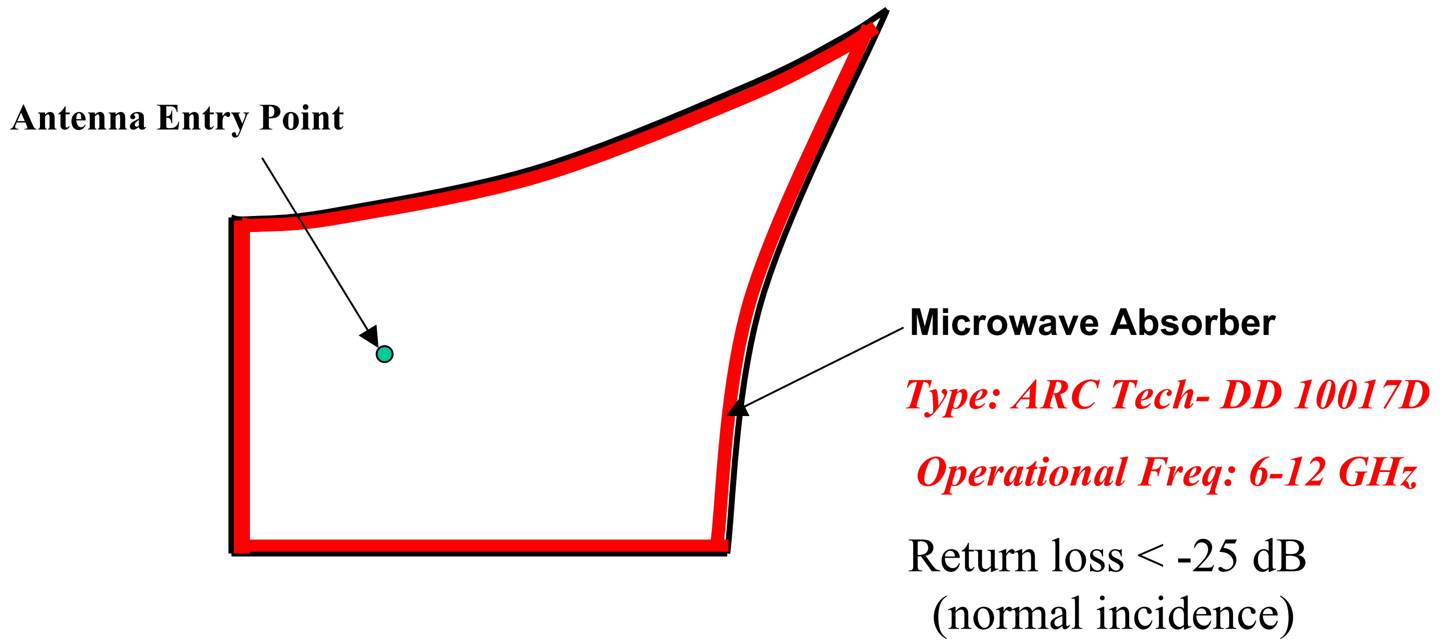
$(ka << 1)$

k = wavenumber

η = impedance of medium in cavity



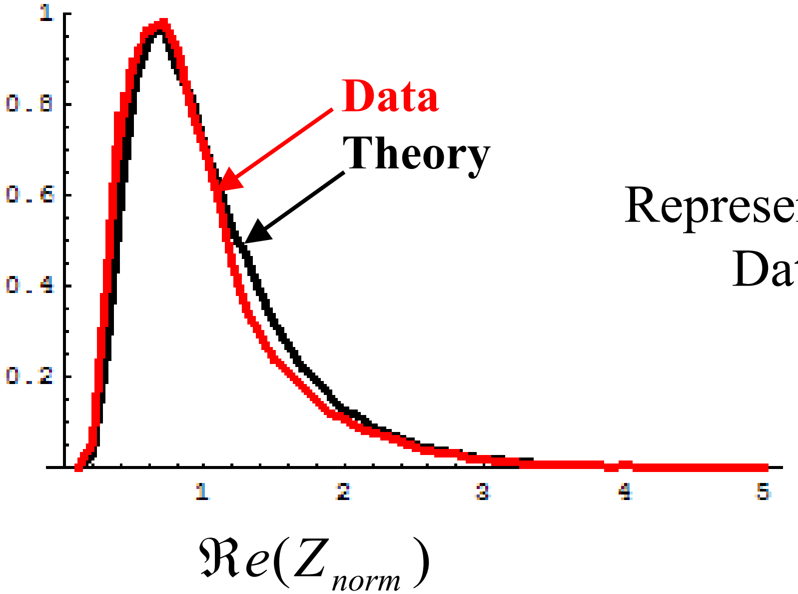
Experimental Setup : Radiation Impedance Measurement



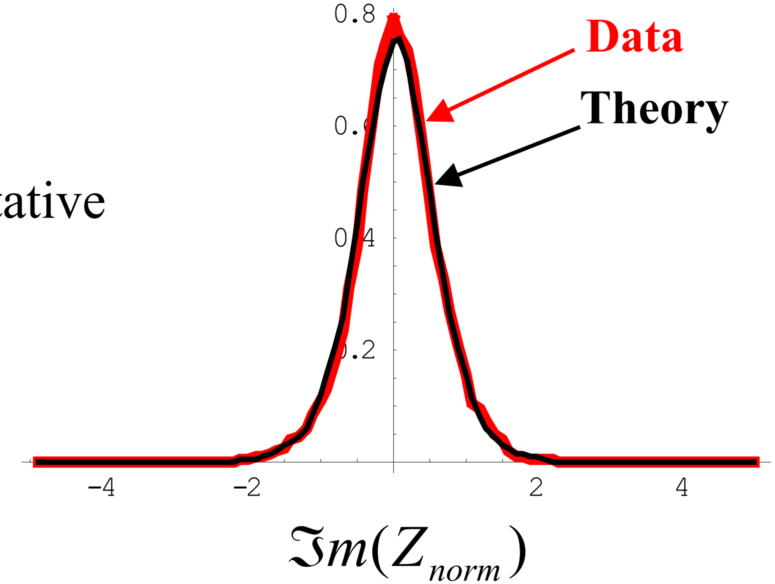
- The microwave absorber serves as an absorbing boundary thereby eliminating internal reflections of the waves within the cavity.
- 1-port S and Z measurements of radiation boundary setup.
- Radiation impedance used to normalize the ensemble cavity impedance.

Z_{Norm} Probability Distribution Functions

$$Z_{Norm} = \frac{R_{Cavity}}{R_{Rad}} + j\frac{X_{Cavity} - X_{Rad}}{R_{Rad}}$$



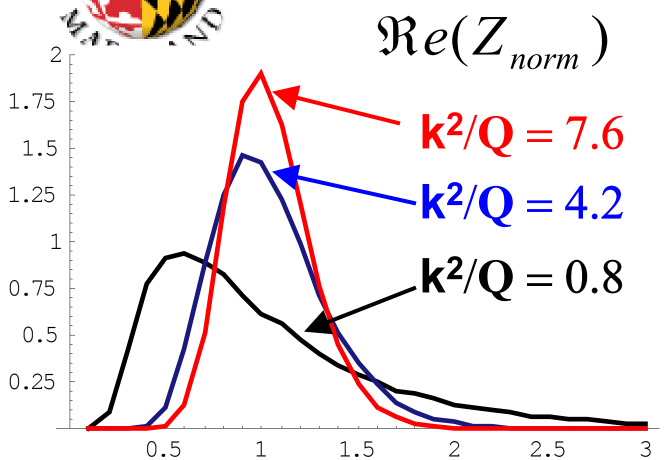
Representative Data



- **Antenna Diameter : 0.025 ”**
- **Cavity Height : 0.310”**
- **Freq. Span => 600 MHz (6 ~ 6.6 GHz)**
- **# of points => 80100**
- **Fit parameter: k^2/Q = 1.05** → $Q = h/\delta$, $\rho_{Cu} = 0.6\ \mu\Omega\text{-cm}$

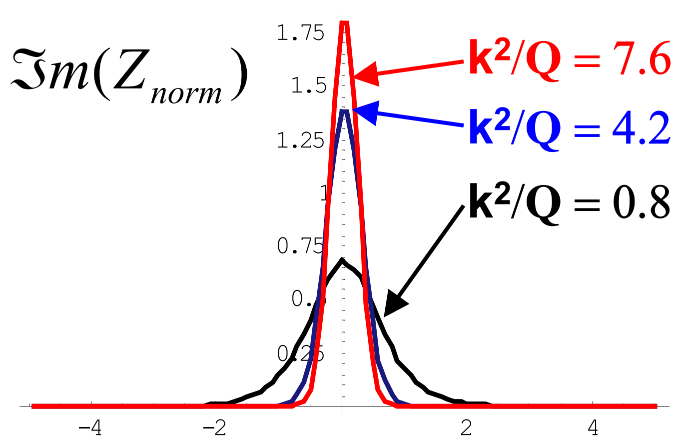
Single parameter simultaneous fit to two PDFs

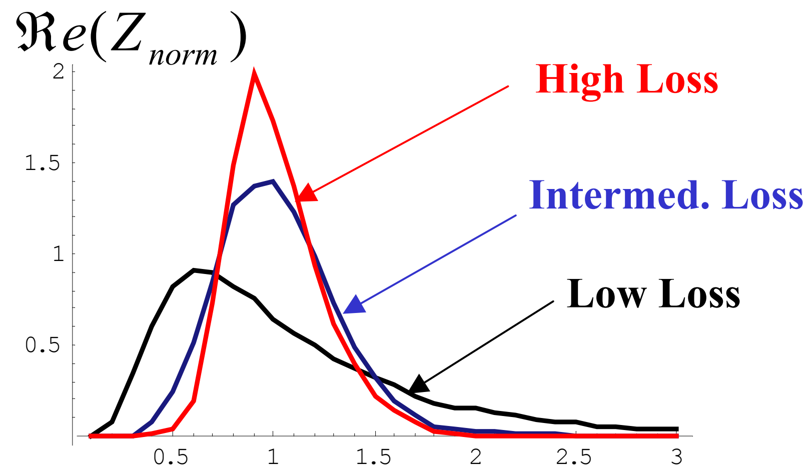
Variation of PDFs with Loss



Model

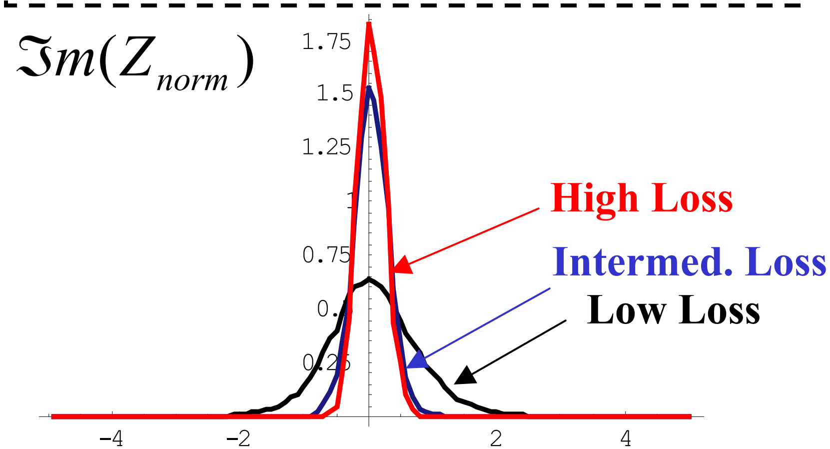
Model one parameter fits (k^2/Q)

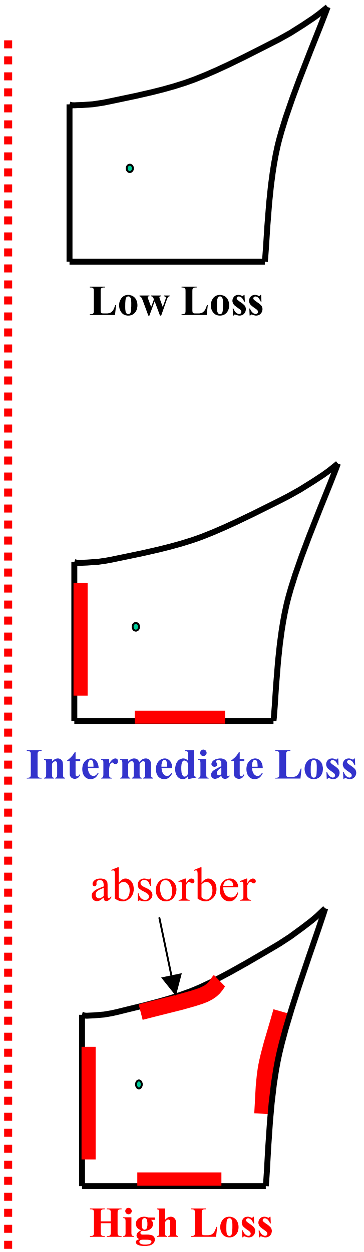




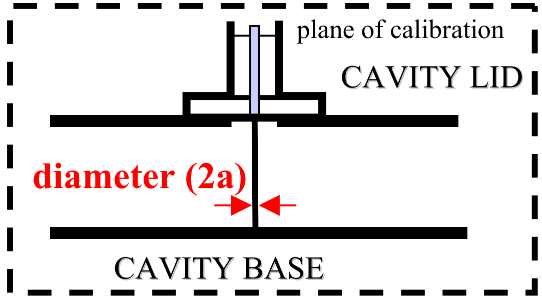
Experiment:

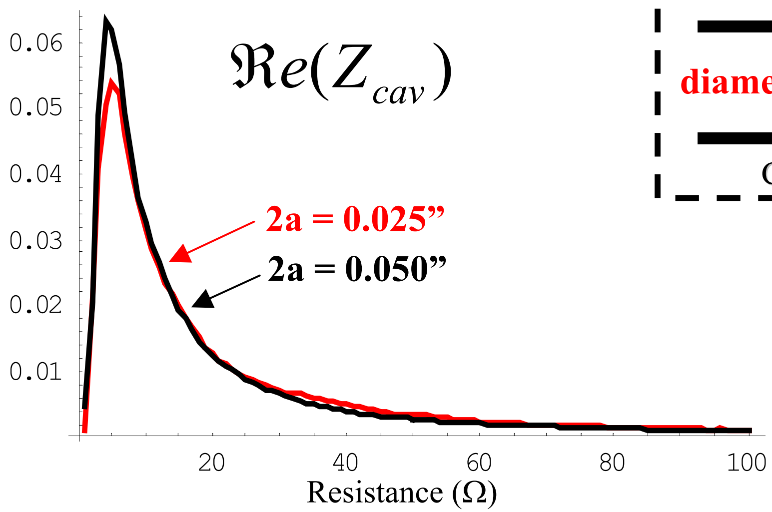
- Freq: 7.2 ~ 8.4 GHz
- Antenna Diameter: 0.05”
- Height of cavity : 0. 31”
- # of pts: 160,200

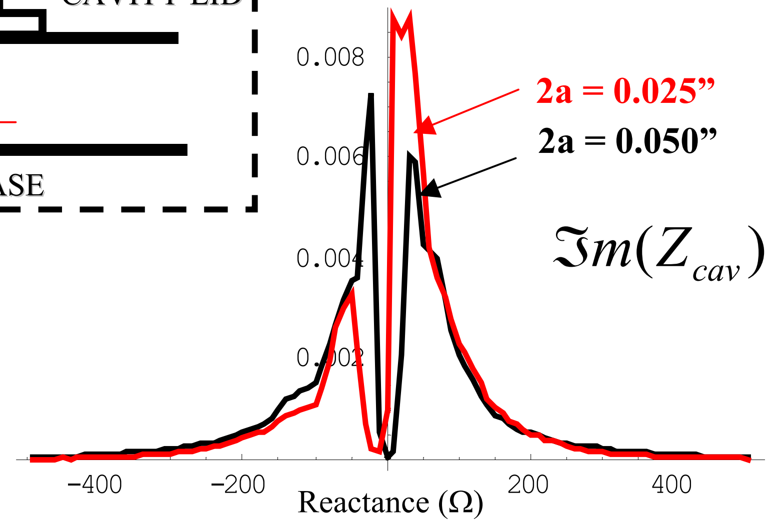




Variation of PDFs with Antenna Radius

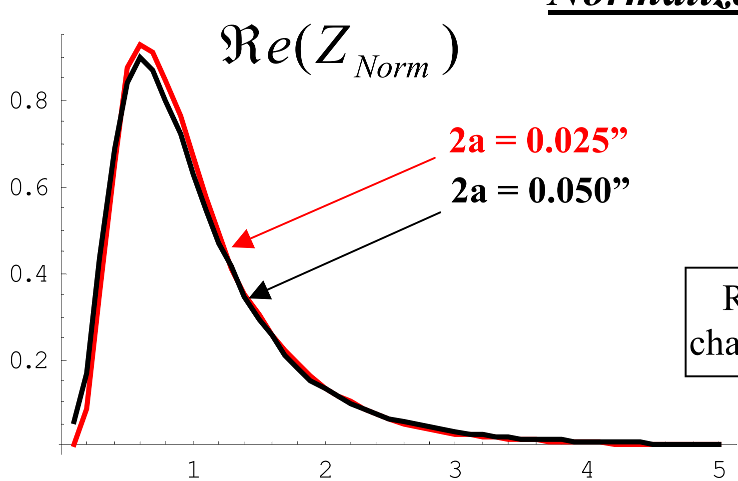




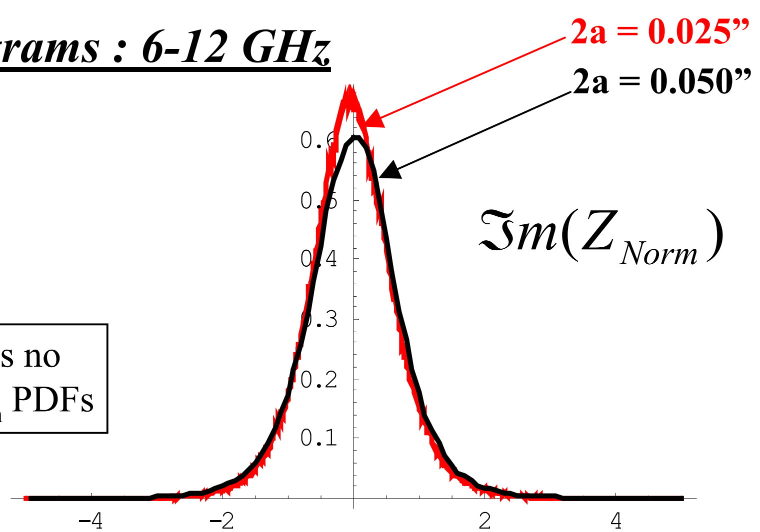


Un-Normalized Histograms : 6-12 GHz

Normalized Histograms : 6-12 GHz

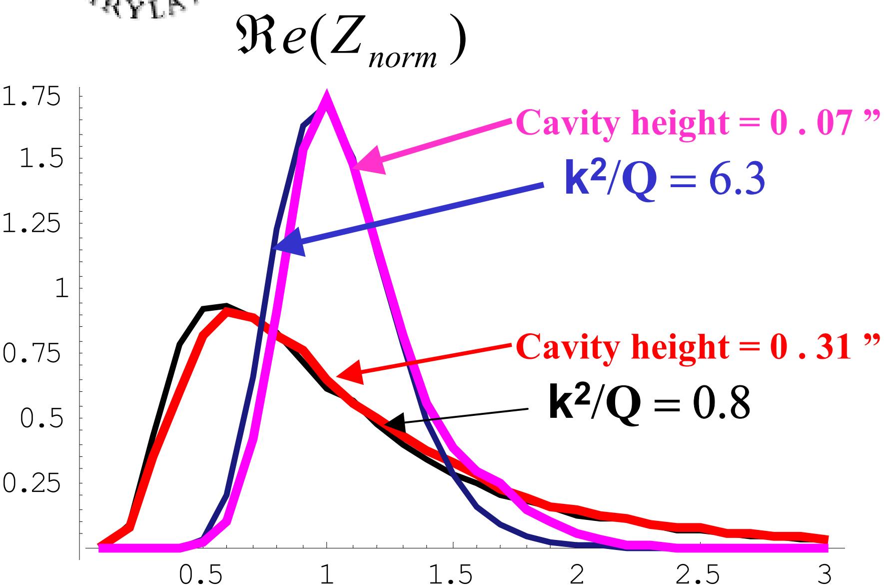


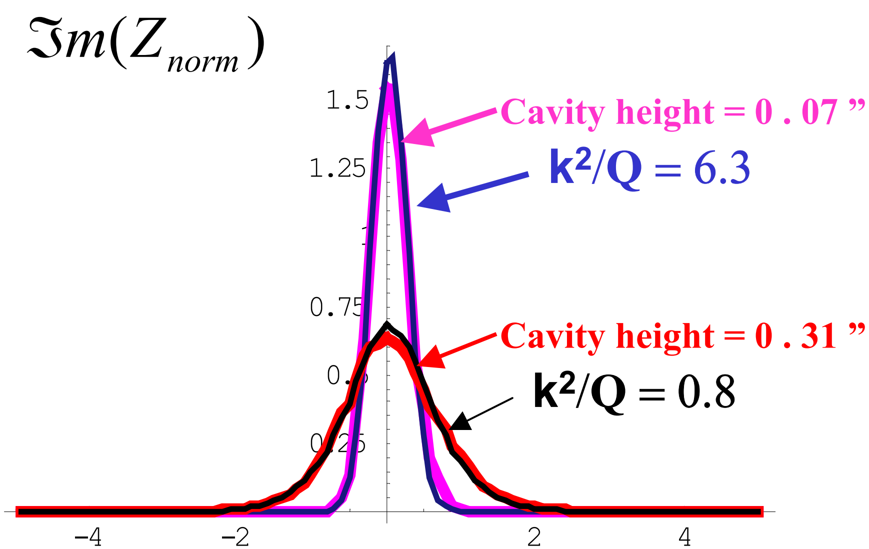
RCM predicts no change in Z_{Norm} PDFs



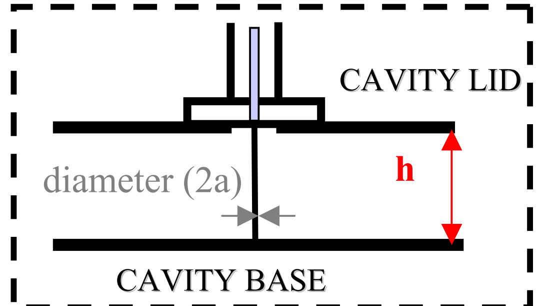
Variation of PDFs with Cavity Dimension: Height

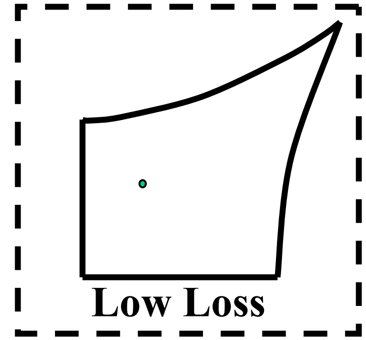
Change in Q due to h





- Frequency range: 7.2 - 8.4 GHz
- Antenna diameter: 0.050”



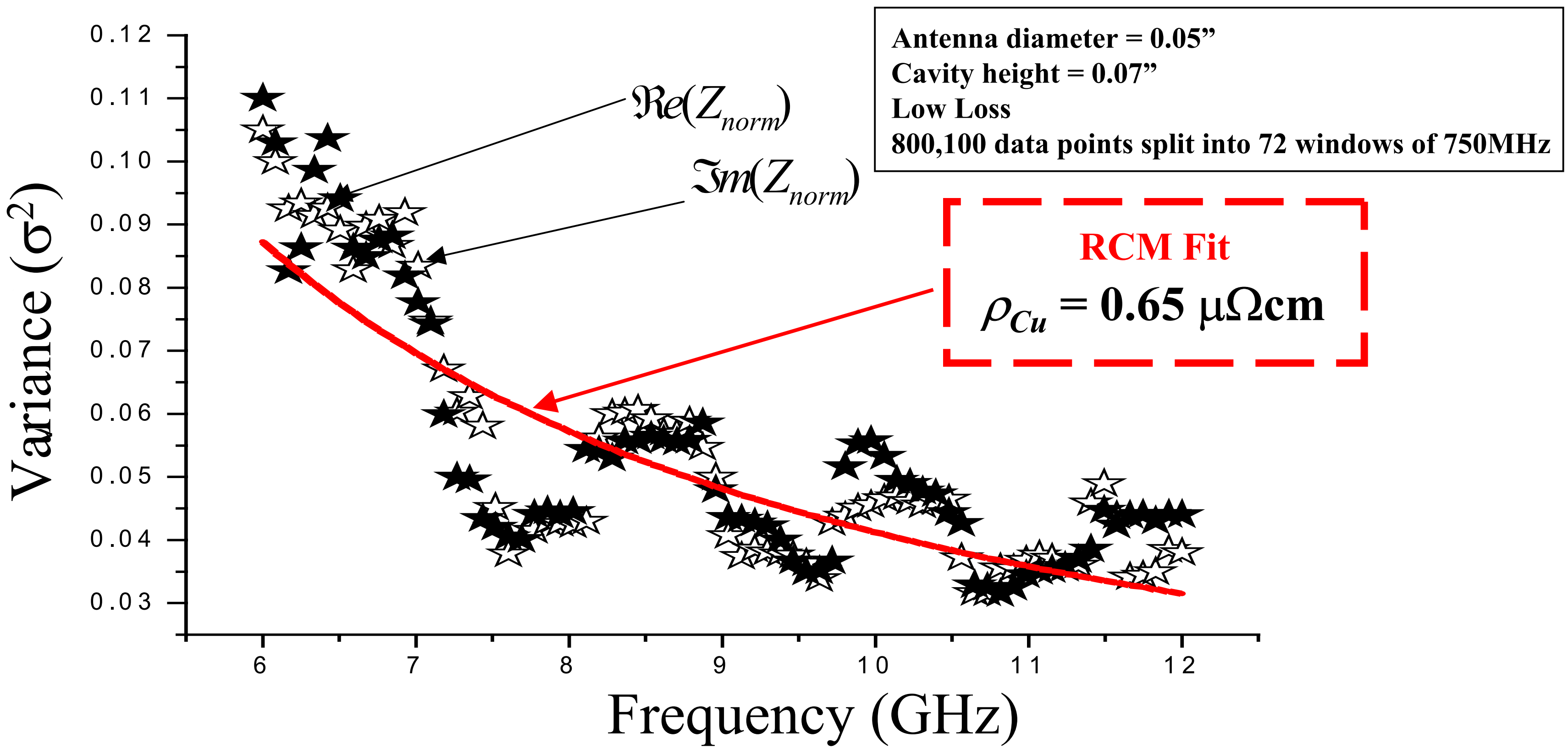


- **RCM Predicts that the Full Width Half Max (width) of $\Im m(Z_{norm})$ PDF α √Q_{cav}**
- **$Q_{cav} = \omega U_{stored}/P_{diss} = h/\delta$**
- **Volume ratio = 4.5**
- **k^2/Q ratio of PDFs = 7.8**
- **Discrepancy due to 2-piece insert (with gap) with poor surface finish, ±0.01” tolerance in h**

Variance of Re[Z_{Norm}] and Im[Z_{Norm}] PDFs

RCM Predictions:

1) Var[Re[Z_{Norm}] PDF] = Var[Im[Z_{Norm}] PDF]
2) Variance $\sigma^2 \cong 3Q/2\pi k^2 \sim 1/f^{3/2}$



Some Practical Implications

The Random Coupling Model should work if the mode density is sufficiently high

It relies on the existence of chaotic ray orbits in the short wavelength limit

Frequency
Losses
Radiation impedance of the ports

} Determine the shape and scales of the Z_{Cavity} PDFs

Statistical predictions of impedance and scattering matrix properties for complicated systems

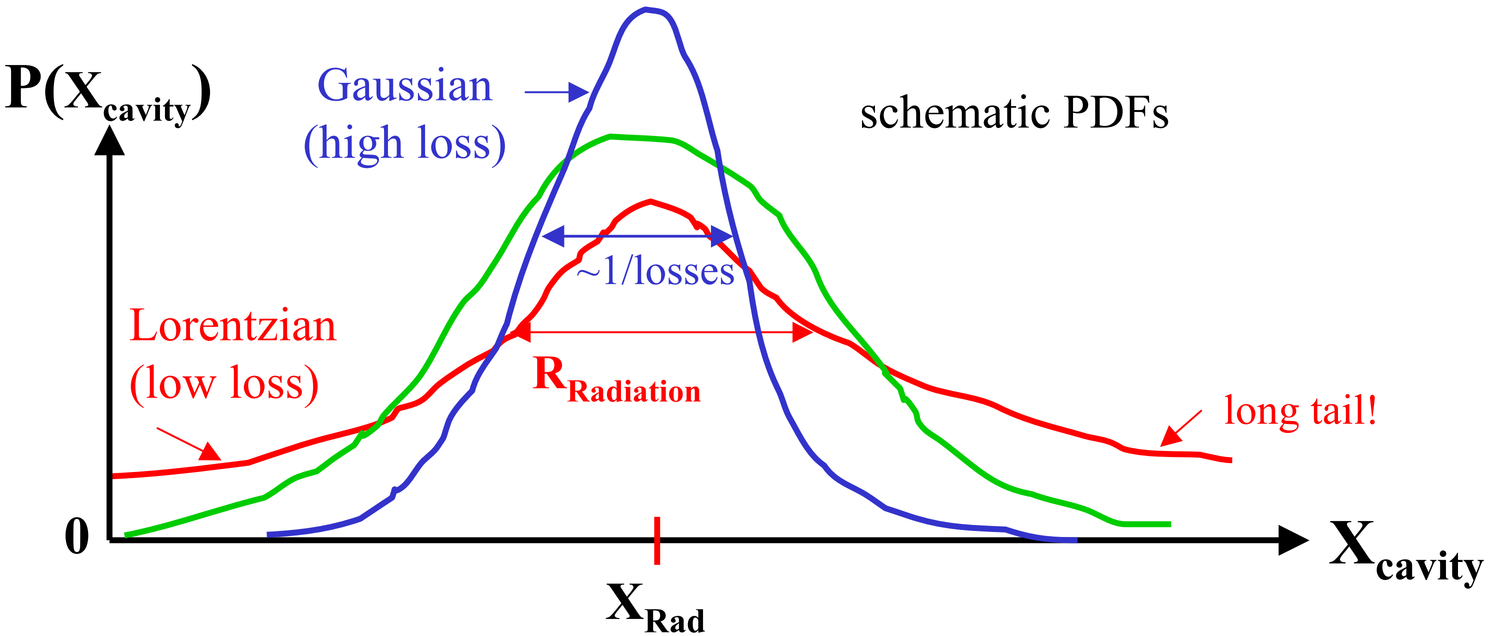
$\rightarrow$ PDFs of field distributions on components inside enclosures

$$\hat{V} = Z\hat{I}$$

Given an excitation current at one port, we find the voltage PDF at all other ports

Clear strategies to engineer the PDFs to suit one's purposes

Cavity Impedance PDF Engineering X_{cavity}

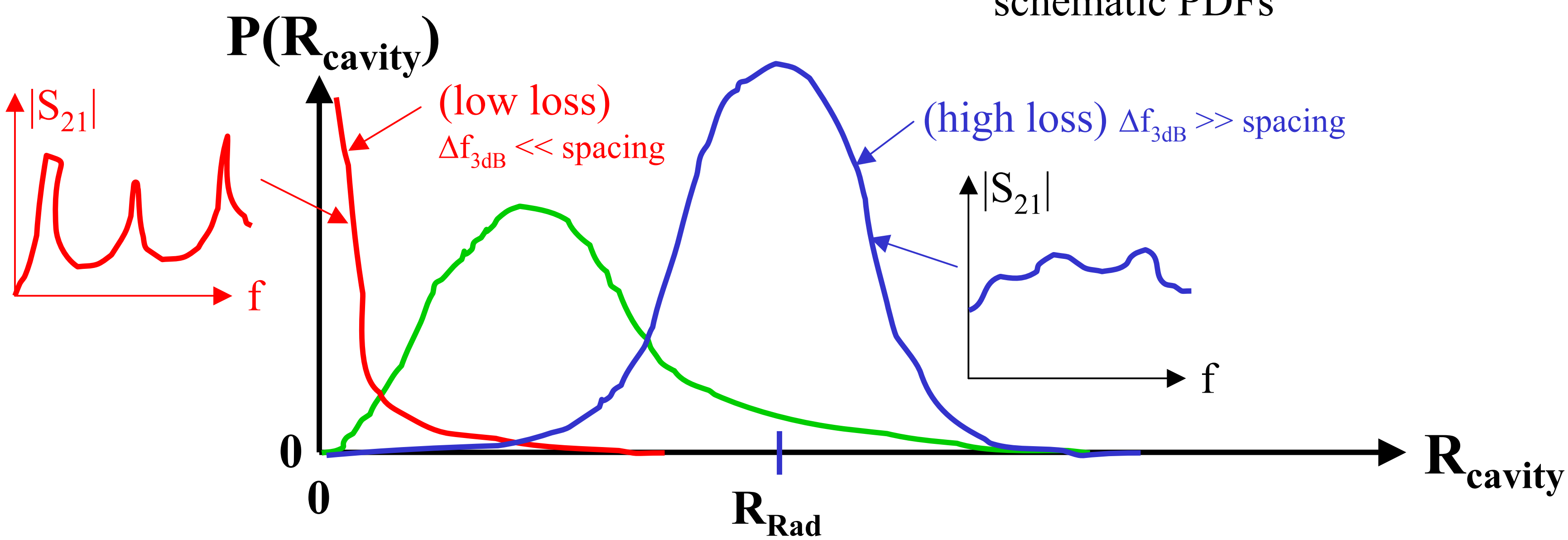


X_{Rad} sets the scale for X_{cavity}

Low-loss case: broad tails, width ~ $R_{Radiation}$

Lossy case: narrow distribution, width ~ $\sqrt{Q}$

Cavity Impedance PDF Engineering R_{cavity}



R_{Rad} sets the scale for R_{cavity}

Low-loss case: $R_{Cavity} < R_{Rad}$

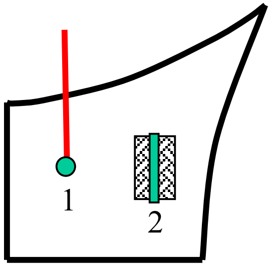
Lossy case: $\Rightarrow$ Gaussian distribution, width $\sim \sqrt{Q}$

Conclusions

- Experimentally tested some basic predictions of the Random Coupling Model (RCM):
  - $Re[Z_{Norm}]$ and $Im[Z_{Norm}]$ PDFs – good agreement with theory
  - Equivalence of the variances of the two PDFs
  - Frequency dependence of the variances of the PDFs {in Low-Q Limit}
  - Loss dependence of the PDF shapes
  - Z_{Norm} PDFs independent of antenna dimension
  - Cavity volume dependence of Z_{Norm} PDFs

- The RCM should provide a solid foundation for quantitative electromagnetic effects studies in enclosures

Future Work



• 2-Port Measurements

>> More realistic case: e.g. use a circuit or patch antenna for port 2

>> Test the effects of time-reversal symmetry breaking

>> Test other predictions of the Random Coupling Model (RCM)

• Low-Loss Cavities

>> LN_2 cooled bow-tie cavity (with cryogenic microwave absorber)

>> Test the RCM in the low-loss limit

• Other Topologies of Interest (R. Prange, S. Fishman collaboration)

>> Include a dielectric slab inside the cavity ⇒ ray splitting

>> Rectangular obstacles inside a rectangular box (sharp corners)

• 3-Dimensional Cavities

>> The RCM should work well

>> The high mode density makes experiment/theory simpler

>> Most realistic case

Comparison of RCM predicted PDFs with measured PDFs

Cryogenic (77 K) Cavity Impedance Statistics Measurement