Statistical Properties of Wave Chaotic Scattering and Impedance Matrices

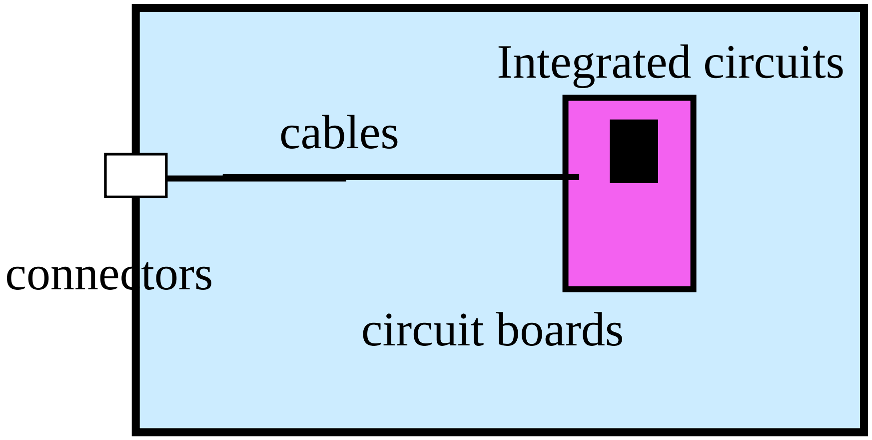
MURI Faculty: Tom Antonsen, Ed Ott, Steve Anlage,

MURI Students: Xing Zheng, Sameer Hemmady, James Hart

AFOSR-MURI Program Review

Electromagnetic Coupling in Computer Circuits

Schematic



• What can be said about coupling without solving in detail the complicated EM problem ?

• **Statistical Description !**

(Statistical Electromagnetics, Holland and St. John)

• Coupling of external radiation to computer circuits is a complex processes:

apertures
resonant cavities
transmission lines
circuit elements

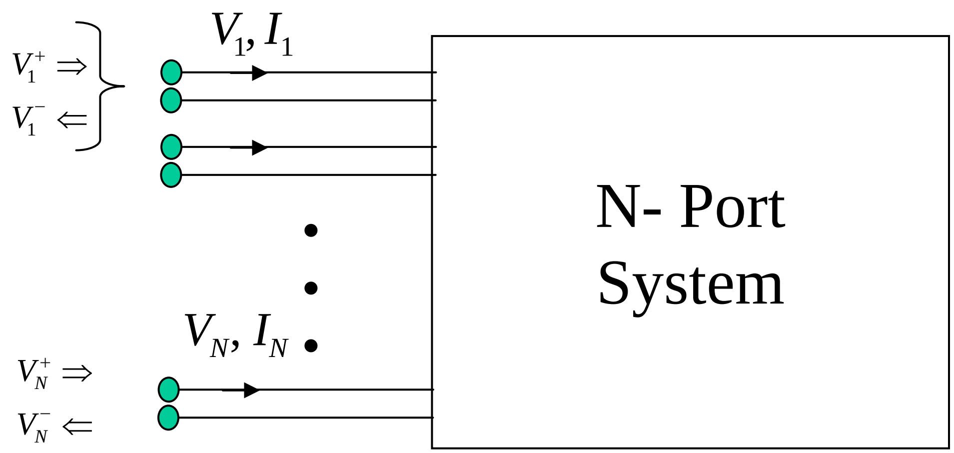
• Intermediate frequency range involves many interacting resonances

• System size >> Wavelength

• Chaotic Ray Trajectories

Z and S-Matrices
What is S_{ij} ?

N ports
• voltages and currents,
• incoming and outgoing waves



Z matrix

$$\begin{pmatrix} V_1 \\ V_2 \\ \vdots \\ V_N \end{pmatrix} = \boldsymbol{Z} \begin{pmatrix} I_1 \\ I_2 \\ \vdots \\ I_2 \end{pmatrix}$$

voltage current

S matrix

$$\begin{pmatrix} V_1^- \\ V_2^- \\ \vdots \\ V_{N1}^- \end{pmatrix} = \boldsymbol{S} \begin{pmatrix} V_1^+ \\ V_2^+ \\ \vdots \\ V_{N1}^+ \end{pmatrix}$$

outgoing incoming

$$\boldsymbol{S} = (\boldsymbol{Z} + \boldsymbol{Z}_0)^{-1}(\boldsymbol{Z} - \boldsymbol{Z}_0)$$

$$\boldsymbol{Z}(\omega)\,,\ \boldsymbol{S}(\omega)$$

• Complicated function of frequency
• Details depend sensitively on unknown parameters

Random Coupling Model

1. Formally expand fields in modes of closed cavity: eigenvalues $k_n = \omega_n/c$

2. Replace exact eigenfunction with superpositions of random plane waves

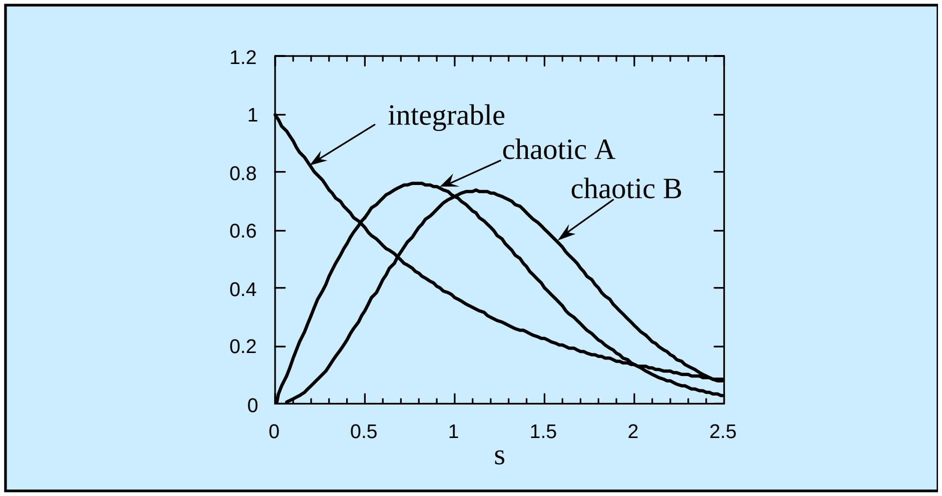
$$\phi_n = \lim_{N\to\infty} \mathrm{Re}\left\{\sqrt{\frac{2}{AN}}\sum_{k=1}^{N} a_k \exp\left[i\left(k_n \boldsymbol{e}_k \cdot \boldsymbol{x} + \theta_k\right)\right]\right\}$$

Random amplitude Random direction Random phase

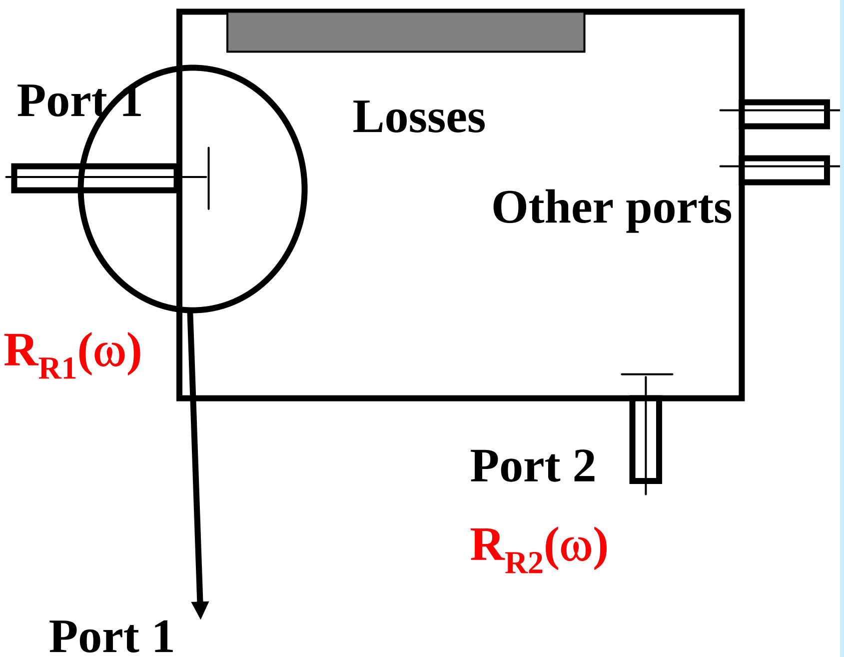
3. Eigenvalues k_n^2 are distributed according to appropriate statistics:

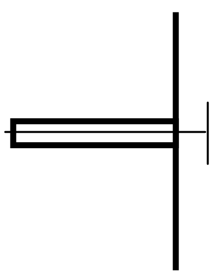
- Eigenvalues of Gaussian Random Matrix

Normalized Spacing $s_n = (k_{n+1}^2 - k_n^2) / \left\langle \Delta k^2 \right\rangle$



Statistical Model of Z Matrix Frequency Domain





Free-space radiation Resistance $R_R(\omega)$

$Z_R(\omega) = R_R(\omega)+jX_R(\omega)$

Statistical Model Impedance

$$Z_{ij}(\omega) = -\frac{j}{\pi}\sum_n R_{Ri}^{1/2}(\omega_n)R_{Rj}^{1/2}(\omega_n)\frac{\Delta\omega_n^2\, w_{in}w_{jn}}{\omega^2(1+jQ^{-1})-\omega_n^2}$$

System parameters
- Radiation Resistance $R_{Ri}(\omega)$
- $\Delta\omega_n^2$ - mean spectral spacing
- Q -quality factor

Statistical parameters
- ω_n - random spectrum
- w_{in}- Gaussian Random variables

Model Validation
Summary

Single Port Case:

Cavity Impedance: $Z_{cav} = R_R z + jX_R$

Radiation Impedance: $Z_R = R_R + jX_R$

Universal normalized random impedance: $z = \rho + j\xi$

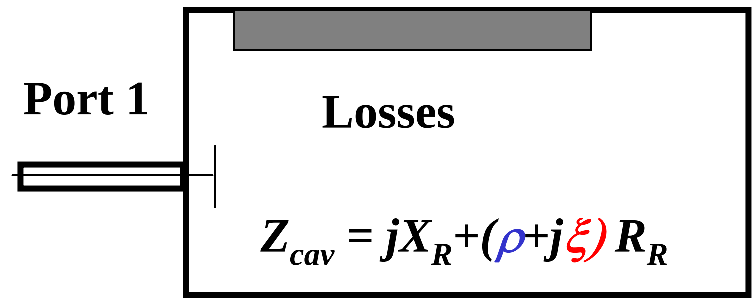
Statistics of z depend only on damping parameter: $k^2/(Q\Delta k^2)$
(Q-width/frequency spacing)

Validation:

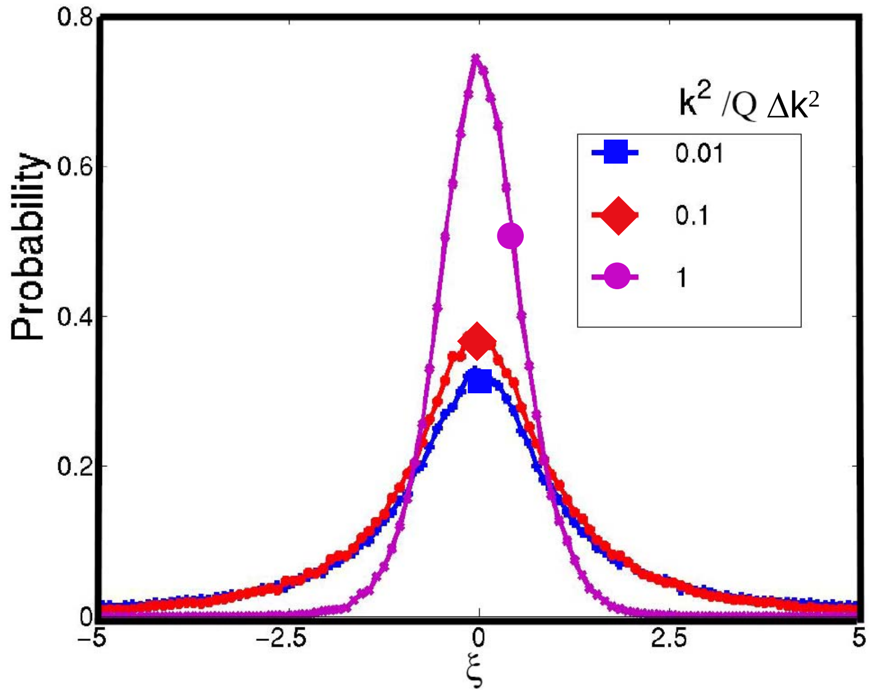
HFSS simulations
Experiment (Hemmady and Anlage)

Normalized Cavity Impedance with Losses

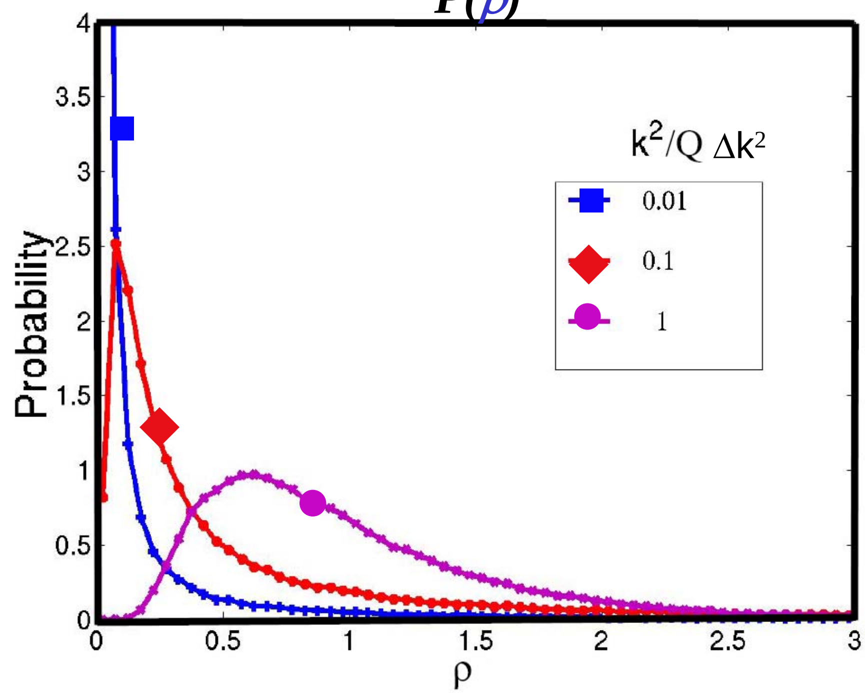
Theory predictions for Pdf's of $z = \rho + j\xi$



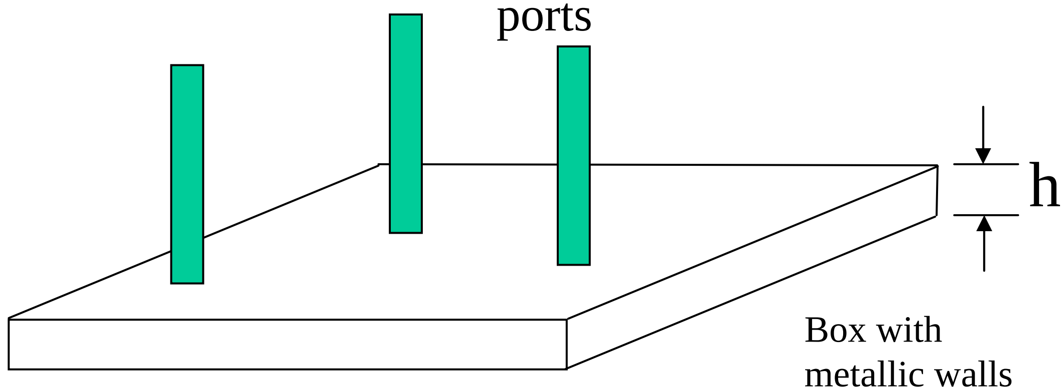
Distribution of reactance fluctuations $P(\xi)$

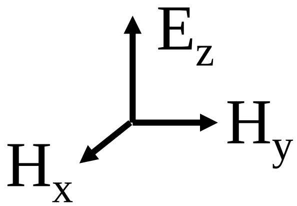


Distribution of resistance fluctuations $P(\rho)$



Two Dimensional Resonators





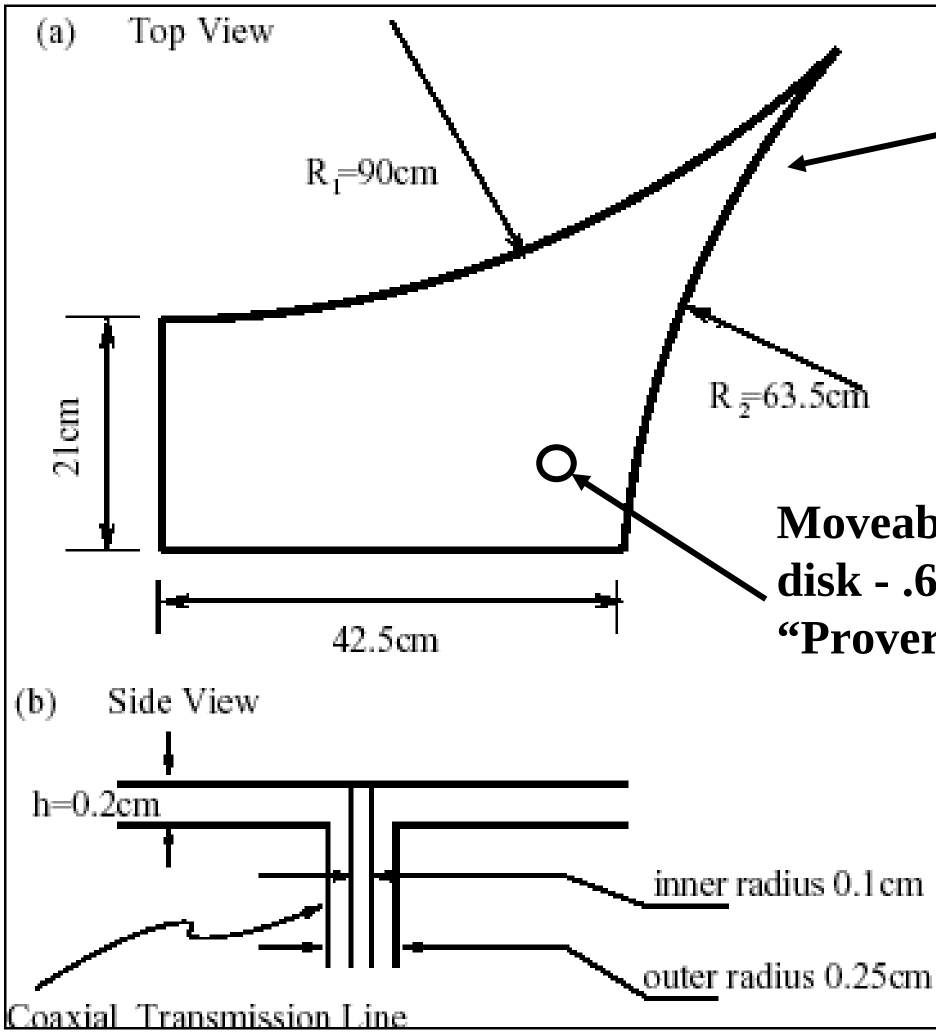
Only transverse magnetic (TM) propagate for $f < c/2h$

- Anlage Experiments
- HFSS Simulations
- Power plane of microcircuit

Voltage on top plate

$$E_z(x,y) = -V_T(x,y)/h$$

HFSS - Solutions
Bow-Tie Cavity



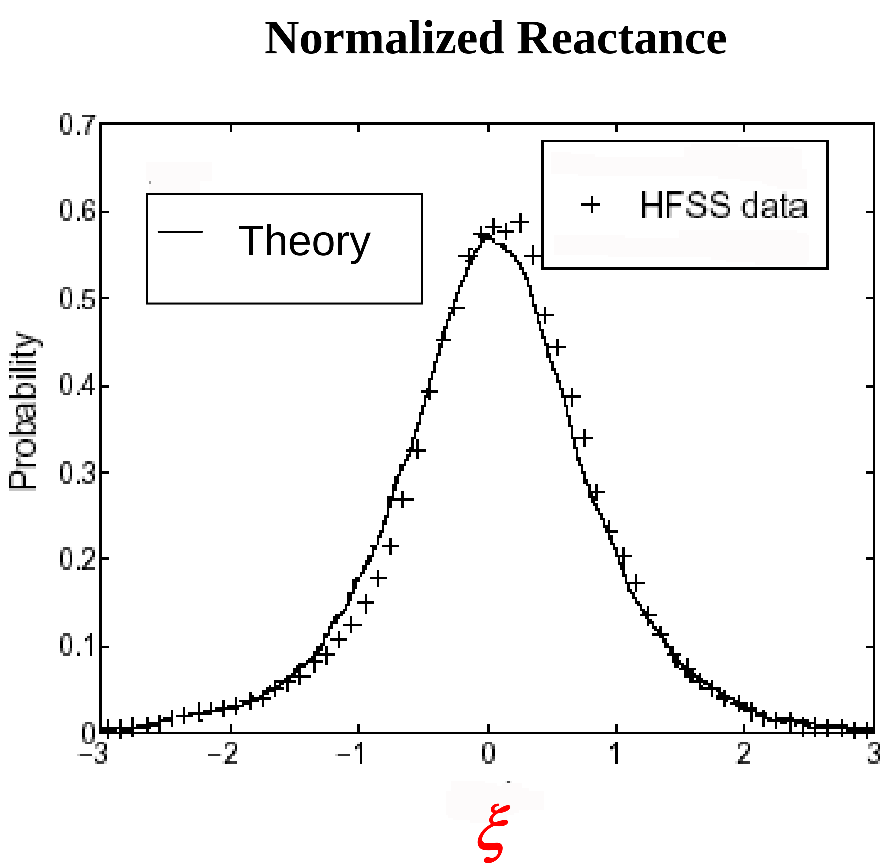
Curved walls guarantee all ray trajectories are chaotic

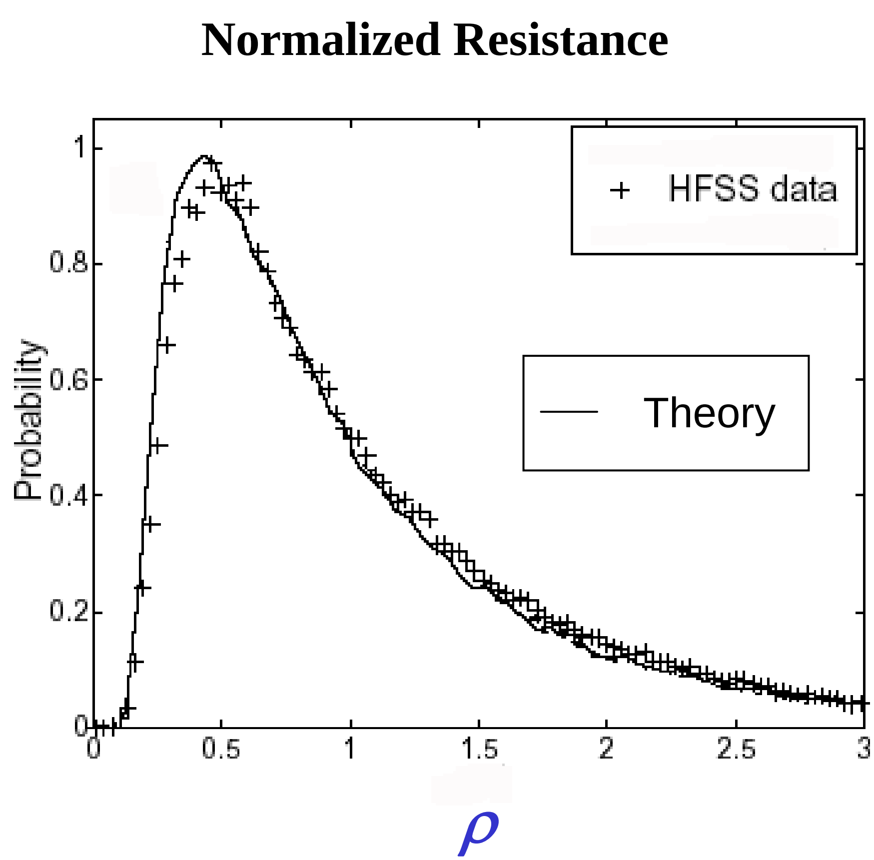
Losses on top and bottom plates

Moveable conducting disk - .6 cm diameter
“Proverbial soda can”

Cavity impedance calculated for
100 locations of disk
4000 frequencies
6.75 GHz to 8.75 GHz

Comparison of HFSS Results and Model for Pdf's of Normalized Impedance

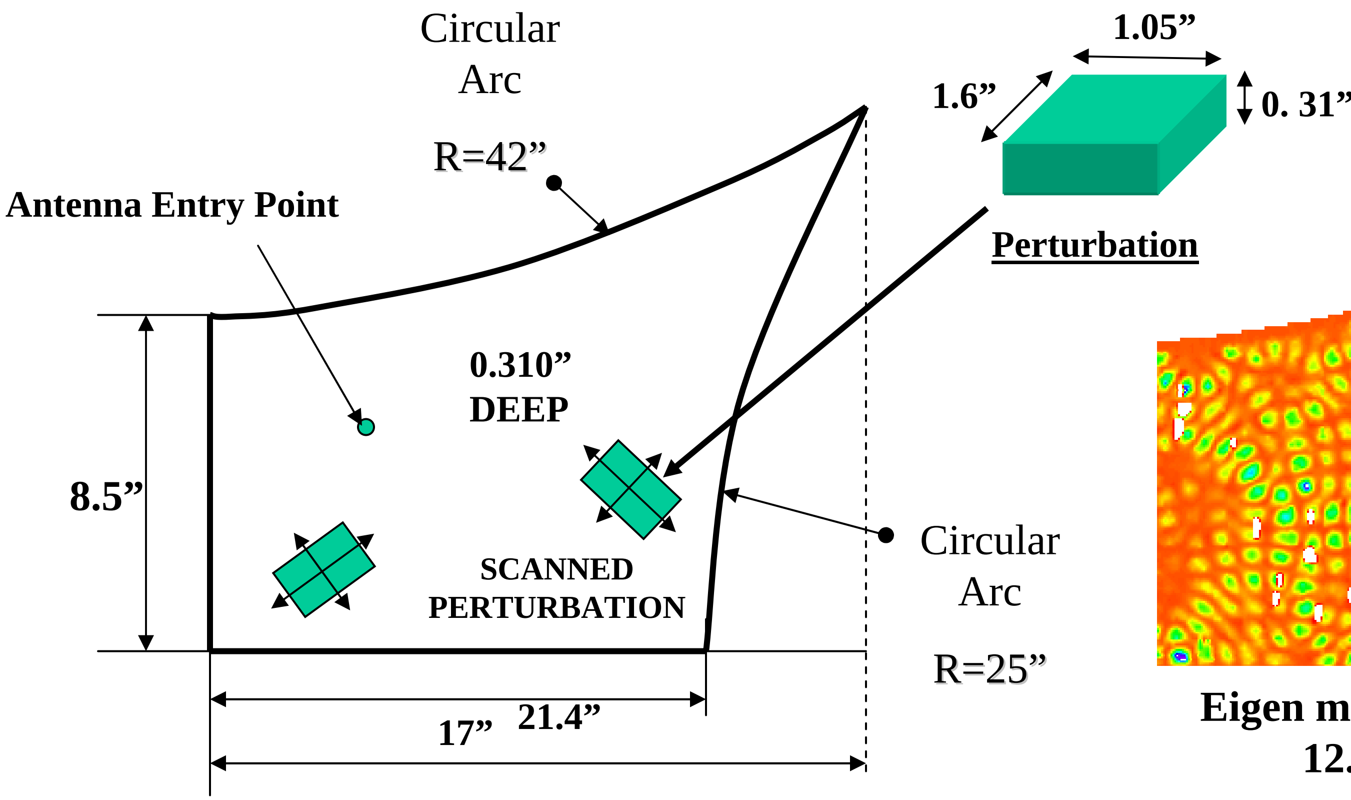


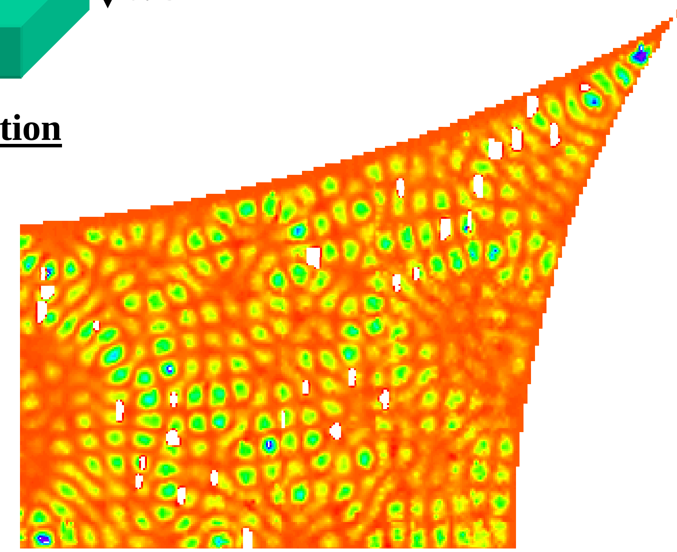


$$Z_{cav} = jX_R + (\rho + j\xi)\, R_R$$

EXPERIMENTAL SETUP

Sameer Hemmady, Steve Anlage CSR





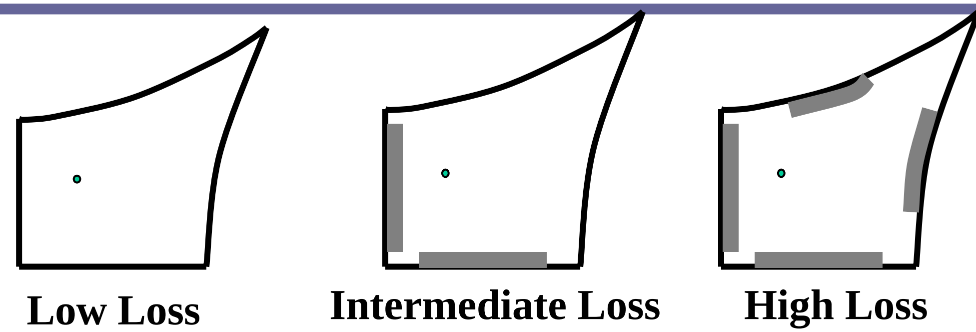
Eigen mode Image at
12.57GHz

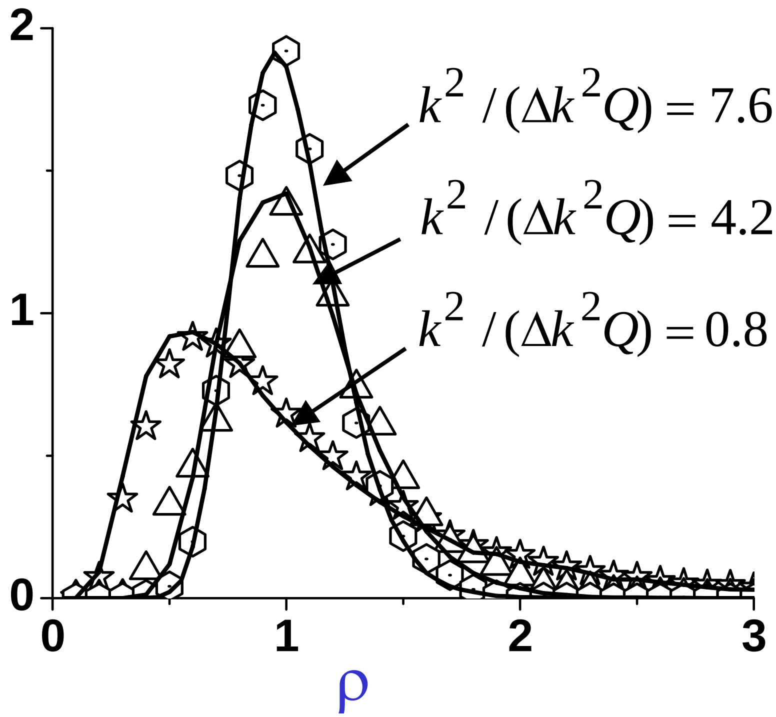
- 2 Dimensional Quarter Bow Tie Wave Chaotic cavity
- Classical ray trajectories are chaotic - short wavelength - Quantum Chaos
- 1-port S and Z measurements in the 6 – 12 GHz range.
- Ensemble average through 100 locations of the perturbation

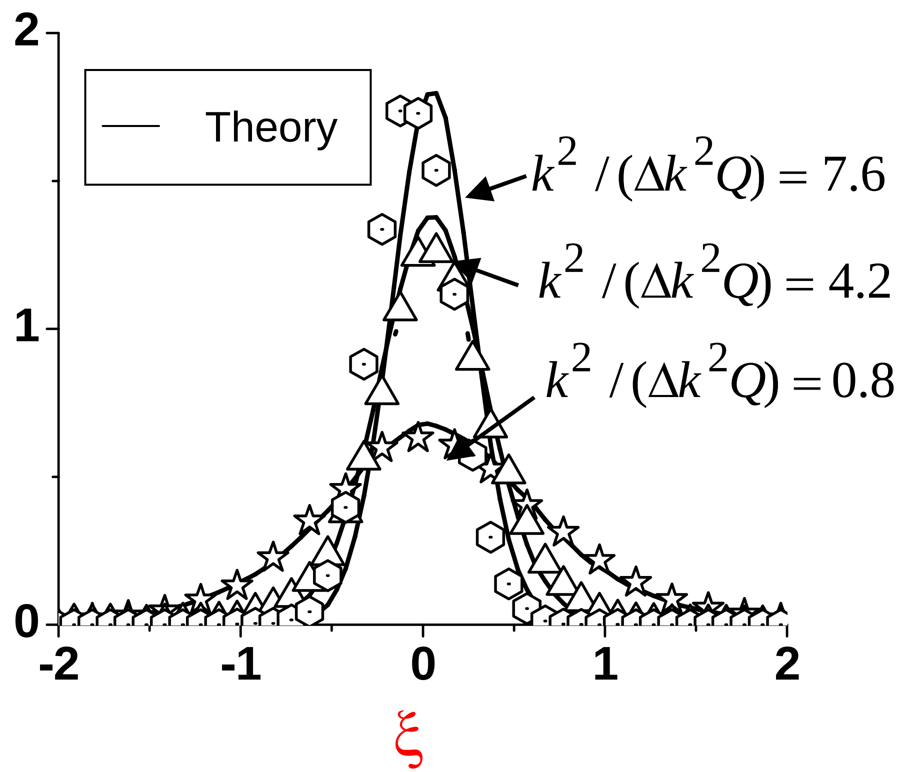
Comparison of Experimental Results and Model for Pdf's of Normalized Impedance

$$\rho = \mathrm{Re}(Z)/R_R$$

$$\xi = (\mathrm{Im}(Z) - X_R)/R_R$$







Normalized Scattering Amplitude
Theory and HFSS Simulation

Actual Cavity Impedance: $Z_{cav} = R_R z + jX_R$

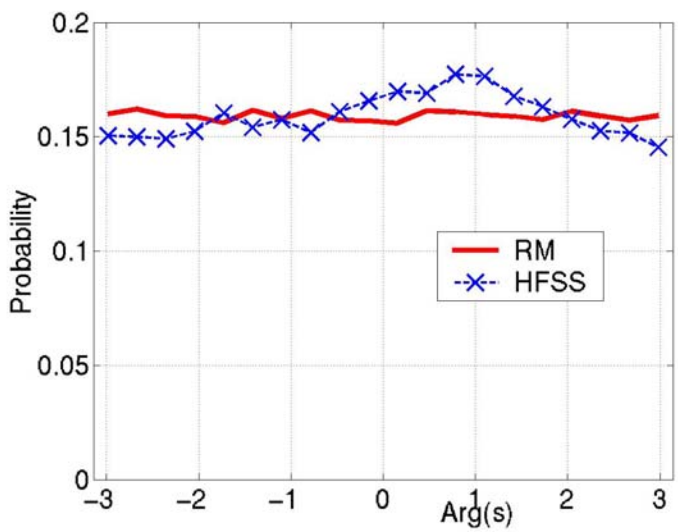
Normalized impedance : $z = \rho + j\xi$

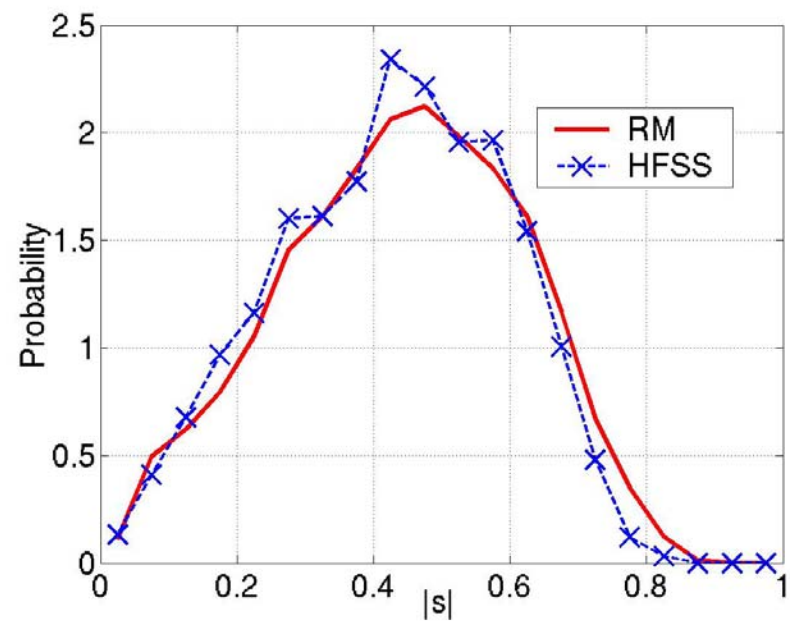
Universal normalized scattering coefficient: $s = (z - 1)/(z + 1) = |s| \exp[i\phi]$

Statistics of s depend only on damping parameter: $k^2/(Q\Delta k^2)$

Theory predicts: $$P(|s|,\phi) = \frac{1}{2\pi} P_{|s|}(|s|)$$

Uniform distribution in phase

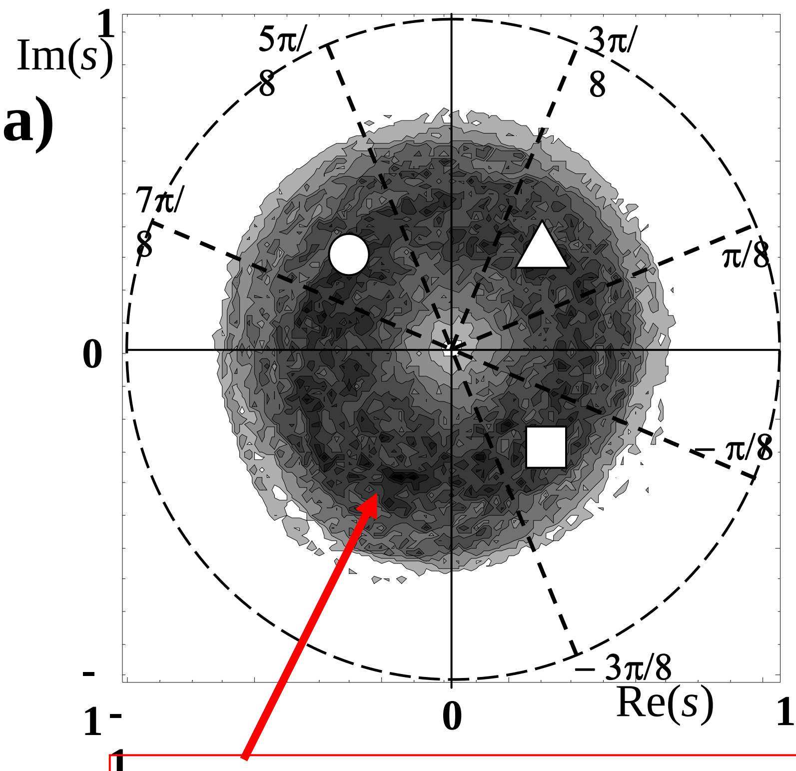




Experimental Distribution of Normalized Scattering Coefficient

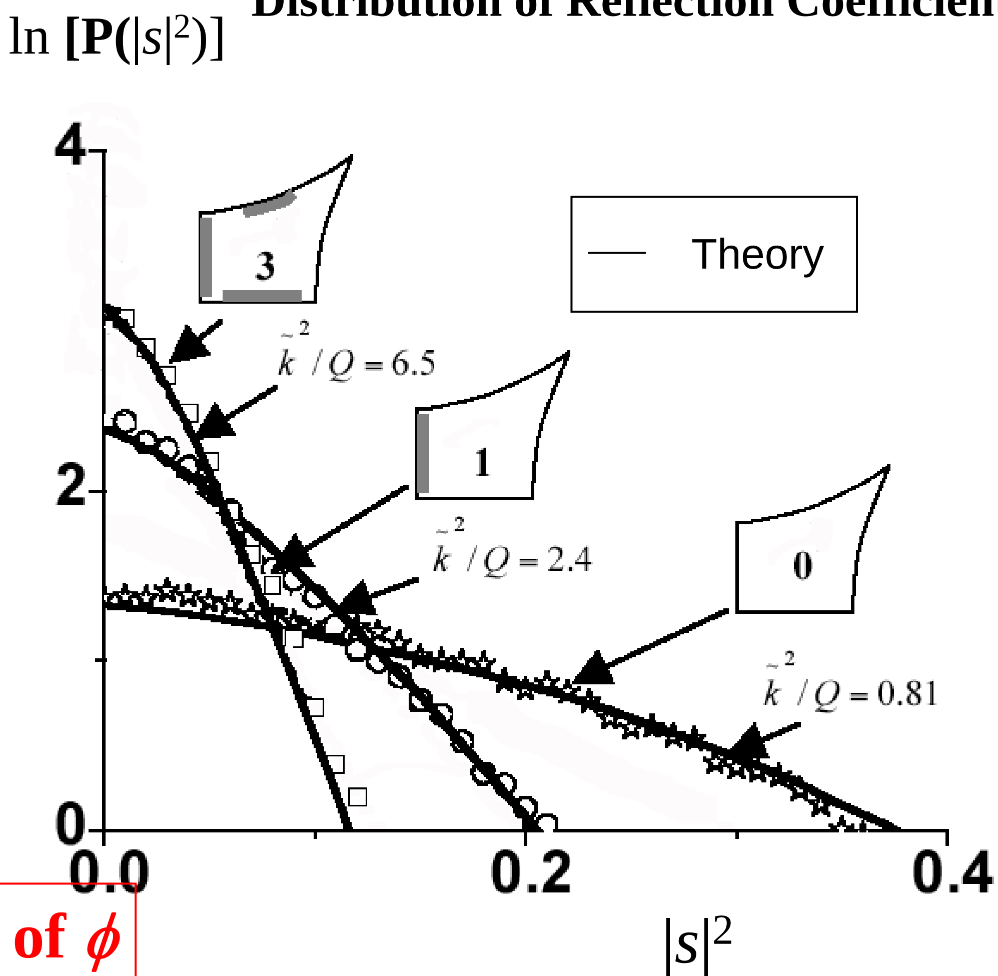
$s = |s| \exp[i\phi]$

a)



Distribution independent of ϕ

Distribution of Reflection Coefficient



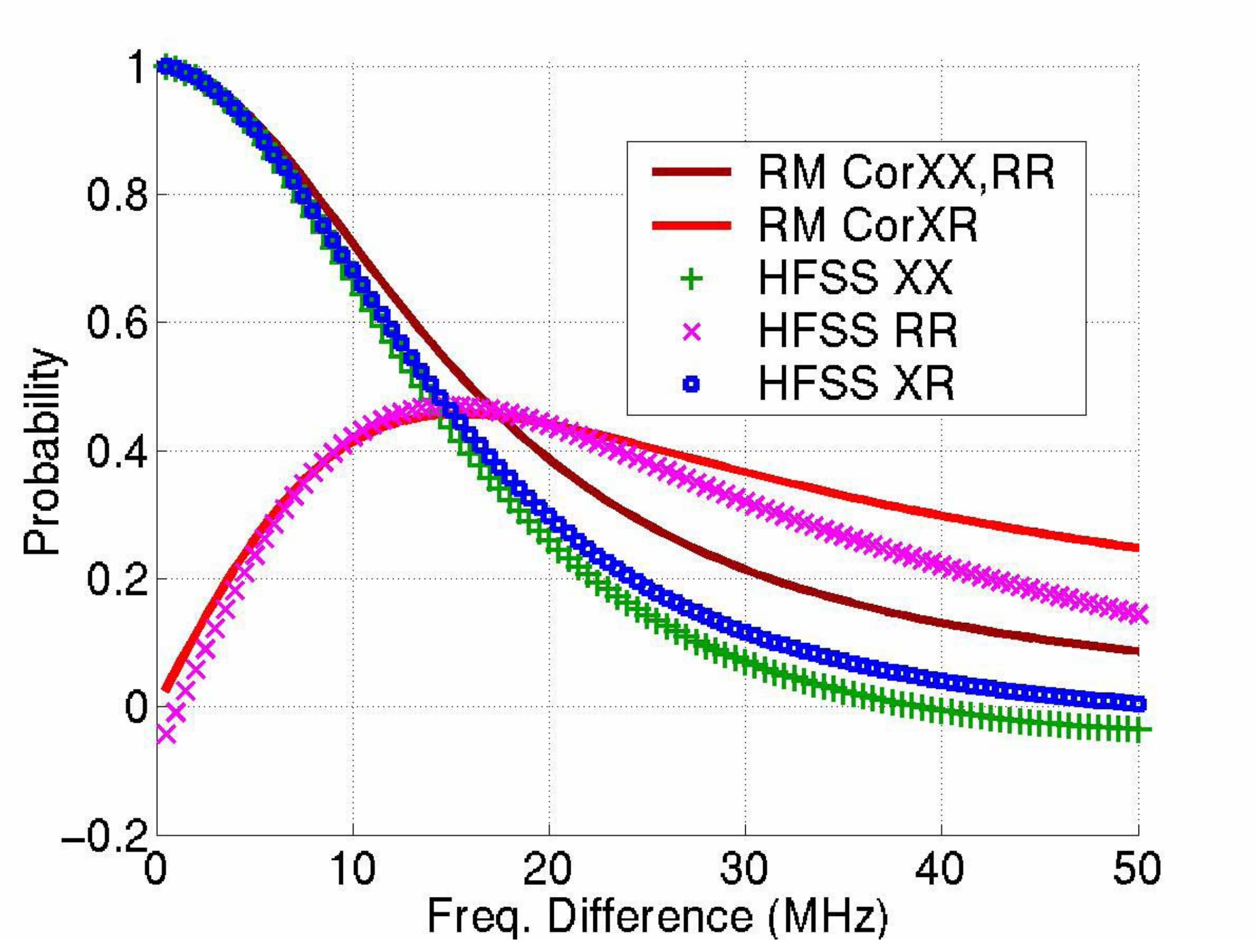
Frequency Correlations in Normalized Impedance Theory and HFSS Simulations

$\boldsymbol{Z_{cav} = jX_R + (\rho + j\xi)\, R_R}$

$RR = \langle(\rho(f_1)-1)(\rho(f_2)-1)\rangle$

$XX = \langle\xi(f_1)\xi(f_2)\rangle$

$RX = \langle(\rho(f_1)-1\,)\xi(f_2)\rangle$



(f_1-f_2)

Properties of Lossless Two-Port Impedance (Monte Carlo Simulation of Theory Model)

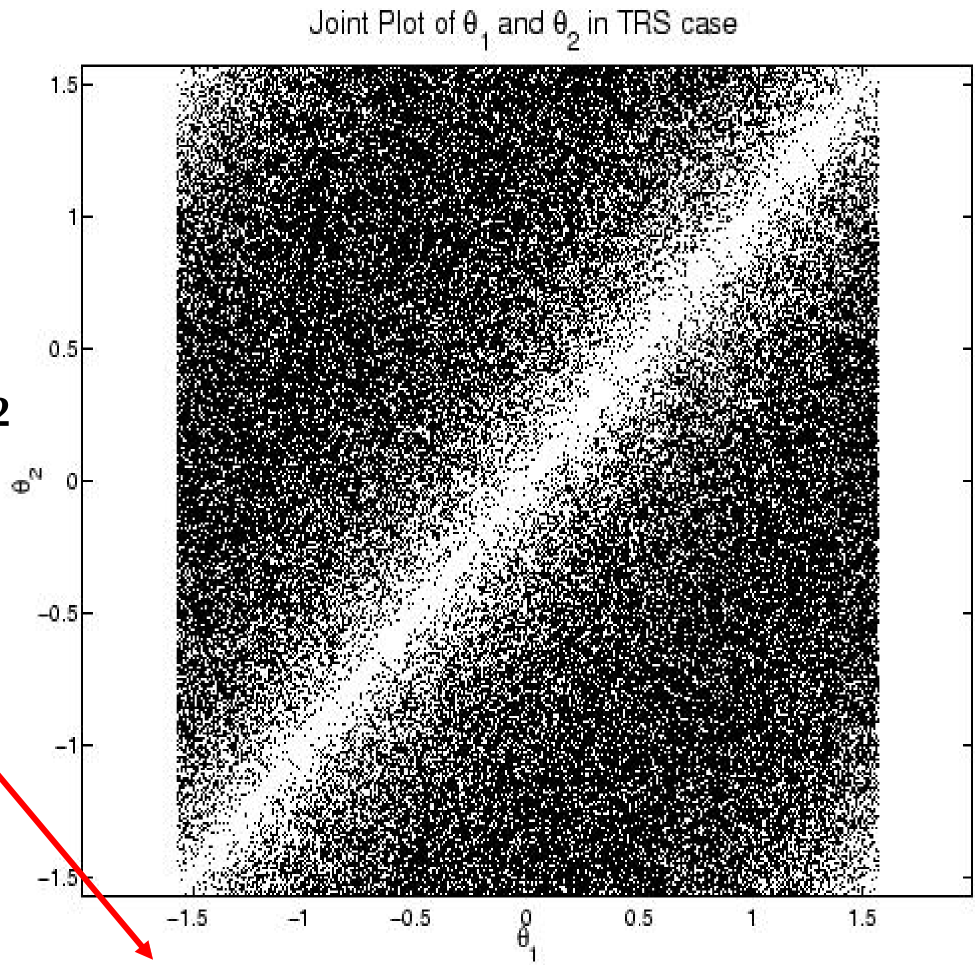
Eigenvalues of Z matrix

$$\det\left|\boldsymbol{Z} - jX\boldsymbol{1}\right| = 0$$

$$X_{1,2} = X_R + \xi_{1,2} R_R$$

$$\xi_{1,2} = \tan\left(\frac{\theta_{1,2}}{2}\right)$$

θ_2

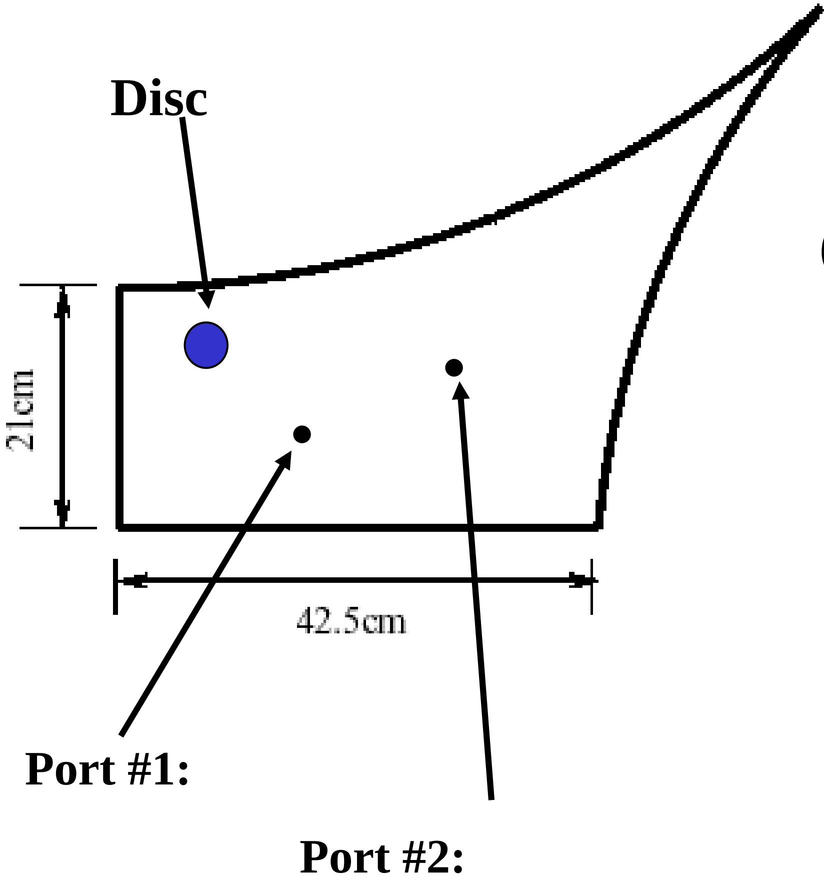


θ_1

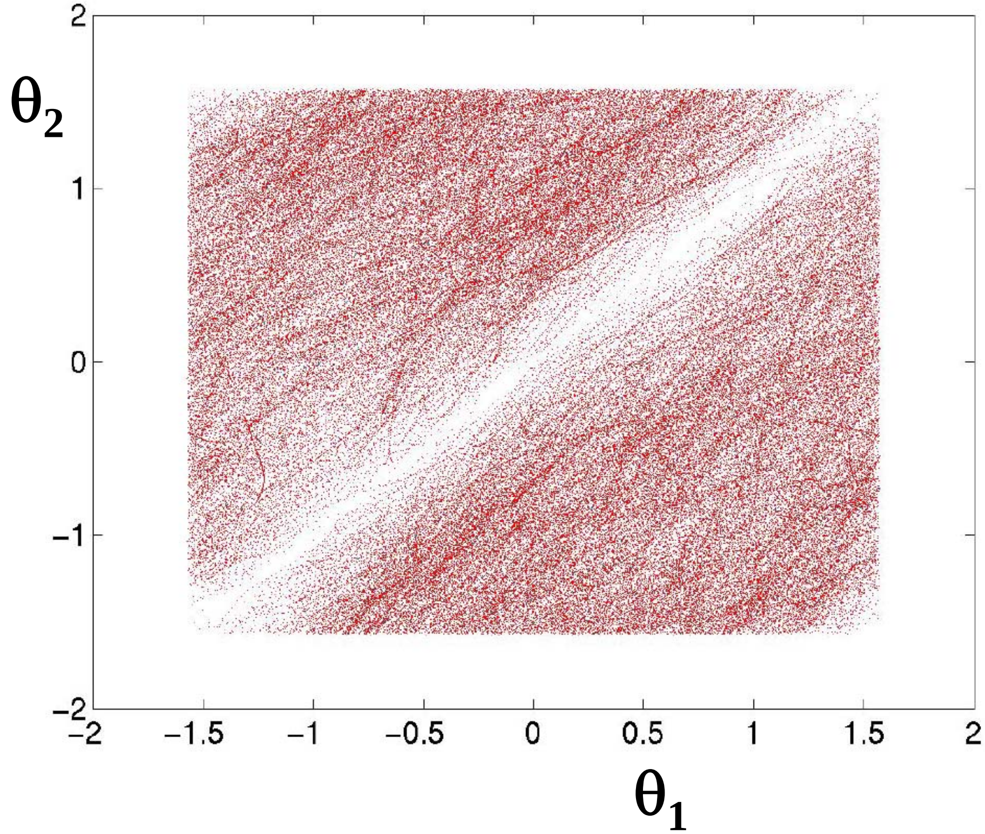
Individually $\xi_{1,2}$ are Lorenzian distributed

Distributions same as In Random Matrix theory

HFSS Solution for Lossless 2-Port

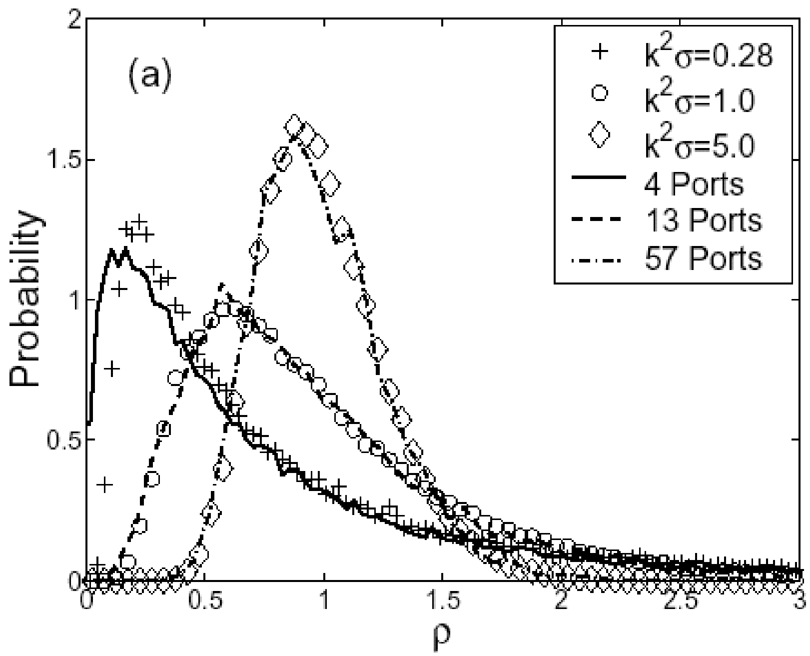


Joint Pdf for θ_1 and θ_2



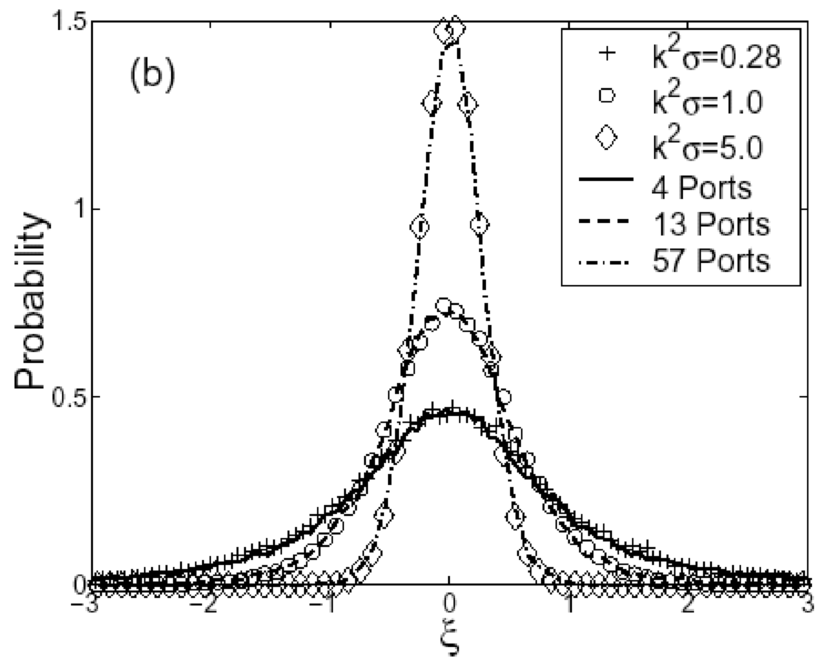
Comparison of Distributed Loss and Lossless Cavity with Ports
(Monte Carlo Simulation)

Distribution of resistance fluctuations
P(ρ)



ρ

Distribution of reactance fluctuations
P(ξ)



ξ

$$Z_{cav} = jX_R + (\rho + j\xi)\, R_R$$

Time Domain Model for Impedance Matrix

Frequency Domain

w_n- Gaussian Random variables

$$Z(\omega) = -\frac{j\omega}{\pi}\sum_n \frac{R_R(\omega_n)}{\omega_n}\frac{\Delta\omega_n^2\, w_n^2}{\omega^2(1-jQ^{-1})-\omega_n^2}$$

Statistical Parameters

ω_n - random spectrum

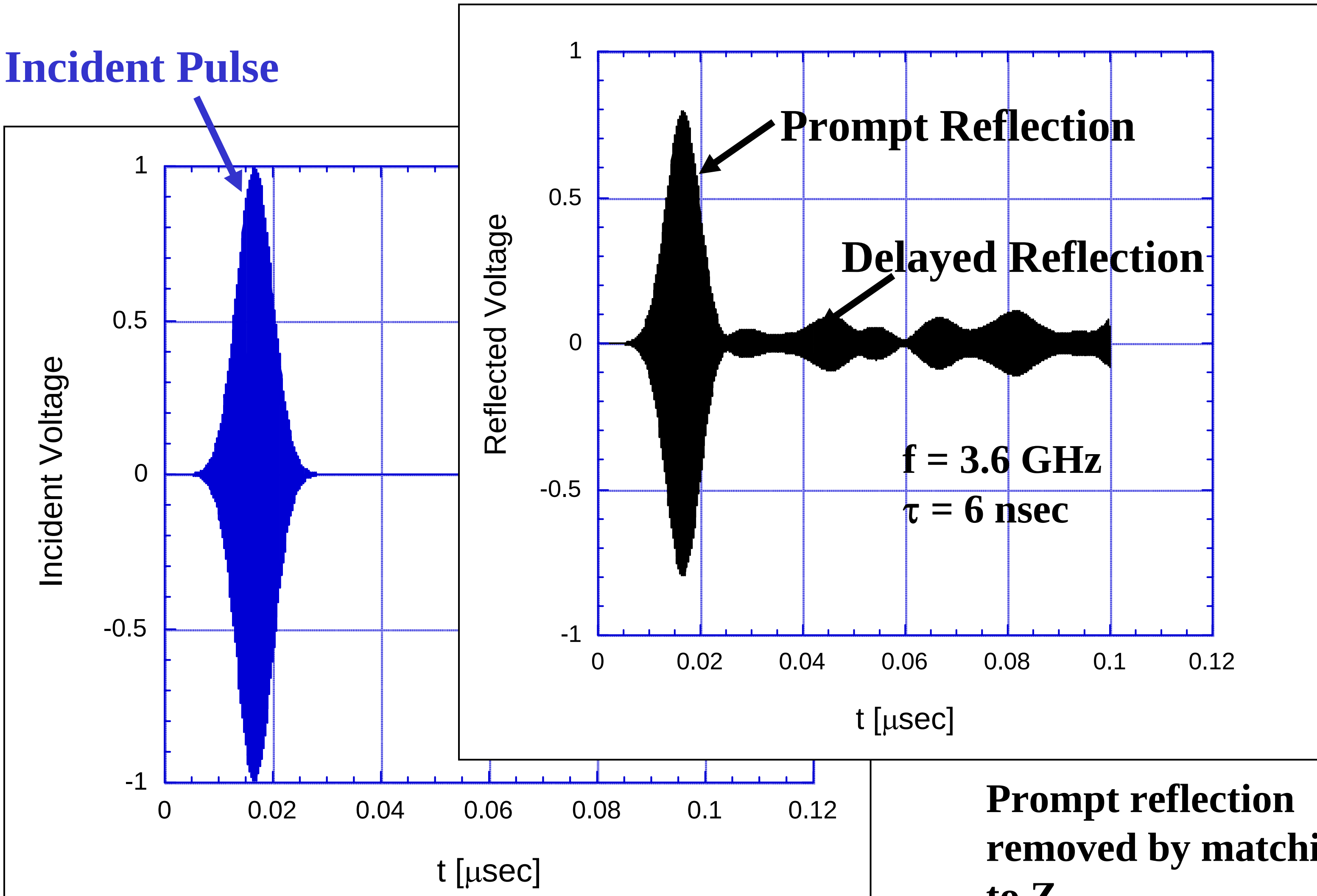
Time Domain

w_n- Gaussian Random variables

$$\left(\frac{d^2}{dt^2}+2\nu_n\frac{d}{dt}+\omega_n^2\right)V_n(t) = -\frac{1}{\pi}\frac{R_R(\omega_n)\Delta\omega_n^2 w_n^2}{\omega_n}\frac{d}{dt}I(t)$$

$$V(t)=\sum_n V_n(t) \qquad \nu_n = \frac{\omega_n}{Q}$$

Incident and Reflected Pulses for One Realization



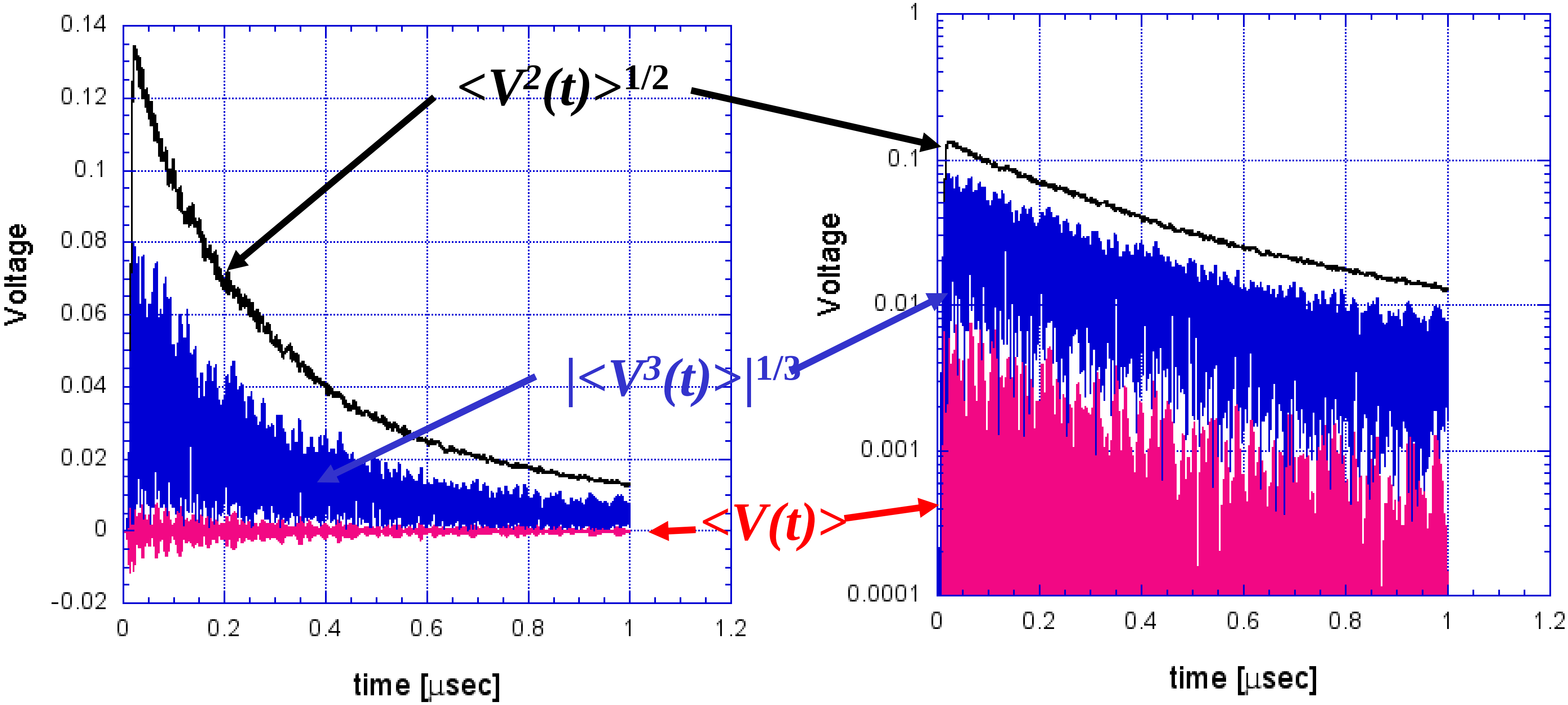
Prompt reflection removed by matching Z_0 to Z_R

Decay of Moments
Averaged Over 1000 Realizations

Prompt reflection eliminated

Linear Scale

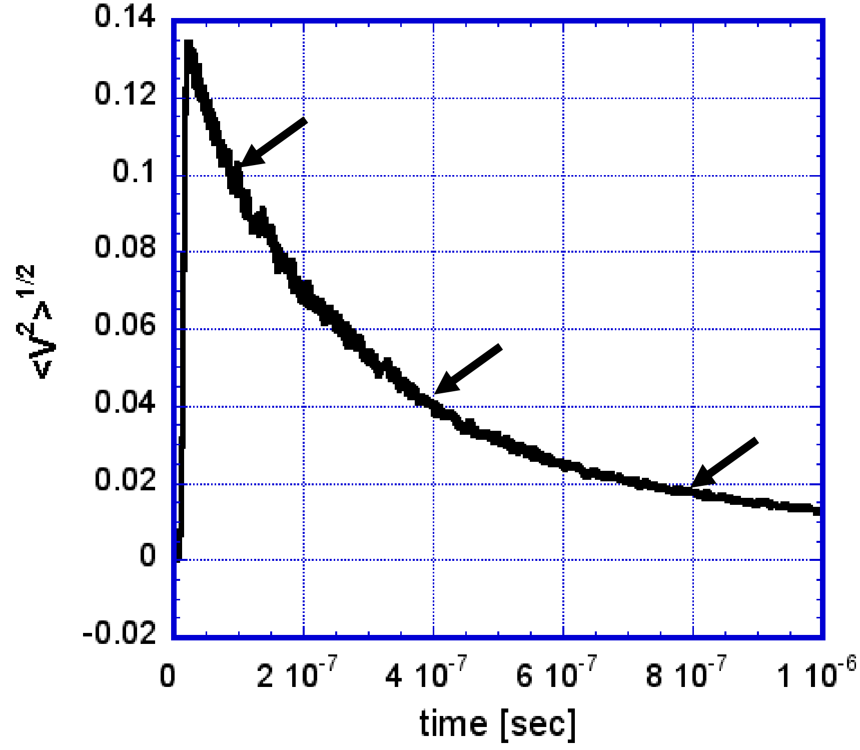
Log Scale



Quasi-Stationary Process

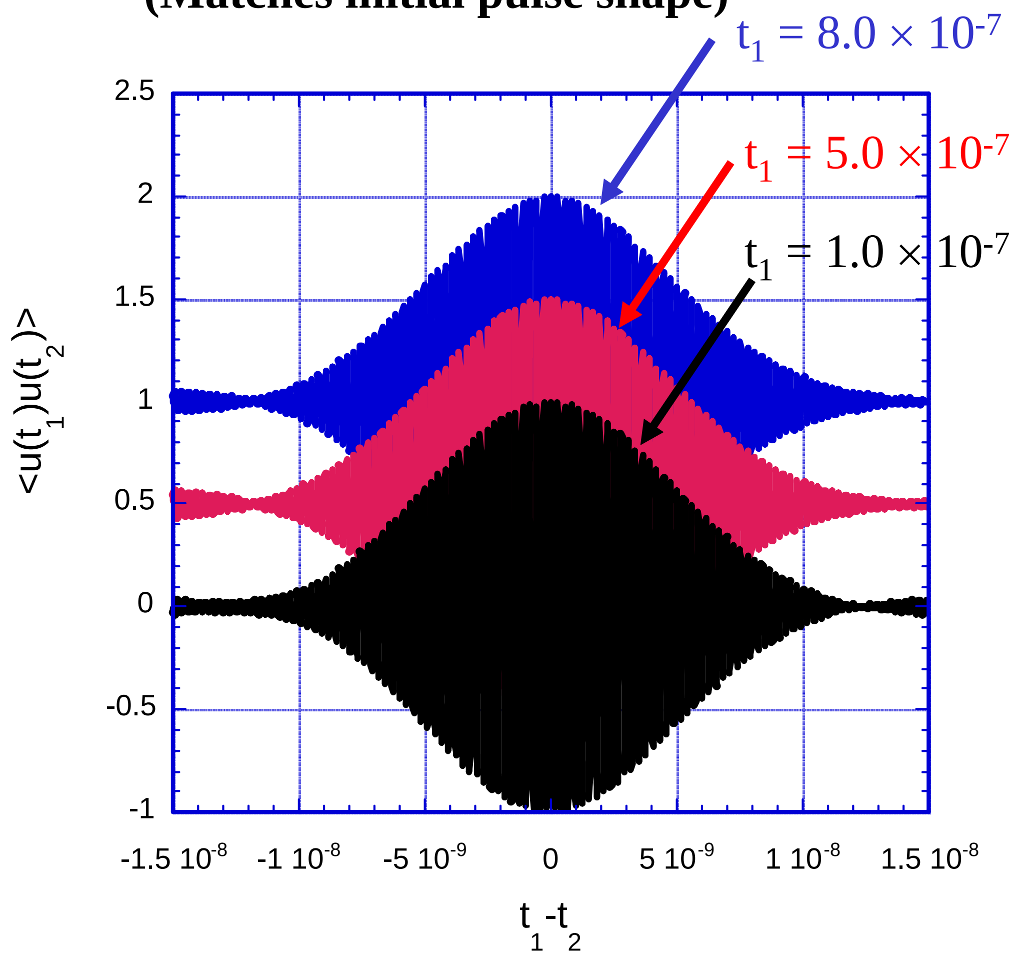
Normalized Voltage

$u(t)=V(t)\,/\,{<}V^2(t){>}^{1/2}$



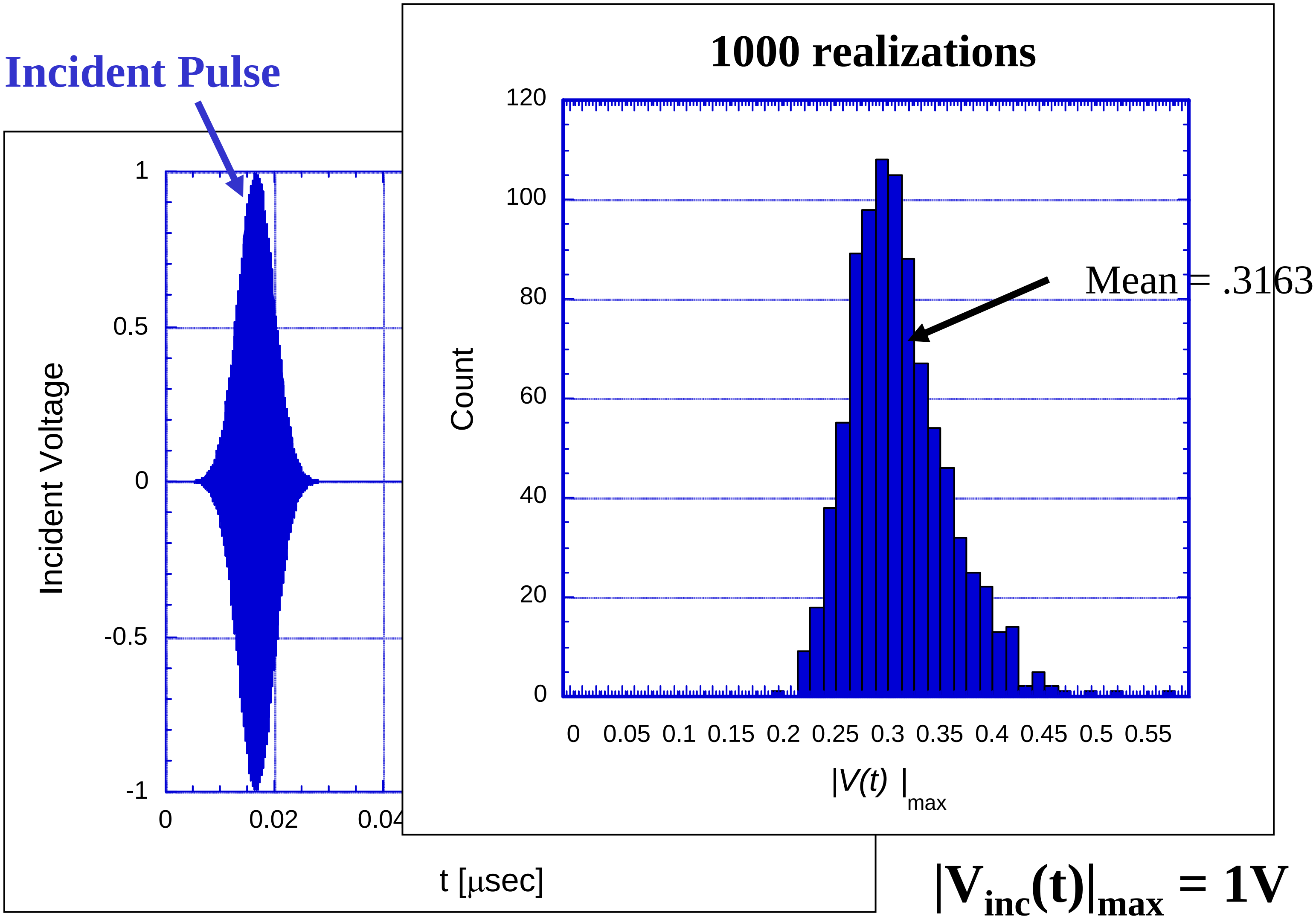
2-time Correlation Function

(Matches initial pulse shape)



Histogram of Maximum Voltage

Incident Pulse



$$|V_{inc}(t)|_{max} = 1V$$

Progress

- Direct comparison of random coupling model with
  - -random matrix theory ✓
  - -HFSS solutions ✓
  - -Experiment ✓
- Exploration of increasing number of coupling ports ✓
- Study losses in HFSS ✓
- Time Domain analysis of Pulsed Signals
  - -Pulse duration
  - -Shape (chirp?)

Current

- Generalize to systems consisting of circuits and fields

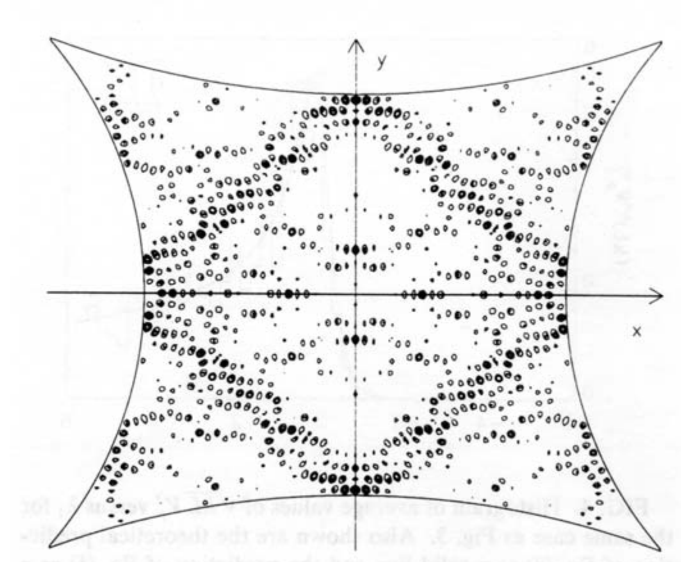
Future

Role of Scars?

- Eigenfunctions that do not satisfy random plane wave assumption

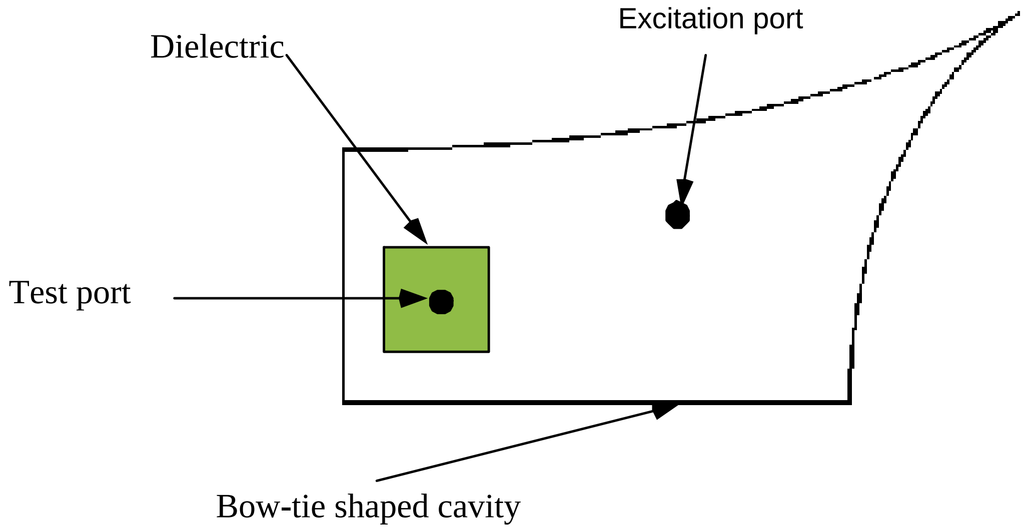
- Scars are not treated by either random matrix or chaotic eigenfunction theory

- Semi-classical methods

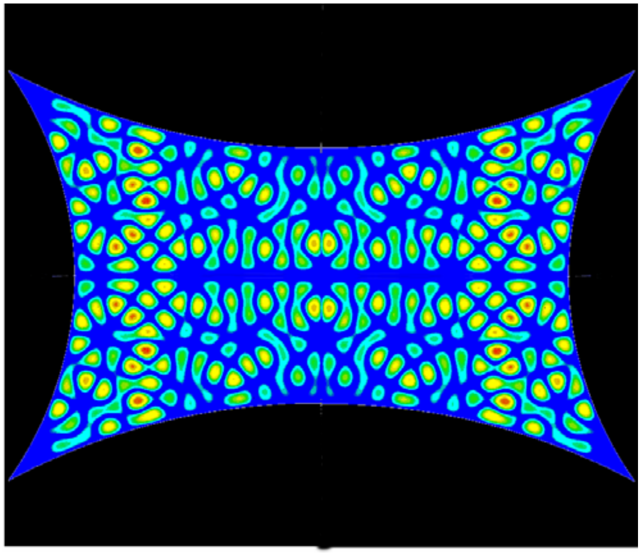


Bow-Tie with diamond scar

Future Directions



- Can be addressed
  - -theoretically
  - -numerically
  - -experimentally



Features:

Ray splitting

Losses

Additional complications to be added later

HFSS simulation courtesy J. Rodgers