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Adjustable resonant cavity for measuring the complex permittivity of dielectric materials

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An adjustable resonance cavity was developed to measure the complex permittivity of dielectric materials. The cavity has an inner diameter of 16.400 cm and an inner height of 2.54 cm. The aluminum stationary wall holder was positioned about 10.8 cm above the top of the cavity. It was fixed into place by three 1.27-cm-diam linear shafts. By suspending from the wall holder, the movable wall moved vertically by sliding on 1.27 cm bore-closed ball bushings. By turning a 1 in.-12 nut, the movable wall could be positioned so that the cavity height equaled the height of the sample. Therefore, this enables the measurement of the permittivity of samples with heights between 0.88 and 1.91 cm and radius between 1.27 and 3.18 cm. The complex permittivity of the sample was calculated based upon the sample dimensions, central frequency of TM$_{ono}$ modes, and Q factor of the resonance curve using an exact solution. The complex permittivity was measured at the three lowest modes, where the frequency span is 1–4 GHz. © *2000 American Institute of Physics.* [S0034-6748(00)01108-4]

I. INTRODUCTION

Resonant cavities have been widely used to measure the complex permittivity of low-loss materials in the microwave frequency range.[1–7] When a dielectric material is placed inside a resonant cavity, the frequency of the resonant modes of the cavity will shift and the resonance curve broadens depending upon the material's complex permittivity and dimensions. This measurement technique is usually classified as destructive because a sample has to be machined to a specific shape and size.[8] Problems arise when one wants to perform permittivity measurements without altering the sample.

Our research required measuring the complex permittivity of alumina composites with variable dimensions. Since there were no commercial or standard measurement techniques that could satisfy our requirements, an adjustable resonant cavity with a movable wall was constructed to accommodate cylindrically shaped samples with variable dimensions. An exact formula was used to determine the material's complex permittivity based on the sample dimensions, resonant frequency, and Q factor. The following describes the design and modeling of a nondestructive resonant cavity.

II. ADJUSTABLE RESONANCE CAVITY: DESIGN AND MODELING

Figure 1 shows the schematic diagram of the resonant cavity, which was made of oxygen-free high-conductivity copper. The cavity has an inner diameter of 16.400 cm and an inner height of 2.54 cm. The aluminum stationary wall holder was positioned about 10.8 cm above the top of the cavity. It was fixed into place by three 1.27-cm-diam linear shafts. By suspending from the wall holder, the movable wall moved vertically by sliding on 1.27 cm bore-closed ball bushings. By turning a 1 in.-12 nut, the movable wall could be positioned so that the cavity height equaled the height of the sample. The sample is placed on the cavity plug and moved into the cavity. Gold contact strips (Instrument Specialties, Delaware Water Gap, PA) were soldered to the edges of the movable wall and cavity plug. These strips provided the high-frequency electrical contact between the cavity walls and plug. The cavity can accommodate samples with a height between 0.88 and 1.91 cm and a radius between 1.27 and 3.18 cm. The mechanical dimensions of the system rather than electrical consideration limit the sample size.

Coupling loops excited and measured H_ϕ modes in the cavity. These antennas, which were placed 90° apart to provide mode selectivity, were made by looping the inner conductor of a 50 Ω semirigid coaxial cable and soldering it to the outer conductor. Coaxial connectors (SMA) were soldered on the other end of these coaxial cables, which were fed through holes in the cavity walls and held into place by modified male Cajon ultra-Torr connectors. As shown in Fig. 2, a vector network analyzer HP 8520C, which was connected to the loop couplers via precision coaxial cables, swept the frequency and measured the transmitted signal.

The network analyzer signal was swept from 1 to 4 GHz. To reduce measurement uncertainty, the measured transmitted signals were averaged from 16 traces. The central frequency of the resonant modes f_o and Q factor of the cavity were found and measured under computer control. In order to determine the cavity losses, the Q factor of the resonant modes was determined by measuring the 3 dB points of the

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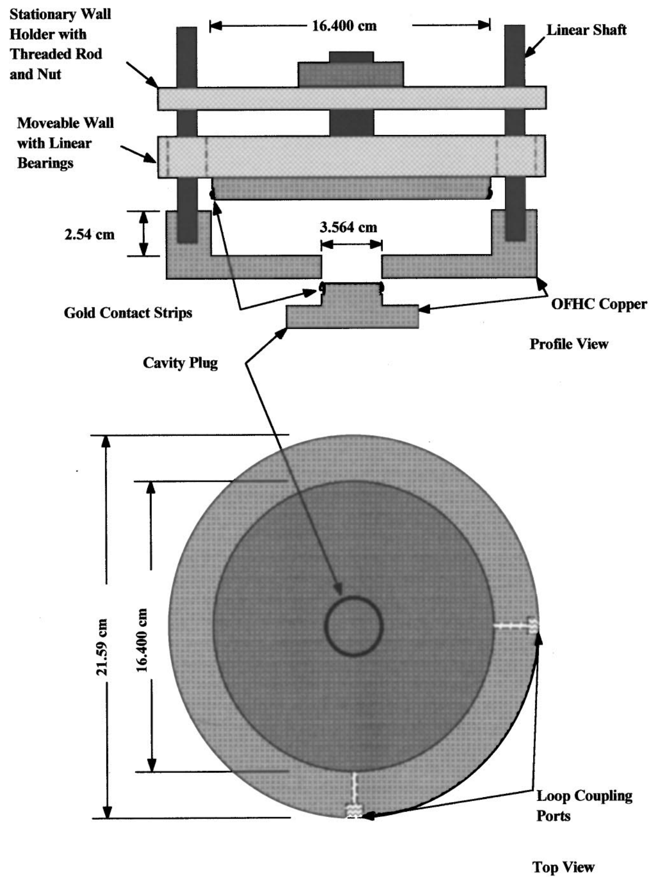

FIG. 1. Schematic diagram of the resonant cavity with a movable wall. The sample is mounted onto the plug which is loaded into the cavity.

resonant curves with and without the sample in the cavity.

The following exact analytical solution allowed computation of complex permittivity of the sample based on these measurements. As shown in Fig. 3, the sample, which is centered in the cavity and labeled as region 1, has material properties of ϵ_1 and μ_1 (ϵ_1 is the permittivity of free space and μ is the permeability of free space). Region 2, which is air, has material properties of ϵ_2 and μ_2. The resonant cavity has an inside diameter of $2b$ and the sample has a diameter of $2a$. The height of the cavity and sample is l.

The electric and magnetic fields within the cavity interact with the material. To relate the shift and broadening of the resonant modes, the electromagnetic fields were matched at the sample-to-air interface. The general expressions for an electric field in axial direction E_z and in region i is

FIG. 2. Picture of the apparatus: the vector network analyzer connected to the resonant cavity with a movable wall.

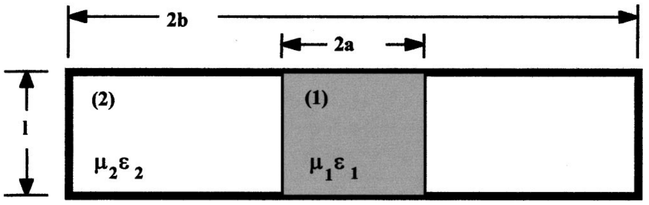

FIG. 3. Profile view of the circular cavity loaded with a sample. The dielectric sample is region 1 (ϵ_1 and μ_1). Region 2 is air (ϵ_2 and μ_2). The thick solid line represents the inner walls of the cavity.

$$E_{zi}=[A_iJ_m(\alpha_i\rho)+G_iN_m(\alpha_i\rho)] \times(C_ie^{im\phi}+D_ie^{-im\phi})[E_i\cos(kz)+F_i\sin(kz)], \tag{1}$$

where α_i is defined by

$$\alpha_i^2=\mu_i\epsilon_i\omega^2-k^2. \tag{2}$$

In Eq. (1) and (2), J_m and N_m are the Bessel and Newman functions of order m. A, G, C, D, E, and F are constants. ρ is the radius and ϕ is the angle around the axis of symmetry, α is the transverse wave number, ω is the angular frequency, and i is the region in the cavity ($i=1,2$), as shown in Fig. 3.

Since the diameter of the cavity is greater than its height, the lowest-order modes in a cylindrical cavity are TM modes, where $B_z=0$. Boundary conditions at the top and bottom ends of the cavity require that the electric field parallel to a conducting surface to be zero. Thus,

$$kl=p\pi, \quad p=0,1,2,\ldots.$$

Applying this boundary condition at the cavity walls and matching the tangential electric and magnetic fields at the sample interface provides the following relationship:

$$\frac{\epsilon_1}{\alpha_1}\frac{J_m'(\alpha_1a)}{J_m(\alpha_1a)} =\frac{\epsilon_2}{\alpha_2}\left[\frac{J_m(\alpha_2b)N_m'(\alpha_2a)-J_m'(\alpha_2a)N_m(\alpha_2b)}{J_m(\alpha_2b)N_m(\alpha_2a)-J_m(\alpha_2a)N_m(\alpha_2b)}\right]. \tag{3}$$

The complex permittivity of the sample can then be calculated from the measured complex angular frequency of the cavity,

$$\omega=2\pi\left(f_o+\frac{f_o}{2Q_s}\right), \tag{4}$$

and numerically solving Eq. (3) for the complex permittivity. In this procedure, Q_s is the Q factor of the sample,

$$\frac{1}{Q_s}=\frac{1}{Q_l}-\frac{1}{Q_e}, \tag{5}$$

where Q_l is the Q factor of a loaded cavity and Q_e is the Q factor of the empty cavity. A two-variable minimization routine determined the complex permittivity of the material for a given estimate of the value. The error in calculating ϵ' was about 0.005% and in calculating ϵ'' is about 0.14%.

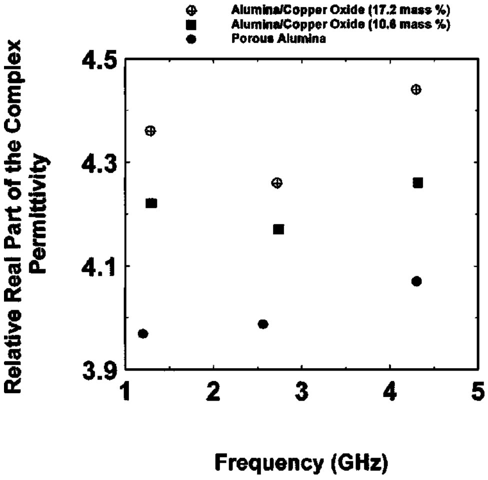

FIG. 4. Measured relative real part of the complex permittivity of porous alumina composites with respect to frequency.

Finally, the movement of the cavity wall and cavity plug change the electrical contact between cavity walls, which affect the accuracy of the permittivity measurements. Good and consistent electrical contact could be indicated by the accuracy and reproducibility of the measured f_o and Q_e. The reproducibility of the measured f_o is within 0.01%. Actually, the measured central frequency of the resonant modes was consistently about 0.4% below the theoretical values due to the resistive wall losses. The Q factor of a cavity depends upon the cavity height and conductivity of the walls. The average reproducibility of the Q_e was about 5%. But, the measured Q_e, which ranged from 2000 to 3000, was generally an order of magnitude larger than Q_s. Thus, the uncertainty in the measured complex permittivity due to variance in electrical contact is greatly diminished.

III. DIELECTRIC MEASUREMENTS

This resonant cavity has been used to measure the complex permittivity of porous alumina composites with various cylindrical dimensions.[9] A full description of sample preparation with various concentrations of copper oxide has been given by Gershon.[9] Figure 4 shows the relative real part of the complex permittivity of some composites with respect to frequency. The relative real part of the complex permittivity increased linearly with concentration of copper oxide.[9] Figure 5 shows the relative imaginary part of the complex permittivity of composites with respect to frequency, which increases with the concentration of copper oxide.

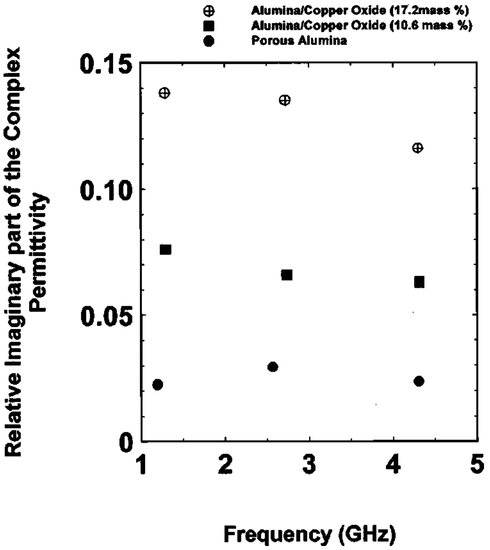

FIG. 5. Measured relative imaginary part of the complex permittivity of porous alumina composites with respect to frequency.

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