Liquid metal flow encasing a magnetic cavity

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A stationary equilibrium of a liquid metal flowing past a cylindrical magnetic cavity is presented. The cavity has an azimuthal magnetic field and can also have an axial field. The liquid metal flow can be maintained by a sufficiently high pressure head. The scheme could be used to support a flowing liquid wall for systems producing high heat fluxes. © *2000 American Institute of Physics.* [S1070-664X(00)02803-2]

A thermonuclear fusion plasma affords an example of a system where the first wall may be exposed to unacceptably high heat fluxes. Conventional cooling techniques and materials appear to not be viable. Flowing liquid walls have been proposed as a means to extract the heat rapidly.[1,2] For a flowing liquid metal wall, one must consider the magnetohydrodynamics (MHD) of a flowing, conducting fluid in the presence of strong magnetic fields. In this paper, we present an analytic MHD equilibrium solution for a liquid metal flowing past a cylindrical magnetic cavity. In essence, we envision a flow fast enough that the poloidal magnetic field of the cavity is excluded from the liquid metal, thus obviating the constraints of the frozen-in theorem and allowing for a steady state (the axial field in the cavity is convected along in frozen-in fashion).

Consider the system shown in Fig. 1. The system is two-dimensional, i.e., the direction into the page $(\hat{z})$ is a symmetry direction. Liquid metal (LM) coats an outer wall and encases a magnetic cavity. The shape of the LM-cavity interface is arbitrary, given by $r=R(\theta)$. The shape of the outer wall will be specified later. LM flows in from the top and emerges from the bottom. Gravity $\mathbf{g}$ points downward. In the zero-resistivity theory, the magnetic field is zero in the LM. For simplicity, we assume that there is a current carrying wire going into the page that accounts for the magnetic field in the cavity.

The equations governing the systems are as follows. In the simplest case, the magnetic field is represented by $\mathbf{B}=\hat{z}\times\nabla\psi$, where ψ is the magnetic flux function. The incompressible LM flow is written $\mathbf{u}=\hat{z}\times\nabla\phi$ where ϕ is the flow stream function. The magnetic flux function in the cavity satisfies $\nabla^2\psi=0$. Accordingly, we let ψ be of the form

$$\psi/B_0=a\ln(r/a)-\epsilon_1 r\cos\theta+\epsilon_2(r^2/a)\cos 2\theta, \tag{1}$$

where (r,θ) coordinates are centered at the wire, the wire current $I_0\equiv B_0ac/2$, and ϵ_1 and ϵ_2 are small parameters to be determined later. In the LM, we assume that the poloidal flux, ψ, is zero in the limit that the resistivity $\eta\rightarrow 0$. This will be justified subsequently. In this limit, the LM flow satisfies $\mathbf{u}\cdot\nabla\nabla^2\phi=0$, where $\phi(|x|\rightarrow\infty)\rightarrow u_0 r\sin\theta$ consistent with $\mathbf{u}\rightarrow -u_0\hat{x}$ at large $|x|$. We have also neglected the viscosity. Since the vorticity is zero at infinity, we have $\nabla^2\phi=0$ everywhere. The general solution is $\phi\sim r^{\pm m}e^{im\theta}$. Finally, at the interface, we must have

$$p(\text{interface})=B^2(\text{interface})/8\pi, \tag{2}$$

where p is the liquid pressure, neglecting surface-tension.

To obtain an analytic solution, we adopt the ordering

$$\rho u_0^2\sim\rho g a\ll p\sim B_0^2/8\pi,$$

i.e., the magnetic field is strong and the liquid pressure builds up to balance the magnetic pressure to lowest order. The inertial and gravitational forces are also assumed weak. Thus, to lowest order, we have a cylindrically circular magnetic bubble

$$\psi_0=B_0a\ln(r/a),$$

the liquid pressure p_0 must be such that it balances the magnetic pressure of the bubble, viz.,

$$p_0=B_0^2/8\pi, \tag{3}$$

and the flow stream function is given by

$$\phi_0=u_0a\left(\frac{r}{a}-\frac{a}{r}\right)\sin\theta$$

corresponding to LM flow past a smooth cylindrical obstacle. The interface is specified as $R_0(\theta)=a$. To this order, p_0 must equal p_∞, the pressure applied to the LM at infinity. For a given current in the cavity, p_∞ determines a, the radius of the cavity. Alternatively, for a given current and a given volume of LM in a closed cavity, p_∞ is the pressure that the LM is placed under by the magnetic field.

To first order, the gravitational field and the Bernoulli forces cause a distortion of the bubble. We assume that ψ distorts according to Eq. (1). Accordingly, the surface $\psi=0$ (correct to first order) is given by the equation

$$r/a=1+\epsilon_1\cos\theta-\epsilon_2\cos 2\theta. \tag{4}$$

We must now satisfy the pressure balance condition Eq. (2) at the distorted surface. We first calculate B^2 at this surface. There, we have $B^2/B_0^2=|\nabla\psi|^2/B_0^2\simeq(a/r)^2-2\epsilon_1(a/r)\cos\theta+4\epsilon_2\cos 2\theta$. When evaluated at the surface given by Eq. (4), we find

$$B^2/B_0^2(\text{interface})\simeq 1-4\epsilon_1\cos\theta+6\epsilon_2\cos 2\theta. \tag{5}$$

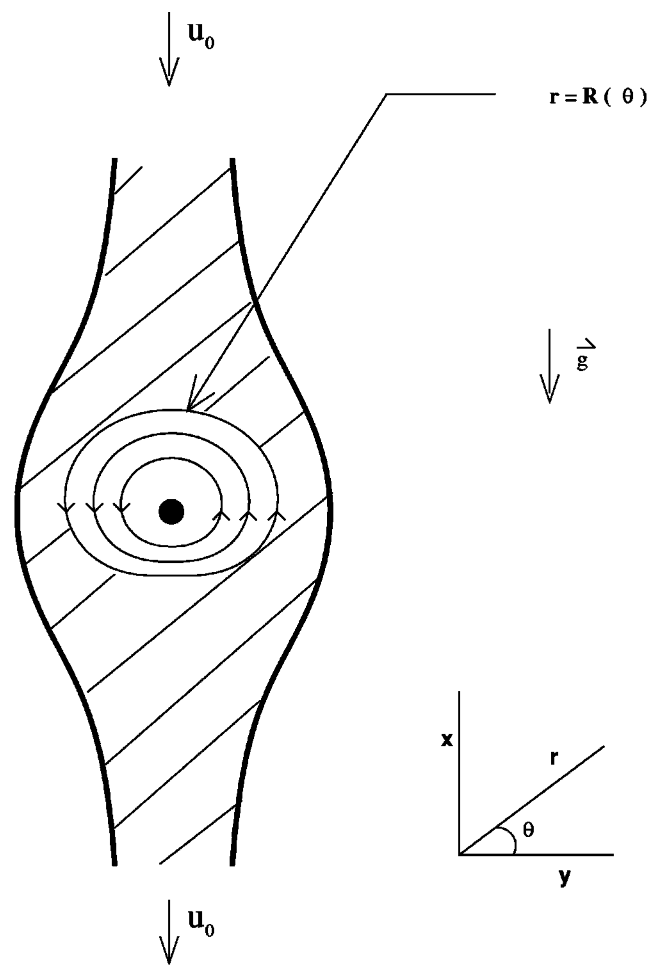

FIG. 1. Schematic depiction of LM flow past magnetic bubble. The distortions of the bubble are slight but otherwise as expected. A central wire prevents the bubble from floating up. In this schematic, the outer wall is placed to be coincident with the flow streamline for the simplest case.

The $B^2/8\pi$ in the above must be equal to $p(\text{interface})$. The hydrodynamic pressure in the LM is given by the equation $\rho\mathbf{u}\cdot\nabla\mathbf{u}=-\nabla p+\rho\mathbf{g}$. Dotting with $\mathbf{u}\cdot$ and integrating along a streamline, we obtain Bernoulli's law as a condition at the bubble-surface: $p+(1/2)\rho u^2+\rho g x=P_\infty$. Thus, the liquid pressure (correct to first order) is

$$p\approx P_\infty-(1/2)\rho u_\theta^2-\rho g x$$

$$=P_\infty-(1/2)\rho u_0^2(1+a^2/r^2)^2\sin^2\theta-\rho g r\cos\theta$$

from which

$$p(\text{interface})\simeq P_\infty-\rho u_0^2(1-\cos 2\theta)-\rho g a\cos\theta. \tag{6}$$

Matching Eqs. (5) and (6), we find

$$\epsilon_1=(4\pi\rho g a/2B_0^2),\quad \epsilon_2=(4\pi\rho u_0^2/3B_0^2), \tag{7}$$

$$B_0^2/8\pi=p_\infty-(1/2)\rho u_0^2+\rho g h, \tag{8}$$

where h is the height at which the flow speed is equal to u_0 and the pressure in the LM is p_∞.

We note that both ϵ_1 and ϵ_2 are positive. From Eq. (4), we conclude that the magnetic cavity shifts upward and distends to an ellipse with the major axis in the $\hat{y}$ direction. The shift upward is due to buoyancy—the cavity is indeed a bubble and will tend to float up. This can be checked by calculating the force on the central wire due to the magnetic field. This upward force, all of which comes from the cos θ term in Eq. (5), exactly equals $(\pi a^2)\rho g$, the weight of the displaced fluid.

In our case, the central wire prevents the cavity from lodging on to the top wall; in a real system, one may have to add ''stabilizer fields'' as discussed further below. The elliptical distention is from the Bernoulli effect — the flow has top and bottom stagnation points, resulting in Bernoulli pressure highs top and bottom (compared to left and right). This leads to an elliptical distention.

An arbitrary constant axial field, B_z, can now easily be added to the above solution. To see this, one needs to examine the complete force balance equation and Faraday's Law to conclude immediately that a constant B_z preserves the solution above. In effect, the axial flux does not change the pressure jump condition, being the same on both sides, and is convected along by the incompressible LM flow, entering and exiting with the flow.

Also, it can be readily shown that if plasma with pressure p_{in} is added to the magnetic cavity, the only effect is to modify Eq. (3) to read $p_0=B_0^2/8\pi+p_{in}$, and a term $-p_{in}$, is added to the right hand side of Eq. (8).

A physical wall may be placed to coincide with one of the streamlines, in the simple analytically tractable case. Of course, a more realistic placement of the outer wall would necessitate using a more complete set of the flow eigenfunctions but would not change the essential physics of our solution.

The above theory is valid provided the LM flows on a time scale short compared with the time scale for the magnetic field inside the cavity to resistively diffuse into the LM. This condition may be expressed quantitatively as

$$\frac{u_0}{\pi a}\gg\frac{\eta}{w^2},$$

where w is the width of the LM layer. The zero-resistivity theory presented above includes a current sheet at the cavity/LM interface. Allowing resistivity broadens this sheet. As a result, some poloidal magnetic field penetrates the LM; this penetration is countered by the rapid flow that, in effect, convects the flux back into the cavity. The resulting steady state gives a penetration depth Δ given by

$$\Delta^2=\pi a\,\eta/u_0\ll w^2.$$

Introduction of resistivity breaks the top–bottom symmetry of the ideal problem.[3] The incoming fluid at the top tends to convect flux back into the cavity whereas the outgoing fluid at the bottom tends to pull the flux further into the LM. The resulting flux in the boundary layer is expected[3] to have the shape shown in Fig. 2.

An analytic solution in the boundary layer is difficult. Because of the ordering $\rho u_0^2<B_0^2$, the flow in the layer is expected to be sub-Alfvénic close to the interface but transits to super-Alfvénic through the layer. The layer ordering is then $\rho u_0^2\sim B^2$, possibly leading to an Alfvén resonance. The

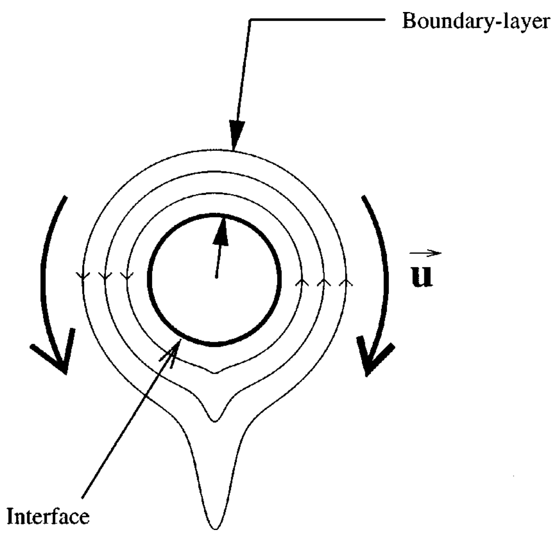

FIG. 2. Conjectured depiction of the magnetic surfaces in the boundary layer between the bubble surface and the LM (exaggerated). The asymmetric stretching out of the field is based on the findings of Ref. 3.

resistivity is an order unity effect in the layer, possibly mitigating any highly reactive resonances. In any case, the layer equations can be written

$$\rho \mathbf{u}\cdot \nabla \nabla^2 \phi = \mathbf{B}\cdot \nabla \nabla^2 \psi, \tag{9}$$

$$\mathbf{u}\cdot \nabla \psi = \eta \nabla^2 \psi. \tag{10}$$

As we move away from the layer, the above set reduces to $\rho \mathbf{u}\cdot \nabla \nabla^2 \phi \rightarrow 0$, $\mathbf{u}\cdot \nabla \psi \rightarrow 0$, which are the correct ''outer'' equations. In the layer, no terms can be neglected. This system is difficult to solve. The second equation, Eq. (10), may be scaled: $u_r B_\theta \sim \eta (B_\theta/\Delta)$. Using $u_r \sim u_0 \Delta/a$ (since $u_r \rightarrow 0$ at the interface) we have

$$\frac{u_0}{a} \sim \frac{\eta}{\Delta^2},$$

the expected scaling for Δ.

In summary, our solution of a mildly distorted cavity has been obtained if the assumptions

$$\rho g a, \quad \rho u_0^2 \ll B_0^2/8\pi$$

are made. In addition, the LM flows relatively freely past the ''obstacle'' in a channel of width w provided the condition

$$\frac{u_0}{\pi a} \gg \frac{\eta}{w^2}$$

is met. We have also assumed that an inviscid solution is valid since the viscosity is much smaller than the resistivity. Surface-tension is also neglected. To devise an experiment to test this, all of the three inequalities must be satisfied. In addition, the heat generated from the current sheet at the interface must not overheat the LM. The generated power is

$$P \simeq \eta \left(\frac{cB_0}{4\pi\Delta} \right)^2 (2\pi a) L \Delta. \tag{11}$$

This does not include the power required to overcome viscous stresses in likely turbulent flow. It may be possible to test the above scenario by a small sized experiment. As representative numbers, consider $B_0 = 3$ kG, $a = 20$ cm, $w = 10$ cm, $L = 10$ cm. Here, B_0 is the poloidal field at the interface, a is the radius of the interface, w is the width of the LM layer about the cavity, L is the length of the cylinder. For these parameters, the relevant dimensionless numbers are $8\pi\rho g a/B_0^2 \simeq 1/18$, $8\pi\rho v_0^2/B_0^2 \simeq 1/2$, where we define v_0 to be given by $v_0/\pi a = \eta/w^2$, and $\eta = 830$ cm^2/s for liquid sodium. Thus, the flow speed would have to exceed v_0. For the above numbers, $v_0 \simeq 6$ m/s. The ohmic power dissipated in the boundary layer works out to $P = 8000$ W. The magnetic field is 3 kG at the interface which corresponds to a wire current of $I_0 = 300$ kA. To maintain the flowing wall, a sufficiently large pressure head is all that is required: this is different from other proposed methods wherein an external voltage is required to maintain the wall.[4]

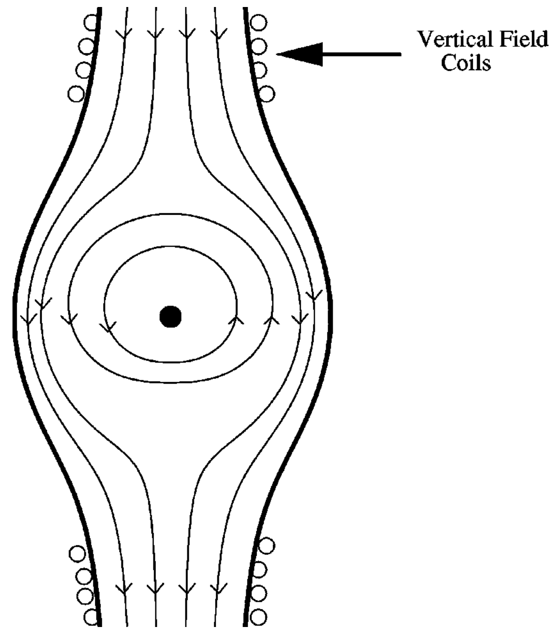

FIG. 3. Schematic depiction of placement of the external ''guide'' magnetic field. This might mitigate the buoyancy and any Bernoulli instabilities. Note that an X-point is introduced at $\theta = 0$ but might be benign given the large flow rates.

Various uncertainties remain that may adversely impact the LM scheme.

(1) The appearance of an upward cos θ shift in the solution indicates buoyancy. In our calculation, the central current coil will prevent an unchecked buoyant rise; in a tokamak-like plasma, such rise can continue until the plasma cavity lodges against the top wall, possibly plugging the flow inlet.

(2) The equilibrium calculation we describe is likely to be unstable to up–down or left–right ''Bernoulli Shifts.'' For example, a leftward shift constricts the flow channel, which speeds up the flow, in turn, leading to a Bernoulli pressure drop, thus accentuating the leftward shift.

Both shifts (1) and (2) could possibly be cured by the addition of a vertical field, depicted in Fig. 3. This externally imposed vertical field is small compared to B_0 since it simply counters first order gravitational and Bernoulli effects. It can readily be added to our calculation above: it can be shown that the sole effect is to shift the definition of the Bernoulli parameter according to $\epsilon_2 \rightarrow \epsilon_2[1-B_v^2/(4\pi\rho u_0^2)]$, where B_v is the size of the vertical field at infinity; although in a finite-sized vessel, we expect this term to also modify ϵ_1 (as a function of the central wire's height in the vessel), and thus modify the force on the central wire, thereby halting the buoyancy at a particular height of the bubble in the vessel. An X-point is necessarily introduced into the boundary layer but should not lead to deleterious behavior on account of the rapid flow.

(3) This flow is at a high viscous Reynolds' number and significant bulk-disturbances and surface turbulent waves can be expected. We have not addressed these issues. The possibility of bubbles ripping from the cavity and advected downstream at the lower hyperbolic point cannot be discounted.

(4) The resistive boundary layer calculation remains to be done. This calculation must be done to settle two outstanding, possibly deleterious effects: first, there is an Alfvén resonance, as pointed out already, and, second, it is not possible to rule out a solution that involves a slow, cylindrically symmetric diffusion of cavity magnetic flux into the LM, thus eroding the LM coating width w. In particular, one outstanding question is whether this erosion will occur until the magnetic energy density at the new flowing-LM/static-LM boundary just equals the kinetic energy of the flowing LM. This would imply that the required LM speed is governed not only by the resistive rate but also by the Alfvén speed. These issues probably will have to be resolved via a 2D numerical simulation.

(5) Our calculation needs to be extended to toroidal geometry. While in the cylindrical geometry done here, an axial field leaves intact the conclusions of our paper, in toroidal geometry the toroidal field enters in a nontrivial manner. In particular, the $1/R$ drop off results in further skin currents driven in the liquid metal to make the toroidal field convect consistently with an LM flowing incompressibly in toroidal geometry. Work in this area is in progress and will be reported elsewhere.

(6) How the system should be started up to get to the steady state needs addressing. We have investigated some scenarios. A possible, first-step experiment might consist of a geometry as in Fig. 1 with all the LM in a (possibly pressurized) tank initially poised at the top. A spigot is now opened and LM blows down, rapidly enough to ensure frozen-in conditions. Flux is swept out with the flow until such a point at which the ram pressure equals the magnetic pressure (akin to the solar wind/magnetosphere interaction). After this point, the flow naturally diverts around the bubble, sweeping out the remaining field. This situation can obviously be made steady-state by appropriately recycling the LM.

(7) The Ohmic power dissipated in the boundary layer is a concern. While this power is quite reasonable for a small experiment, as discussed above, it comes to represent significant circulating power when scaled to, say, a tokamak reactor. While this is a concern, we note that the power scaling varies strongly with B, possibly B^3, so that extrapolations to reactor regimes, especially given future innovations and advanced fusion schemes, may be premature.

A flowing LM wall would, as already mentioned, greatly mitigate the ''first wall'' problem of fusion reactors. In addition, LM walls could also work to ameliorate the tokamak disruption problem. In a tokamak disruption, magnetic energy is released very rapidly, resulting in large mechanical stresses from eddy currents excited in the support structures. If there were a LM wall, a significant part of the energy would be dumped into splashing the LM, in effect putting a damper on the rapid energy release.

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[1]R. W. Moir, Nucl. Fusion **37**, 557 (1997).
[2]N. B. Morley and M. A. Abdou, Fusion Technol. **31**, 135 (1997).
[3]Z. Chacko and A. B. Hassam, Phys. Plasmas **4**, 3031 (1997).
[4]L. Zakharov, private communication (1999).