Velocity Shear Stabilization of Centrifugally Confined Plasma

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A magnetized, centrifugally confined plasma is subjected to a 3D MHD stability test. Ordinarily, the system is expected to be grossly unstable to "flute" interchanges of field lines. Numerical simulation shows though that the system is stable on account of velocity shear. This allows consideration of a magnetically confined plasma for thermonuclear fusion that has a particularly simple coil configuration.

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In essence, nearly all magnetic schemes to confine plasmas for thermonuclear fusion are based on the idea that energetic charged particles gyrate tightly about a magnetic field line which is then configured to close on itself inside the system [1]. This requirement on closure of field lines (at least six confinement schemes are based on this idea) implies significant constraints in coil design. It would be desirable to relax this constraint, say, by allowing "open" field configurations (wherein the field lines are not confined but the particles are). One well-known open configuration is the magnetic mirror [1]. This scheme relies on the mirror forces to reflect particles at the mirror throats and so contain plasma. But mirror reflection can contain plasma only up to a collision time, beyond which particles scatter into a "loss cone" and are lost on the open field line. Another issue for mirrors is the MHD stability of the mirror: the magnetic configuration is inherently unstable to "flute" interchanges of field lines. Basically, a field line loaded with hot particles can interchange with one of cold particles, thus releasing net potential pressure energy, akin to the Rayleigh-Taylor gravitational energy release in ordinary fluids. While it is possible to suppress this interchange in advanced mirror schemes, the latter come with greater magnetic coil complexity and, in any case, do not necessarily resolve the loss-cone issue mentioned.

The centrifugally confined plasma scheme [2,3] is an open field line configuration which holds the promise of overcoming these drawbacks. In the centrifuge scheme, a magnetic mirror-type plasma is made to rotate azimuthally at supersonic speeds, in accordance with frozen-in $\vec{E} \times \vec{B}$ motion. The resulting centrifugal forces, given the field line curvature, prevent the escape of ions along the open lines—the mirror forces become irrelevant and the loss cone is erased. The MHD flute stability issue, however, is intricate and constitutes the subject of this Letter. A quick assessment of flute stability is as follows: at first glance, it would seem that the outward centrifugal force adds to the unfavorable gravitational acceleration and makes the interchanges even more potent. There is, however, a new ingredient, shear in the angular frequency of rotation (a sheared flow is inevitable for plasma situations): it has become increasingly clear over the past decade that flow shear can stabilize interchanges (among other plasma instabilities), basically by introducing a shearing frequency that tears apart convection cells before they can release energy [4–6]. Thus, the overall flute stability is a result of these competing effects. To make matters more complicated, gradients in the flow shear might introduce Kelvin-Helmholtz instabilities: the quick assessment is that the latter would likely, at worst, be slowly growing because of the Rayleigh inflection theorem [7]. Evidently, the issue of whether rotation shear would iron out the interchange needs resolution.

In this Letter, we show by numerical simulation that a centrifugally confined plasma in a mirror-type configuration is stable to the flutes, at Mach numbers of rotation of about 4. If this conclusion holds for a fusion-grade plasma (expected to be in the same dimensionless parameter range as our simulation), it allows consideration of a fusion device with a very simple coil configuration (among other advantages) [3]. We solve numerically the 3D MHD and transport equations [8] in cylindrical (R, ϕ, Z) coordinates. The governing equations are

$$\frac{\partial n}{\partial t} + \vec{\nabla} \cdot (n\vec{u}) = 0\,, \tag{1}$$

$$\frac{\partial (nM\vec{u})}{\partial t} + \vec{\nabla} \cdot (nM\vec{u}\vec{u}) = -\vec{\nabla}\left(2nT + \frac{B^2}{8\pi}\right) + \frac{\vec{B}}{4\pi} \cdot \vec{\nabla}\vec{B} + \vec{\nabla} \cdot (nM\mu\nabla\vec{u}) + \vec{F}\,, \tag{2}$$

$$\frac{\partial T}{\partial t} + \vec{\nabla} \cdot (T\vec{u}) = \frac{1}{3}T\vec{\nabla} \cdot \vec{u} + \frac{1}{n}\vec{\nabla} \cdot (n\kappa_\perp \vec{\nabla} T) + \frac{1}{n}\hat{b} \cdot \vec{\nabla}(n\kappa_\parallel \hat{b} \cdot \vec{\nabla} T) + \frac{2}{3}M\mu R^2 \left|\vec{\nabla}\left(\frac{u_\phi}{R}\right)\right|^2, \tag{3}$$

$$\frac{\partial \vec{B}}{\partial t} = \vec{\nabla} \times (\vec{u} \times \vec{B}) + \eta \frac{c^2}{4\pi}\nabla^2 \vec{B}\,. \tag{4}$$

Standard notation is used. The viscosity is assumed isotropic; the thermal conductivity is anisotropic with conduction along the field dominating that cross field. Viscous heating is included, as this is the means by which centrifugal schemes could be heated [3]: for simplicity, we keep only the most significant term in the viscous heating (since the plasma rotates supersonically in the ϕ direction, we expect $u_\phi \gg u_z, u_r$).

We use normalized units as follows: lengths are normalized to the simulation radial dimension L, the magnetic field B is normalized to a reference field B_0, and electron number density n is normalized to a reference density n_0. Thus, speeds are normalized to the reference Alfvén speed $V_{A0} \equiv (B_0^2/4\pi n_0 M)^{1/2}$, and time is normalized to the Alfvénic time scale L/V_{A0}. It follows that energies and temperature are normalized to MV_{A0}^2, the viscosity μ, and the thermal conductivities $\kappa_\perp$ and $\kappa_\parallel$ are each normalized to LV_{A0}, and resistivity η is normalized to $4\pi LV_{A0}/c^2$.

Our simulation box is within two concentric cylindrical walls. The width of the box is 1, the inner cylinder is at radius 0.45, and the elongation in the z direction is 5. (For efficient centrifugal confinement, it is desired to have the ratio of the outermost to the innermost radius of a field line to be at least 3 [3]. The inner radius of 0.45 was picked for this reason and for numerical ease.) The external magnetic field is, dominantly, a uniform field in the z direction plus the field of two additional "mirror" coils of radius 1.75 placed at the top and bottom of the box. The latter coils produce the throats of the mirror. (Since we impose periodic boundary conditions in the z direction, in practice we also place additional coils in periodic fashion along z, separated by a distance of five units. The latter coils are subdominant to the main field described earlier; for the simulation, we terminated the series at 20 extra coils above and below the box.) The number of grid points in the simulations reported below was $60 \times 40 \times 100$.

As mentioned, periodic boundary conditions are imposed in z, as well as in the ϕ direction. The boundaries in R are assumed to be perfectly conducting hard walls: since field lines cut these walls in general, we assume zero flow at and into the walls, we let the perturbed normal magnetic field, $\tilde{B}_R$, be zero, and we assume that the perturbed transverse magnetic field satisfies $\partial_R \tilde{B}_Z = 0$, $\partial_R(R\tilde{B}_\phi) = 0$, consistent with zero current at the walls. The growth rate of the interchange instability is much larger than resistive penetration rates through a conducting vessel wall, thus the conducting wall boundary conditions used ("line tying" and no flux penetration for the magnetic field) are reasonable. (For current-driven kink modes, it is well known [1] that close fitting conducting shells reduce the growth rate; interchange mode growth rates are independent of the wall radius since these modes are well localized.) In addition, in a real system, a low temperature plasma with attendant high density of neutral atoms close to the walls provides a strong drag on plasma flow, thus the no-slip boundary conditions on the flow are also reasonable from this standpoint.

The temperature T at the radial walls, as well as at the z boundaries, is kept at "room temperature" T_0. This is achieved by putting in a heat sink term of the form $-(T - T_0)Ae^{-\alpha(\Delta x)^2}$ on the boundary, where A is a large constant, $\alpha \simeq 1/(\text{grid size})^2$ and Δx is the distance to the wall. This simulates radiation close to the walls, which would be expected and would keep the temperature low there.

The numerical algorithm is described in detail in Guzdar *et al.* [9]. We began the simulation with uniform density and temperature at room temperature ($n = 1$ and $T = T_0$). The initial magnetic field was all due to the external coils. There was no rotation in the initial state. Further, we did not "seed" any noise in the toroidal direction initially, i.e., we first used the 3D code to attain a 2D azimuthally symmetric laminar state. For this run, we took the viscosity μ, the perpendicular thermal conductivity $\kappa_\perp$, and the resistivity η all to be 0.002. The parallel thermal conductivity $\kappa_\parallel$ was set to be $200\kappa_\perp$. The room temperature T_0 was set to be 0.002. With this initial condition, we applied a force, $F_\phi = 8\mu$, in the ϕ direction to model the external $I \times B$ forces on the plasma (other methods of "start-up" were tried, e.g., imposed radial currents at the top and the bottom also spun up the plasma). Because of the applied force, the plasma started rotating in the ϕ direction. By building speed, the centrifugal force was then seen to push the plasma towards the midplane. The temperature rose due to viscous heating, especially in the flanks, with heat conducting toward the midplane. After about 300 time units from the onset of the driving force, the system came to an approximate steady state.

Figure 1 shows the temperature and the pressure gray scale contour plots of this 2D laminar state, with magnetic field lines overlaid. The pressure is localized to a peak in the center. All the temperature rise results from viscous heating. Temperature contours tend to match magnetic field lines because of the much higher thermal conductivity along the field line. The angular frequency of rotation, Ω, self-consistently ended up being a flux function, as predicted by theory [3]. A profile is shown in Fig. 1(c), with shear in Ω clearly evident given the no-slip boundary conditions. The central Mach number is a key parameter. We define the Mach number M_s by $M_s^2 = Mu_\phi^2/T$. For this run, we achieved a maximum Mach number of $M_s = 4$ at the center. The pressure drop $p_{\max}/p_{\min} = 86$, and the Alfvén Mach number was $M_A = 0.3$. This laminar state shows that centrifugally confined plasmas can provide reasonable profiles for a fusion device.

This steady state was then seeded with random noise, in all coordinates, of the size 10^{-4} in density, and all flow variables. The fastest growing instability was expected to have a toroidal wavelength of the order of $\pi/6$. Thus, for numerical ease, we reduced the box size in ϕ and imposed periodic boundary conditions over the range $\phi = [0, \pi/3]$ (this was remedied in other runs, wherein we confirmed that longer wavelengths were not an issue, by increasing the box size). This 3D random noise test was allowed to run for more than 60 time units, much longer than the expected growth times (of the order of tens of time units). There was no sign of the characteristic interchange instability and breakup of the laminar state: the random noise was initially seen to smooth out, some mild undulation in ϕ was then seen, at wavelengths of the order of $\pi/3$ rad,

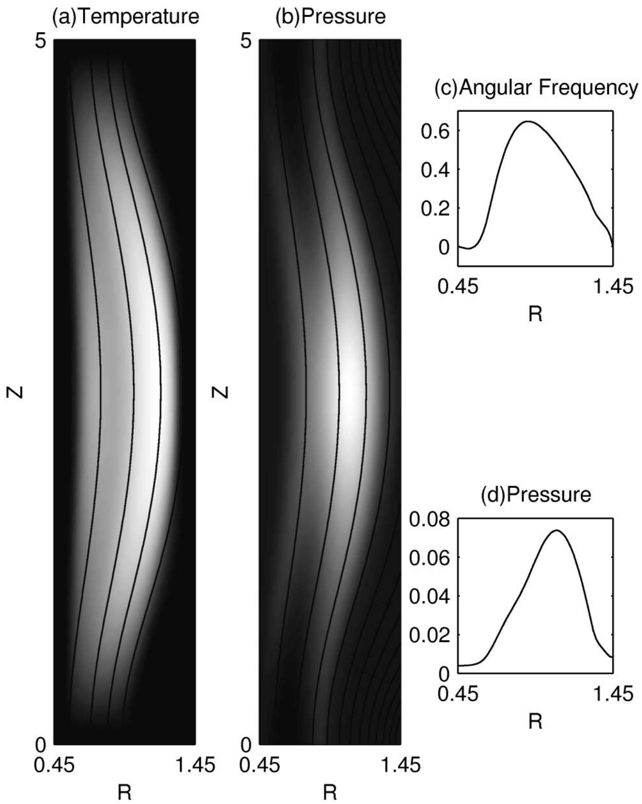

FIG. 1. The 2D laminar profiles: (a) temperature and (b) pressure, with magnetic field lines overlaid. A cut of angular frequency at midplane is shown in (c). Likewise, (d) is a pressure cut.

and this undulation then settled down to a small wobble with amplitude less than 1%. A wobble had been observed in a z pinch simulation done earlier [10] and so was expected in our simulation, but the size of the wobble was found to be considerably smaller than expected. For all practical purposes, this simulation indicated that the laminar state was maintained. As mentioned above, we experimented with longer box sizes in ϕ ($\pi/2$, π, and 2π) as well as differing resolution. We also started the simulation from scratch (no rotation), but included random noise. No evidence was found in any case that would equivocate the conclusion that the system is stable.

Because we saw no breakup of the system, it was important to find a counterexample where the code did produce a characteristic interchange breakup. To be sure, Cartesian versions of this code have shown large scale, nonlinear, turbulent behavior [10]. (In addition, our stable result is consistent with theoretical analysis which suggests stabilization at Mach numbers somewhat larger than unity [5,11].) Nonetheless, we looked for an unstable situation to test in this case. One possibility was to rerun the simulation with no shear in the angular frequency, i.e., configure the system such that Ω' was zero and the entire plasma was rigidly rotating. Then, there would be instability (since one can then transform to a frame in which the centrifugal force would go as R and there would be no velocity shear). This test, unfortunately, cannot be implemented for the centrifuge system without changing too many things that would then make the comparison meaningless: if we set up an equilibrium with $\Omega' = 0$, there would be no viscous heating [see Eq. (3)], thus the temperature would be a constant and the density profile would adjust to compensate. As a result, we would be comparing two different situations. In order to carry out a test that would maintain the density and temperature profiles but minimize the velocity shear, we settled on an "artificial" test. We took the final output frame of the 3D seeded code above and "froze" the centrifugal and Coriolis accelerations as well as the viscous heating as follows: In the momentum equation [Eq. (2)] we froze the terms corresponding to the Coriolis and centrifugal accelerations, set the applied force $\vec{F}$ to zero, and reset all the remaining flow terms to zero as the initial condition. The new momentum equation then looked like

$$\frac{\partial(nM\vec{u})}{\partial t} + \vec{\nabla}\cdot(nM\vec{u}\vec{u}) = -\vec{\nabla}\left(2nT + \frac{B^2}{8\pi}\right) + \frac{\vec{B}}{4\pi}\cdot\vec{\nabla}\vec{B} + \vec{\nabla}\cdot(nM\mu\nabla\vec{u}) - 2nM\vec{\Omega}_0\times\vec{u} - nM\vec{\Omega}_0\times(\vec{\Omega}_0\times\vec{R}). \tag{5}$$

Here, Ω_0 is the rotation frequency function frozen from the previous run. Note that the variable $\vec{u}$, while set to zero initially, is free to evolve. Concomitantly, the density n in front of the centrifugal term is free to evolve. Thus, we are starting from a situation where there are destabilizing accelerations but no flow shear—and no possibility of flow shear buildup since the applied force $\vec{F}$ is zero. Note also that in the initial state the above equation keeps the system in equilibrium, and no equilibrium pressure profile adjustments will occur at restart. Likewise, we also froze the heating terms in the temperature equation. In particular, in Eq. (2), we froze the viscous heating term to keep it at $(2/3)M\mu R^2|\vec{\nabla}\Omega_0|^2$. This form of the heat equation ensures that there will be no temperature adjustments on the transport time scale.

We restarted with random noise as before. The discharge became unstable. Figure 2 shows the pressure on the $R-\phi$ cut through the midplane at $t = 0$, 70, 83, 89, 95, and 101. The characteristic "mushrooms" associated with an interchange are clearly visible and the entire discharge is effectively destroyed. Continuation of the run at this stage would produce turbulence. We then restored velocity shear: we reintroduced the force $\vec{F}$, at the same level as before, and restarted from the last frame of Fig. 2, except that u_ϕ was set to $R\Omega_0$ as an initial condition. The Coriolis and the centrifugal terms were turned off. The discharge then recovered. The pressure profiles on the $R-\phi$ midplane cut at $t = 0$, 3.5, 7, 10.5, 17.5, and 63 are shown in Fig. 3.

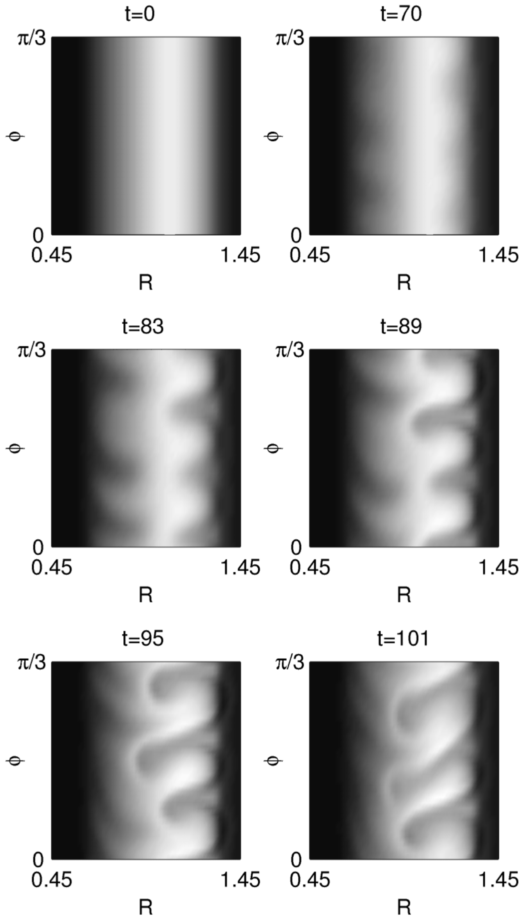

FIG. 2. The time evolution of the pressure on the $R - \phi$ midplane of the test without velocity shear.

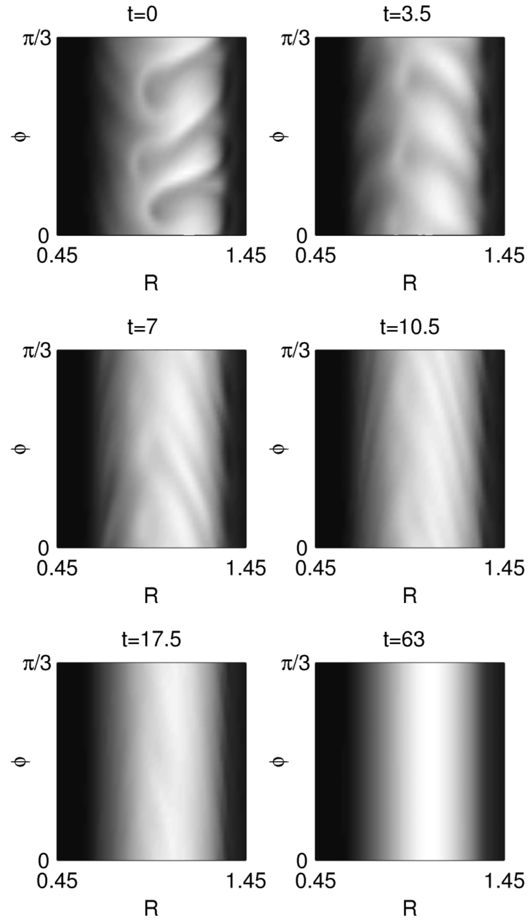

FIG. 3. Stabilization from restoring velocity shear.

The discharge was stabilized and the laminar state was almost completely recovered at $t = 63$.

In conclusion, our numerical experiment demonstrates the existence of a stable, centrifugally confined plasma within a magnetic configuration that is relatively simple. It is incontrovertible that a simple magnetic mirror is grossly flute unstable, and would be even more so under rigid rotor azimuthal rotation. We have shown that strong velocity shear renders the system laminar. Analytic calculations in progress support this numerical finding [11]. This is a very attractive idea for a fusion device. Supersonic rotation is required but this is precisely what is also required for the containment of the plasma by centrifugal forces [3]. The system we consider is of small Larmor radius and, accordingly, the simulation is based on ideal MHD. Drift instabilities, by definition for this system, have lower growth rates and shorter wavelengths and are not included in this description. The lower growth rate, however, means our large velocity shear would be strongly stabilizing; the shorter wavelengths would imply that these instabilities would not disrupt the discharge but, at worst, cause turbulent transport. All frequencies considered are below the ion-cyclotron frequency, at least for long wavelengths. Thus, kinetic cyclotron effects, again, would not be grossly disruptive. Clearly, however, both drift and kinetic effects would have to be included in a more encompassing study.

An experiment currently under construction should be able to test this result [3]. The experimental plan allows for extra coils to produce an azimuthal field to assist velocity shear stabilization of interchange instability, if needed. The present simulation, albeit at Reynolds numbers smaller than expected in the experiment, suggests the azimuthal field may not be needed.

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