Comment on "Creation of Magnetic Energy in the Solar Atmosphere"

In a recent Letter [1], Woods and Ashbourn propose that coincident, but opposing, magnetic and thermal gradients can create magnetic energy, and in particular that this process can occur within stationary magnetic flux tubes in the solar atmosphere. We argue that although local enhancements in the magnetic energy of a flux tube are permitted by their assumptions, an overall increase is not.

After discarding small terms in Ohm's law and allowing only radial spatial variations in a cylindrical flux tube geometry, Woods and Ashbourn arrive at the following equation governing the time evolution of the $\hat{z}$ component of the magnetic field:

$$\frac{\partial B}{\partial t} = \xi' B' + \xi B'' - (v_r B)', \tag{1}$$

where ξ is proportional to the resistivity, v_r is the bulk fluid motion in the radial direction, and primes indicate derivatives with respect to the radial coordinate r.

This is merely the familiar magnetic induction equation—the first two terms on the right-hand side represent resistive diffusion of the field, while the third corresponds to field transport by the bulk motion of the fluid. Although this final term is the standard explanation for the creation of magnetic energy in solar dynamo theory, Woods and Ashbourn claim that even in its absence B can increase in time if ξ is chosen properly, an assertion with which we disagree.

Multiplying both sides of Eq. (1) by B, taking $v_r = 0$, performing some algebraic manipulations, and integrating over the volume of the flux tube, we have

$$\frac{1}{2}\frac{\partial}{\partial t}\int B^2\,dV = -\int \xi\left(\frac{\partial B}{\partial r}\right)^2 dV + \int \frac{\partial}{\partial r}\left(\xi B\,\frac{\partial B}{\partial r}\right) dV\,, \tag{2}$$

where dV is the volume element in cylindrical coordinates. The left-hand side is merely the time derivative of the tube's magnetic energy; for it to increase as Woods and Ashbourn propose, the right-hand side must be positive. Only the final term can contribute with the proper sign, but it can be rewritten, using the divergence theorem, as a surface integral:

$$\int \frac{\partial}{\partial r}\left(\xi B\frac{\partial B}{\partial r}\right) dV = \int \xi B\,\frac{\partial B}{\partial r}\,dA\,. \tag{3}$$

For reasonable flux-tube models, including the one suggested in [1], not only does $B \rightarrow 0$ on the surface of the tube—making this term negligibly small—but $B' < 0$ there as well; an increase in the magnetic energy arising from this term is therefore impossible.

Although we have shown the magnetic energy of the entire tube must decrease with time, regardless of the functional form of B and ξ, one can find regions of increase by making careful choices of both variables and the volume over which Eq. (2) is integrated. Such increases are ultimately transitory, an unsurprising result as Eq. (1) with $v_r = 0$ is closely related to the diffusion equation. Only systems with $v_r \neq 0$ can provide a steady, global source of magnetic energy.

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[1] L. C. Woods and J. M. A. Ashbourn, Phys. Rev. Lett. **84**, 3748 (2000).

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