Velocity shear stabilization of interchange modes in elongated plasma configurations

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Interchange modes in magnetized plasmas can be stabilized by cross-field velocity shear. This effect is re-examined for systems with elongated cross-sections. For large elongations, E, the interchange growth drops as $E^{-1/2}$ while the velocity shear scale is still determined by the short scale size. Consequently, velocity shear stabilization of elongated plasmas is shown to be more efficient by $E^{-1/2}$. [S1070-664X(99)02310-1]

I. INTRODUCTION

It is well recognized that velocity shear has a strong stabilizing influence on the gravitational interchange instability of fluids and plasmas.[1,2] The interchange mode taps its energy from density gradients inverted with respect to a gravitational field, or, in the case of plasmas, pressure gradients inverted with respect to curvature of the confining magnetic field. If, however, the ambient fluid is flowing transverse to the pressure stratification, and there is a shear in the flow in the direction of stratification, the interchange mode can be stabilized. A stability criterion is given by[2]

$$V' > \gamma_g [\ell n R]^{1/2}. \tag{1}$$

Here, V' is the shear in the flow velocity, γ_g is the growth rate of the interchange mode, and R is a Reynolds number, essentially the ratio of γ_g to a visco-resistive dissipation rate. The stability criterion essentially says that the shearing rate must exceed the growth rate;[3,4] the logarithmic factor reflects the fact that the stabilization ultimately stems from a phase-mixing process in which the viscosity and resistivity play an essential, albeit weak, role.[2]

Velocity shear stabilization of the interchange mode has important implications for the stability of thermonuclear fusion plasmas. Experimental evidence and its theoretical basis show that such stabilization is operative in hot plasmas.[5] Fusion experiments, utilizing the stabilizing effects of velocity shear, are planned or under construction.[6–8] In this paper, we address whether and how plasma shaping can aid the stabilization from velocity shear. In particular, we address the effect of elongation on the velocity-shear stabilization of the interchange.

To orient the discussion, consider the Z pinch shown in Fig. 1. This is an elongated Z pinch with dimensions a and b. As is well-known, Z pinches are unstable to the interchange (the ''sausage'' instability). The interchange growth rate, γ_g, scales as $\gamma_g \sim c_s/(L_n r_c)^{1/2}$, where c_s is the sound speed, r_c is the radius of curvature, and L_n is the density gradient scale size. In the case of a roughly circular cross section (i.e., $a=b$), we have $r_c \sim a$ and $L_n \sim a$ in which case $\gamma_g \sim c_s/a$. If there is an axial flow (see Fig. 1) then the velocity shear, V', scales as V/a. Using these in Eq. (1), we may re-express the stability criterion as

$$M_s > [\ell n R]^{1/2}, \tag{2}$$

where $M_s \equiv V/c_s$ is the Mach number. For typical fusion parameters, the Reynolds number, based on classical transport, is of order $R \sim 10^8$. This means that a critical Mach number of about 4.3 would be necessary for stabilization. It is important to state that criterion (1) is arrived at conservatively[2] — the actual Mach number requirement could be somewhat less. Nonetheless, supersonic flow seems to be required.

The interchange growth rate, however, weakens with increasing elongation $E \equiv b/a$. The question then arises as to how $E>1$ affects criterion (1). We provide here a heuristic estimate, to be confirmed by the calculation in this paper. The interchange growth rate is reduced by $E>1$ as follows: In the ''straight portion'' of the elongated field shown in Fig. 1, the radius of curvature, r_c, is weak, scaling as $r_c \sim b$. In the ''curved'' portion, $r_c \sim a$. The average curvature can then be estimated by

$$\frac{\langle (1/r_c) \rangle}{\langle 1 \rangle} \sim \frac{((1/b)\times b)+((1/a)\times a)}{(a+b)}, \tag{3}$$

where $\langle\,\rangle$ denotes flux surface averaging and, for instance, $(1/b)$ is multiplied by b to take into account the length of the straight portion. In effect, then, $r_c \sim (a+b) \sim b$, for $b \gg a$. The density scale, L_n, scales as a. Thus, $\gamma_g \rightarrow c_s/(ab)^{1/2}$. The velocity shear scale, however, always scales with the smaller scale size, i.e., $V' \sim V/a$. Using these in criterion (1), we estimate the critical Mach number for stabilizing elongated interchanges to be

$$M_s > \left[\frac{\ell n R}{E}\right]^{1/2}. \tag{4}$$

For an elongation of $E=4$, the critical Mach number is halved to $M_s > 2.1$. To reiterate, large E lowers the interchange growth rate but velocity shear is always based on the smaller scale, leading to an overall reduction of the Mach number requirement by $E^{1/2}$.

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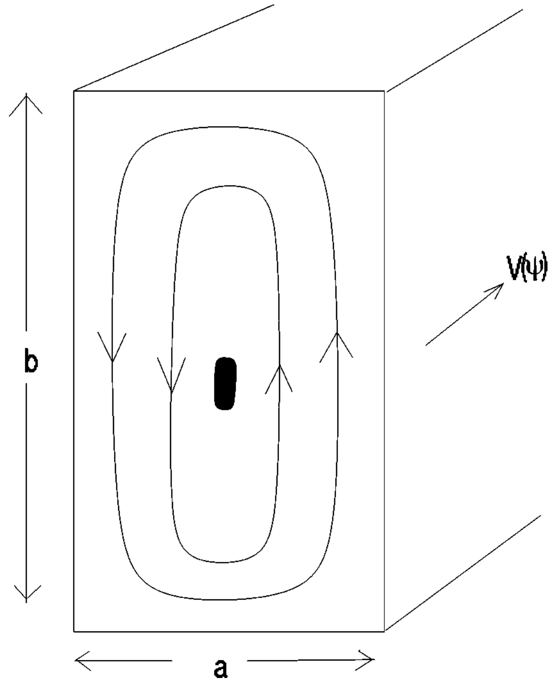

FIG. 1. A Z-Pinch with elongated cross section.

In this paper, we present an analytical calculation that confirms the foregoing heuristic estimate. In the next section, we set up the model and the governing equations. In Sec. III, we linearize the equations about the flowing Z-pinch equilibrium. In Sec. IV, we discuss the long wavelength stability, wavelengths $\sim O(a)$: These are primarily velocity shear driven Kelvin–Helmholtz modes. In Sec. V, we address the short wavelength interchange modes. In Sec. VI, we discuss the extension of our results to other plasma shapes. We conclude in Sec. VII.

II. EQUATIONS

We will solve for the stability of the interchange in an elongated Z-pinch such as the one depicted in Fig. 1. We assume that the magnetic field is only poloidal, given by

$$\vec{B}=\hat{z}\times\nabla\psi. \tag{5}$$

For simplicity, we will assume that the magnetic energy is dominant, i.e., we assume that the plasma β is small and the kinetic energy is small, viz.

$$(\tfrac{1}{2})nMu^2\sim(\tfrac{3}{2})p\ll B^2/2, \tag{6}$$

where $\mathbf{u}$ is the flow velocity, n is the density, M is the ion mass, and $p\equiv nT$ is the pressure with temperature T. This assumption, along with the assumption of no toroidal field, simplifies the analysis because the interchange between flux tubes is perfect in the sense of being approximately electrostatic. The assumption of large magnetic field is easily realized in low β systems, for example in the case where a central axial conductor and/or external conductors carry the currents that generate the magnetic field.[6] In the case of a field-reversed configuration,[9] the analysis is strictly valid only for the outer surfaces where the pressure is small. Nonetheless, the conclusions of interest are unlikely to change qualitatively.

In the electrostatic limit, the magnetic field is given, for all times, by Eq. (5) and the equilibrium equation for ψ in the plasma is

$$\nabla^2\psi\simeq 0. \tag{7}$$

An important quantity is the radius of curvature, $\boldsymbol{\kappa}\equiv\hat{b}\cdot\nabla\hat{b}$, where $\hat{b}\equiv\mathbf{B}/B$. This can be expressed in terms of B by the equilibrium relationship

$$\boldsymbol{\kappa}\cdot\nabla\psi=B\partial B/\partial\psi. \tag{8}$$

Within the electrostatic assumption, the governing equations are

$$\partial n/\partial t+\nabla\cdot(n\mathbf{u})=0, \tag{9}$$

$$\partial p/\partial t+\mathbf{u}\cdot\nabla p+\gamma p\nabla\cdot\mathbf{u}=0, \tag{10}$$

$$nM\mathbf{B}\cdot(\partial\mathbf{u}/\partial t+\mathbf{u}\cdot\nabla\mathbf{u})=-\mathbf{B}\cdot\nabla p, \tag{11}$$

$$\mathbf{B}\cdot\nabla\phi=0, \tag{12}$$

$$\langle\nabla\cdot\mathbf{J}_\perp\rangle=0, \tag{13}$$

where

$$\mathbf{u}\equiv\hat{b}u_\parallel+\mathbf{B}\times\nabla\phi/B^2, \tag{14}$$

$$\mathbf{J}_\perp\equiv\mathbf{B}\times[nM(\partial\mathbf{u}/\partial t+\mathbf{u}\cdot\nabla\mathbf{u})+\nabla p]/B^2, \tag{15}$$

$$\langle f\rangle\equiv\oint(d\ell/B)f. \tag{16}$$

Equations (9)–(13) constitute a closed set for the four variables n, p, $u_\parallel$, and ϕ, with $\mathbf{u}_\perp$ and $\mathbf{J}_\perp$ defined by Eqs. (14) and (15), $\mathbf{B}$ given by Eq. (5), and the flux surface average defined by Eq. (16). Since $\mathbf{B}$ is specified by ψ, a convenient coordinate system is (ψ,ℓ,z), where ℓ is the distance along a closed field line. A useful identity is obtained by the volume average over a flux tube using Gauss' theorem

$$\langle\nabla\cdot\mathbf{A}\rangle\equiv\frac{\partial}{\partial\psi}\langle\nabla\psi\cdot\mathbf{A}\rangle+\frac{\partial}{\partial z}\langle\widehat{z\cdot}\mathbf{A}\rangle. \tag{17}$$

An axisymmetric equilibrium with axial flow, obtained from Eqs. (9)–(13), is given by:

$$\mathbf{u}=\hat{z}V(\psi),\quad V(\psi)\equiv-d\phi/d\psi, \tag{18}$$

$$p=p(\psi), \tag{19}$$

$$n=n(\psi). \tag{20}$$

III. LINEARIZED EQUATIONS

We now linearize Eqs. (9)–(13) about the equilibrium given in Eqs. (18)–(20). The linearized system for the perturbations $\tilde{\phi}$, $\tilde{p}$, and $\tilde{u}_\parallel$ can be written as

$$\frac{d}{dt}\left[\frac{\partial}{\partial\psi}\left[\langle nM\rangle\frac{\partial\tilde{\phi}}{\partial\psi}\right]+\frac{\partial}{\partial z}\left[\left\langle\frac{nM}{B^2}\right\rangle\frac{\partial\tilde{\phi}}{\partial z}\right]\right]-\frac{d}{d\psi}\left[\langle n\rangle\frac{dV}{d\psi}\right]\frac{\partial\tilde{\phi}}{\partial z}$$

$$=-2\left\langle\frac{\boldsymbol{\kappa}\cdot\nabla\psi}{B^2}\frac{\partial\tilde{p}}{\partial z}\right\rangle, \tag{21}$$

$$\frac{d\tilde{p}}{dt}+p'F\frac{\partial\tilde{\phi}}{\partial z}+\gamma p\mathbf{B}\cdot\nabla\left(\frac{\tilde{u}_\parallel}{B}\right)=0, \tag{22}$$

$$nM\frac{d\tilde{u}_\parallel}{dt}=-\frac{\mathbf{B}}{B}\cdot\nabla\tilde{p}, \tag{23}$$

where

$$\frac{d}{dt}\equiv\frac{\partial}{\partial t}+V\frac{\partial}{\partial z}, \tag{24}$$

$$F\equiv 1-\frac{2\gamma p}{B^2}\frac{\boldsymbol{\kappa}\cdot\nabla\psi}{p'}, \tag{25}$$

and primes denote differentiation with respect to ψ. In obtaining Eq. (21), we have used the low β approximation, specifically, $|\nabla\times\mathbf{B}|\ll|\nabla_\perp B|$. We have also used Eq. (17) and made repeated use of Eqs. (8), (12), and (14).

As written above, Eqs. (21)–(23) describe all wavelengths, from $\partial_z\sim 1/b$ to ∂_z large. The various modes that can be obtained include the interchange modes as well as the Kelvin–Helmholtz.[10,11] In particular, the important interchange term is the $\boldsymbol{\kappa}\cdot\nabla\psi$ term in Eq. (21), and the term responsible for the Kelvin–Helmholtz (KH) is the $d^2V/d\psi^2$ term in Eq. (21). In this paper, we will investigate the short wavelength (interchange) modes fairly thoroughly; the long wavelength stability is more difficult to assess. We will assess the latter only in the case of large elongation, $E\gg 1$.

IV. LONG WAVELENGTH MODES

The long wavelength modes may be driven both by V'' as well as n', i.e., they are a coupling of the KH and interchange modes. A complete stability analysis involves solving the entire system [Eqs. (21)–(23)] as is — this is involved. We restrict ourselves to the special case of large elongation, i.e., $b\gg a$. In this limit, we expect the Kelvin–Helmholtz growth rate to scale as $\gamma_{KH}\sim V/a$. However, the sound frequency, γ_s, scales as $\gamma_s\sim c_s|\hat{b}\cdot\nabla|\sim c_s/b$. Thus, $\gamma_s\ll\gamma_{\mathrm{KH}}$, in which case one may neglect $\tilde{u}_\parallel$ in Eq. (22). For the resulting system, we assume the eikonal solution $\tilde{\phi}\rightarrow\tilde{\phi}\exp[-i\omega t+ikz]$. Equation (21), when divided by $\bar{\omega}$, becomes

$$\frac{d}{d\psi}\left[\langle n\rangle\frac{d\tilde{\phi}}{d\psi}\right]-k^2\left\langle\frac{n}{B^2}\right\rangle\tilde{\phi}=-\frac{\langle(nV')'\rangle k\tilde{\phi}}{\bar{\omega}}$$

$$+2k\left\langle\frac{\boldsymbol{\kappa}\cdot\nabla\psi}{B^2}\frac{\tilde{p}}{\bar{\omega}}\right\rangle, \tag{26}$$

and Eq. (22) becomes

$$\bar{\omega}\tilde{p}\approx p'Fk\tilde{\phi}, \tag{27}$$

where $\bar{\omega}\equiv\omega-kV$. We now operate on Eq. (26) with $\int d\psi\tilde{\phi}^*$, where the integral is taken over the entire volume. By an integration by parts, using homogenous boundary conditions on $\tilde{\phi}$, we obtain the quadratic form

$$\left\{\langle n\rangle\left|\frac{d\tilde{\phi}}{d\psi}\right|^2+k^2\left\langle\frac{n}{B^2}\right\rangle\left|\tilde{\phi}\right|^2\right\}$$

$$=\left\{\frac{\langle(nV')'\rangle|\tilde{\phi}|^2}{|\bar{\omega}|^2}\bar{\omega}^*\right\}-2\left\{\left\langle F\frac{\boldsymbol{\kappa}\cdot\nabla\psi}{B^2}\right\rangle\frac{|\tilde{p}|^2\;\bar{\omega}^{*2}}{p'F^2|\bar{\omega}|^2}\right\}, \tag{28}$$

where we have used Eq. (27) and the fact that $\tilde{p}/F$ is a function of ψ, and we have defined $\{f\}\equiv\int d\psi f$. We have assumed that $\bar{\omega}\neq 0$, i.e., we assume ω is not real. We now take the imaginary part of Eq. (28). We get

$$\gamma\left[\left\{\frac{k\langle(nV')'\rangle|\tilde{\phi}|^2}{|\bar{\omega}|^2}\right\}+4\omega_r\left\{\left\langle\frac{F\boldsymbol{\kappa}\cdot\nabla\psi}{B^2}\right\rangle\frac{|\tilde{p}|^2}{p'F^2|\bar{\omega}|^2}\right\}\right]=0, \tag{29}$$

where we have let $\omega\equiv\omega_r+i\gamma$. Now, as $b\rightarrow\infty$, the first term remains finite but, as argued in the Introduction, the average curvature $\langle\boldsymbol{\kappa}\cdot\nabla\psi/B^2\rangle\rightarrow 0$ as $1/b$: thus the second term becomes small for large b. If $(nV')'\neq 0$ anywhere in the plasma, then the first term is nonzero and the above equation cannot be satisfied for any value of $|\omega_r|\lesssim V/a$ (since $\gamma\neq 0$ has been assumed). The only possibility then is that $\gamma=0$, to all orders in (a/b). Thus, we conclude that for $(nV')'\neq 0$, long wavelength modes are stable if $a\ll b$. This condition is known as Rayleigh's Criterion[10,11] and it is readily satisfied for flow driven by a unidirectional external force.

V. SHORT WAVELENGTH MODES

In the limit of $\partial_z\gg 1/a$, all the perturbed variables have variations in ψ and z which are of short scale compared with a. In that case, several terms in Eq. (21) can be neglected including the KH term. Making these approximations, Eq. (21) becomes

$$nM\frac{d}{dt}\langle\nabla^2\tilde{\phi}\rangle\simeq-2\left\langle\frac{\boldsymbol{\kappa}\cdot\nabla\psi}{B^2}\frac{\partial\tilde{p}}{\partial z}\right\rangle, \tag{30}$$

$$\nabla^2\equiv\partial_\psi^2+B^{-2}\partial_z^2. \tag{31}$$

Before solving the system (30), (22), and (23) for the V' stabilized interchange, we examine the case $V=0$ first.

A. Interchanges without flow shear

We assume an eikonal solution $\tilde{\phi}\rightarrow\tilde{\phi}\exp(-i\omega t+ikz)$. As $k\rightarrow\infty$, it can be shown that $|\partial_\psi|\ll k$, i.e., a local dispersion is obtainable. In that case, Eqs. (30) and (22) become

$$nM\omega\langle 1/B^2\rangle k\tilde{\phi}=-2\left\langle\frac{\boldsymbol{\kappa}\cdot\nabla\psi}{B^2}\tilde{p}\right\rangle, \tag{32}$$

$$[\omega^2+c_s^2\mathbf{B}\cdot\nabla(B^{-2}\mathbf{B}\cdot\nabla)]\tilde{p}=\omega p'Fk\tilde{\phi}, \tag{33}$$

where $c_s^2 \equiv \gamma p/nM$ and we used $\tilde{u}_\parallel$ from Eq. (23) into Eq. (22) to get the second order magnetodifferential equation [Eq. (33)]. The system (32)–(33) is hard to solve in general. The essential behavior can be discerned by taking two artificial limits, namely, $c_s \to 0$ and $c_s \to \infty$. For $c_s \to 0$, the second term on the left-hand-side of Eq. (33) can be neglected. The resulting expression for $\tilde{p}$ inserted into Eq. (32) yields the local dispersion relation

$$\omega^2 \simeq -2\left(\frac{p'}{nM}\right)\left\langle \frac{\boldsymbol{\kappa}\cdot\nabla\psi}{B^2}F\right\rangle/\langle 1/B^2\rangle, \quad c_s \to 0. \tag{34}$$

As discussed earlier, the $\boldsymbol{\kappa}\cdot\nabla\psi$ average scales as $1/b$ yielding $\omega^2 \sim c_s^2/ab$. The system is unstable for $p'<0$. ''Compressional stabilization'' is obtained from the F term.[12]

For $c_s \to \infty$, we have, from Eq. (33), $\tilde{p} \to \tilde{p}(\psi)$, to lowest order. Now, annihilating the $\mathbf{B}\cdot\nabla$ term in Eq. (33) by a flux surface average, we get

$$\omega\tilde{p}(\psi) \simeq p'k\tilde{\phi}<F>/<1>. \tag{35}$$

Inserting this into Eq. (32), we obtain the local dispersion relation

$$\omega^2 \simeq -2\left(\frac{p'}{nM}\right)\frac{\langle(\boldsymbol{\kappa}\cdot\nabla\psi/B^2)\rangle}{\langle 1/B^2\rangle}\frac{\langle F\rangle}{\langle 1\rangle}, \quad c_s \to \infty, \tag{36}$$

which scales similarly to the dispersion (34).

We have thus obtained useful information on the usual interchange.

B. Interchanges with flow shear

We now include the effects of flow shear. For simplicity, we will only consider the artificial limit $c_s \to 0$. In this case, the relevant equations are Eqs. (30) and (22) with $\tilde{u}_\parallel$ set to zero. We assess stability in this case by employing the methods of Ref. 2. Accordingly we first make a coordinate transformation from (ψ,z,t) to (x,ζ,τ) defined by

$$x=\psi \tag{37}$$

$$\zeta = z - V(\psi)t, \tag{38}$$

$$\tau = t. \tag{39}$$

The transformed equations are

$$nM\frac{\partial}{\partial\tau}\langle\nabla^2\tilde{\phi}\rangle = -2\left\langle\frac{\boldsymbol{\kappa}\cdot\nabla\psi}{B^2}\frac{\partial\tilde{p}}{\partial\zeta}\right\rangle, \tag{40}$$

$$\frac{\partial\tilde{p}}{\partial\tau} \simeq -p'F\frac{\partial\tilde{\phi}}{\partial\zeta}, \tag{41}$$

$$\nabla^2 \equiv (\partial_x - V'\tau\partial_\zeta)^2 + B^{-2}\partial_\zeta^2. \tag{42}$$

We now note that x and ζ are ignorable in the local approximation. Thus, we let $\partial_\zeta \to ik$, and we let $\partial_x \to 0$ since the fastest growing interchanges are ones with $\partial_x = 0$. Eliminating $\tilde{p}$ in the set (40) and (41), we obtain an evolution equation for $\tilde{\phi}$

$$\frac{\partial^2}{\partial\tau^2}[\langle V'^2\tau^2 + B^{-2}\rangle\tilde{\phi}] = 2\left\langle\frac{\boldsymbol{\kappa}\cdot\nabla\psi}{B^2}F\right\rangle\left(\frac{p'}{nM}\right)\tilde{\phi}. \tag{43}$$

This equation can be solved exactly in terms of Legendre functions.[2] However, the following is readily seen: For early times, $V'^2\tau^2 \ll \langle B^2\rangle/\langle 1\rangle$, an exponentially growing solution is obtained (if $p'<0$) with a growth rate given by Eq. (34). However, in the opposite limit of long times, $V'^2\tau^2 \gg \langle B^2\rangle/\langle 1\rangle$, the mode grows only algebraically [the $\tau\to\infty$ limit of Eq. (43) is an equidimensional equation]. Consequently, mode growth is retarded considerably. If viscous and resistive dissipation were included in the analysis as in Ref. 2, the algebraically growing mode phase mixes rapidly and damps. A conservative estimate of negligible growth leads to the ''stability criterion''[2]

$$\left(\frac{dV}{d\psi}\right)^2 > 2\ell nR\left|\frac{p'}{nM}\left\langle\frac{\boldsymbol{\kappa}\cdot\nabla\psi}{B^2}\right\rangle/\langle 1\rangle\right|. \tag{44}$$

This scales, for $a \ll b$, as

$$\left(\frac{V}{a}\right)^2 > \frac{2c_s^2}{ab}\ell nR, \tag{45}$$

or

$$M_s > \left[\frac{2\ell nR}{E}\right]^{1/2}. \tag{46}$$

As argued in the Introduction, the Mach number requirement goes down as $E^{1/2}$ if E is large.

VI. OTHER CONFIGURATIONS

The calculation herein has been done for an elongated plasma such as the one in Fig. 1. The conclusion, however, is general in that plasma elongation generally aids velocity shear stabilization. As an example of practical interest, consider the configuration of Fig. 2. In this system, plasma is contained along the lines by insulators as end-plates in y. Insulators are necessary, if there is an axial flow, to prevent ''line-tying.'' This system may be unstable to the interchange on account of the magnetic curvature (although the average curvature needs to be evaluated). Clearly, however, an elongated system is better stabilized by velocity shear. While the example of Fig. 2 is somewhat contrived, such systems are practical if z is periodic, as in a toroidal geometry. In that case, the toroidal rotation plays the additional role of centrifugally confining the plasma to the outboard side.[6] The centrifugally confined plasma now makes for a practical fusion device. Evidently, elongated centrifugally confined systems would be more stable.

While the latter statement is essentially correct, it is worthwhile to place it in a fuller context. The same, toroidally outward, centrifugal force that provides centrifugal confinement also tends to increase the drive for interchange instabilities. That is to say, the effective ''gravity'' that drives the interchange modes now includes, in addition to the magnetic curvature, the centrifugal force from rotation. In particular, for supersonic flows, the gravity is apparently dominated by the centrifugal force. The latter effects have

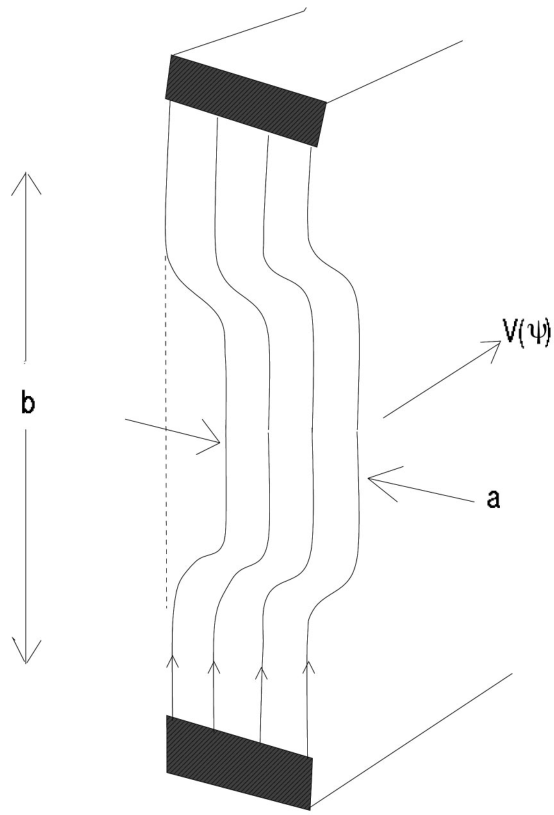

FIG. 2. An open system with an elongated poloidal field configuration.

been examined in detail in Ref. 13 — we summarize the results here. Primarily, it important to point out that the centrifugal acceleration acts only on the mass density of the plasma, as opposed to the acceleration from magnetic curvature which acts on the plasma pressure. Thus, as far as temperature gradients go, centrifugal effects are not operative and, as such, the conclusions of the present paper apply — elongation should be a stabilizing effect for maintaining temperature gradients. Now as far as density gradients go, we have shown[13] that the stabilization that comes from elongation is now replaced by a corresponding stabilization that comes from large aspect ratio. This correspondence is quite close in that, for fixed peak rotation speed, the centrifugal gravity decreases inversely with major radius R, as $1/R$, while the velocity shear, again for fixed peak rotation speed, still scales inversely with the minor radius a, as $1/a$. Thus, for both density as well as temperature gradients, the general idea that velocity shear continues to act over the minor radius while destabilizing effects can be nudged downward by geometrical shaping, is operative.

Finally, we point out that the entire discussion in this paper is focused on systems in which there is no magnetic shear. In particular, the basic criterion that motivates the paper, Eq. (1), was calculated for a magnetic configuration with no magnetic shear, in which case the ideas of shearing rates versus growth rates and phase mixing as being the underlying physics of stabilization are relatively easy to define. Systems without magnetic shear are of interest in devices for magnetic fusion such as field-reversed-configurations, dipoles, and centrifugally confined plasmas. If there is magnetic shear in the system, however, then the idea behind this paper, that stabilization results from plasma shaping, must be revisited. This is because the operative physics of velocity shear stabilization in magnetic sheared systems is significantly different from that underlying criterion (1). To give one example,[14] velocity shear coupled with the magnetic shear causes convection cells to propagate along the mean magnetic field: If the mean field moves to and from regions of favorable and unfavorable gravity, the stabilization comes from the average time spent by the cell in the good gravity region, rather than from the phase mixing physics that is of importance in criterion (1). Clearly, how shaping affects this physics needs examination.

VII. CONCLUSION

Velocity shear stabilization of plasma instabilities is desirable in fusion devices. For low β systems with insignificant magnetic shear, the strongly unstable ideal modes are the interchange modes: These modes can be stabilized by velocity shear provided V' exceeds the growth rate γ_g by a factor of $[\ell n R]^{1/2}$, R being the Reynolds number. This translates to Mach numbers of the order of $[\ell n R]^{1/2}$, ~ 4 for fusion parameters. Any reduction in this critical Mach number is desirable. One way of achieving this reduction, shaping the plasma cross section, is investigated in this paper. By examining elongated plasmas, we have shown that the Mach number requirement is no longer independent of geometry, in fact scales as $E^{-1/2}$: The critical Mach number is of $O[(\ell n R/E)^{1/2}]$. Our analysis applies to short wavelength interchanges. Long wavelength analysis indicates that as long as the flow profiles do not have an inflexion point, i.e., $(d/d\psi)[n dV/d\psi] \neq 0$ [Rayleigh's theorem], the system should be stable for $1 \ll E$. Such ''Rayleigh-flow'' profiles always obtain for flows driven by a unidirectional external force.

The conclusion of this paper would be best borne out by three-dimensional magnetohydrodynamic simulations of interchange unstable elongated Z-pinches. Work along these lines is in progress.

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[1] H. L. Kuo, Phys. Fluids **6**, 195 (1963); R. A. Brown, Rev. Geophys. Space Phys. **18**, 683 (1980).
[2] A. B. Hassam, Phys. Fluids B **4**, 485 (1992).
[3] A. B. Hassam, Comments Plasma Phys. Control. Fusion **14**, 275 (1991).
[4] R. E. Waltz, R. L. Dewar, and X. Garbet, Phys. Plasmas **5**, 1784 (1998).
[5] R. Groebner, Phys. Fluids B **5**, 2343 (1993).
[6] A. B. Hassam, Comments Plasma Phys. Control. Fusion **18**, 263 (1997).

[7]U. Shumlak and R. Hartmann, Phys. Rev. Lett. **18**, 3285 (1995).
[8]R. J. Taylor (private communication, 1998).
[9]M. Tuszewski, Nucl. Fusion **28**, 2033 (1988).
[10]S. Chrandrasekhar, *Hydrodynamic and Hydromagnetic Stability* (Oxford University Press, Oxford, England, 1961).
[11]P. G. Drazin and W. H. Reid, *Hydrodynamic Stability* (Cambridge University Press, Cambridge, 1981), p. 131.
[12]J. P. Freidberg, *Ideal Magnetohydrodynamics* (Plenum, New York, 1987).
[13]A. B. Hassam, to appear in Phys. Plasmas (1999).
[14]F. L. Waelbroeck and L. Chen, Phys. Fluids B **3**, 601 (1991).