NLD SENSOR NETWORKS:

THEORETICAL STUDIES

Theory Faculty:

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Areas of Theoretical Work

- Modeling and simulation of devices and systems in direct support of specific experiments underway or planned.

- Development of applicable theory.

- Theoretical and computational exploration of generic sensor concepts.

This Talk Briefly Discusses Three Current Research Projects

- **Adaptive Learning Network Sensor Using Chaos Sync**
  Postdoc: Francesco Sorrentino
  Faculty: E. Ott

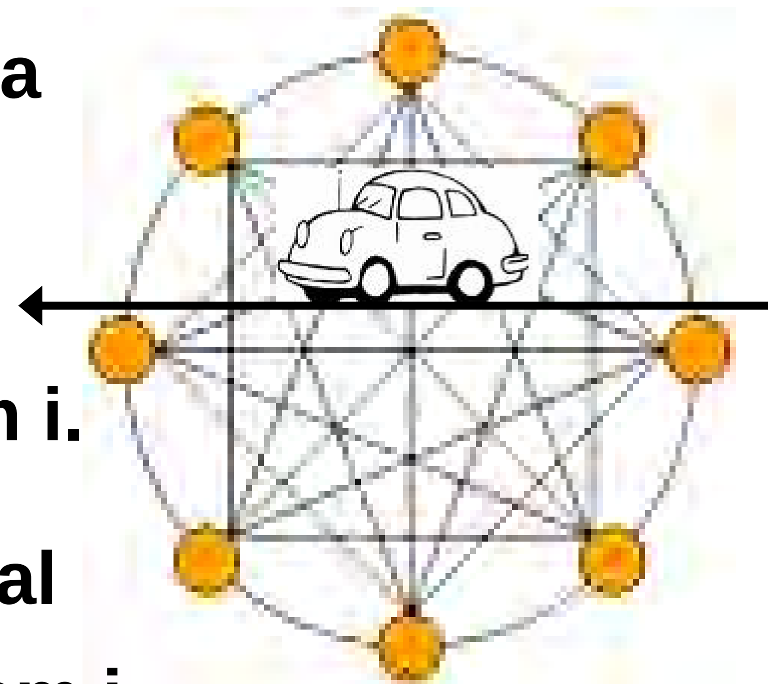
- **Modeling Chaotic TWT's**
  Student: W. S. Lee
  Faculty: T. M. Antonsen, J. Rodgers, and E. Ott

- **Stability and Chaos in Boolean Networks**
  Student: Andrew Pomerance
  Faculty: E. Ott, M. Girvan, and W. Losert

Adaptive Network Sync. of Chaos

A_{ij}: Strength of j →i link.

$\bar{x}_i(t)$: State of system i (e.g., a laser or TWT).



$s_i(t) = H(\bar{x}_i(t))$: signal broadcast by system i.

$r_i(t) = \sum_{j \neq i} A_{ij}(t) s_j(t)$: signal received by system i.

GOAL: Determine $\{A_{ij}\}$

Ref: Sorrentino & Ott, PRL, 2008 & PRE (submitted)

Evolution of System i

$$\dot{\bar{x}}_i(t) = \underbrace{F(\bar{x}_i(t))}_{\text{Chaotic system}} + \gamma\underbrace{\left(r_i(t) - \sum_{j\neq i} \bar{A}_{ij}(t) H(\bar{x}_i(t-\tau_{ij}))\right)}_{Y_i(t)}$$

$$r_i(t) = \sum_{j\neq i} A_{ij}(t) H(\bar{x}_j(t-\tau_{ij}))$$

TIME DELAYS

If $A_{ij}(t) = \bar{A}_{ij}(t)$, then there is a synchronized solution $\bar{x}_1(t) = \bar{x}_2(t) = \ldots = \bar{x}_N(t) = \bar{x}_s(t)$

where $\dot{\bar{x}}_s(t) = F(\bar{x}_s(t))$

Q. Is this solution stable?

A. Maybe

Learning the Ntk: $\underline{A}_{ij}(t) \rightarrow A_{ij}(t)$

- **Define a 'potential':** $\Psi_i = \langle Y_i(t)^2 \rangle_\nu$

where $\langle G(t) \rangle_\nu = \int^t \nu dt' G(t') e^{-\nu(t-t')}$

- **Evolution of $A_{ij}(t)$:**

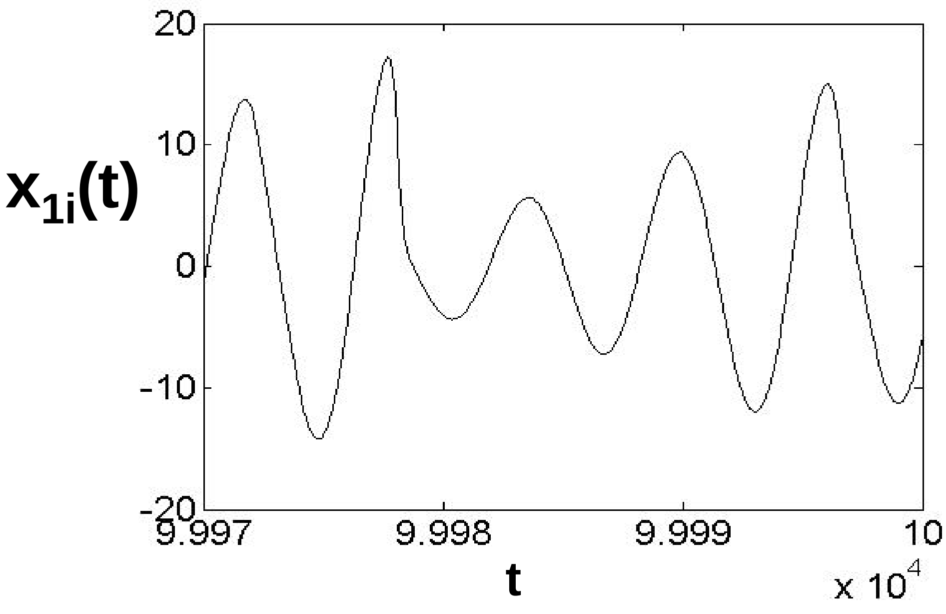
For $\bar{A}_{ij}(t)$ evolution slow w.r.t. ν^{-1}.

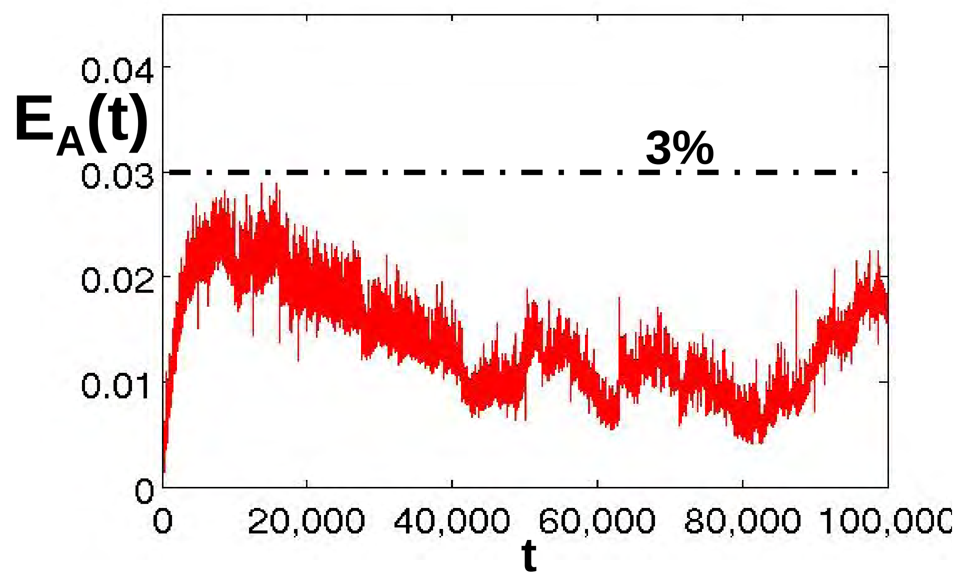
$$\frac{d\bar{A}_{ij}}{dt} = -\beta \frac{\partial \Psi_i}{\partial \bar{A}_{ij}} = 2\beta \left\{ c_{ij}(t) - \sum_{k \neq i} \bar{A}_{ik}(t) C_{ijk}(t) \right\}$$

$$\langle s_i(t-\tau_{ij}) r_i(t) \rangle_\nu \qquad \langle s_i(t-\tau_{ij}) s_i(t-\tau_{ik}) \rangle_\nu$$

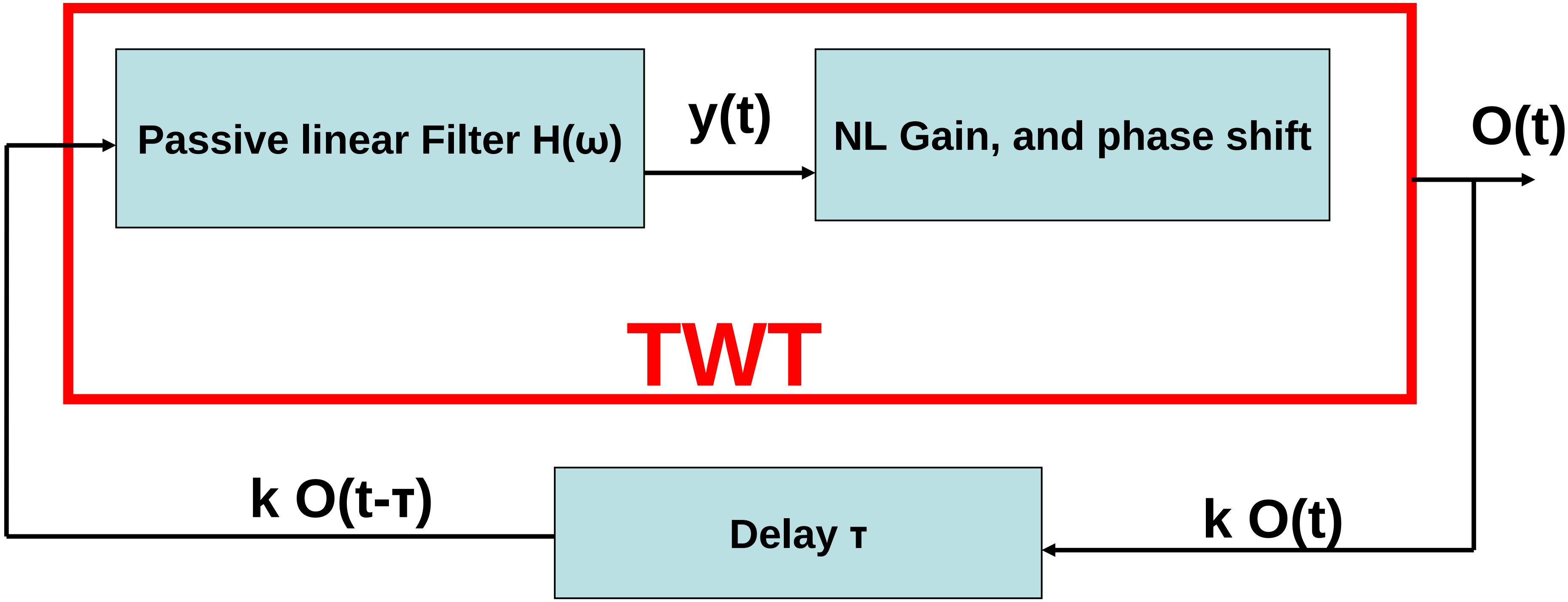
Preliminary Numerical Test

- Rössler system: $\dot{\bar{x}}(t) = F(\bar{x}(t))$ (uncoupled)
- 4 node network, 12 links
- Links: $A_{ij}(t) = [1 + \varepsilon_{ij} \sin(\omega_{ij} t + \phi_{ij})]$
- Error: $E_A(t) = (\# links)^{-1} \sum_{links} \left| A_{ij}(t) - \bar{A}_{ij}(t) \right|$





Modeling Chaotic TWTs



Previous model (normalized)

Linear part: $y(t) = \hat{H}[kO(t-\tau)]; H(s) = G\dfrac{\Delta}{s+\Delta}$ (Low-pass)

NL part: $$O(t) = y(t)\frac{\exp(i\eta|y(t)|^2)}{1+|y(t)|^2}$$

Improved Model

- Linear:

$$H(s) = G\frac{a}{s^2 + bs + c}$$

- NL description of phase shift:

$$\eta|y(t)|^2 \rightarrow \frac{\eta|y(t)|^2}{1 + d|y(t)|^2}$$

- Fitting of parameters to experiments gives good description of device.

Current and Future Work

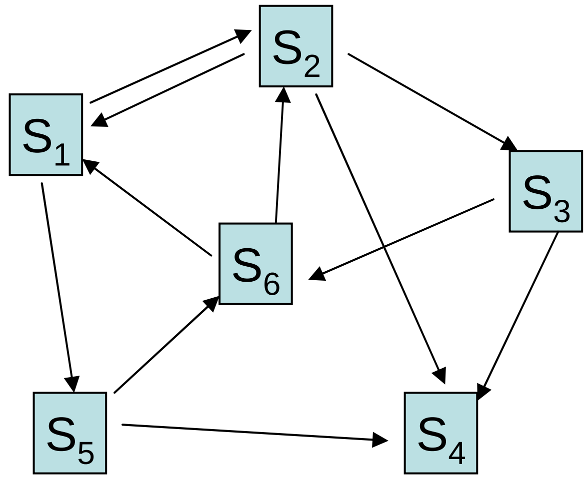
- Current: Modeling experiments on synchronization of two coupled TWT chaotic oscillators.

- Future: Use model to simulate coupled systems of several chaotic oscillators acting in a sensor network.

- Talk on experiments by John Rodgers.

- Poster presentation by Wai-Shing Lee (advised by E. Ott, T. Antonsen and J. Rodgers)

Stability of Boolean Networks



- State of node i at time t, $S_i(t)=0$ or 1
- Output of node i depends on its inputs with delays:

$S_i(t)=F[S_1(t-\tau_{i1}), S_2(t-\tau_{i2}),\ldots]$

E.g., $S_1(t)$ depends on $S_2(t-\tau_{12})$ and $S_6(t-\tau_{16})$

Our Motivation for looking at these systems:

Sensor motivated experiments of Lathrop at UMD and of Gauthier at Duke

Previous Motivation (S.A. Kaufman '69): Gene ntks

Stability/Robustness

- **Large networks of this type can have many attractors.**
- **How stable are these attractors?**
- **I.e., if we flip the state of a randomly chosen S_i is the orbit likely to go to some far away attractor?**
- **Kaufman: ‘Life at the edge of chaos’. Evolution prefers gene networks to be at the border of stability.**

Our Work

- **The stability question also has implications for our exps., particularly wrt the number of attractors and robustness to perturbations.**

- **Stability has previously been addressed only for a rather trivial class of networks. We show how to analize stability for general networks.**

- **See poster by Andrew Pomerance (grad student) E. Ott, M. Girvan and <u>W. Losert</u>.**