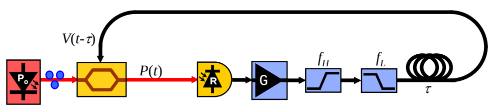
Using synchronization for prediction of high-dimensional chaotic dynamics

Adam B. Cohen, Bhargava Ravoori, Karl R. B. Schmitt, Thomas E. Murphy, Rajarshi Roy

Introduction

We show that synchronization of a numerical model to experimental measurements provides a new way to assimilate data and forecast the future of a time-delayed high-dimensional optoelectronic feedback loop. We will use this feedback loop as a modular element of our nonlinear photonic sensor networks.

Experimental schematic



= semiconductor laser diode providing optical power P_o

= Mach-Zehnder electrooptic modulator

optical signal $P(t)$ is modified by an electrical input $V(t)$ as:

$$P(t) = P_o \cos^2\left(\frac{\pi V(t)}{2V_\pi} + \phi_o\right)$$

= high-speed photodetector outputs an electrical voltage proportional to incident optical signal, RP

= electrical amplifier providing gain G

= high-pass electrical filter described by 7th-order Butterworth transfer function with cut-on frequency f_H = 1 MHz

= low-pass electrical filter described by 7th-order Butterworth transfer functions with cut-off frequency f_L = 100 MHz

= delay τ due to optical fiber and electrical cabling τ = 22.5 ns

Mathematical model

Our electronic band-pass filter can be described by:

$$\frac{d\mathbf{u}(t)}{dt} = \mathbf{A}\mathbf{u}(t) + \mathbf{B}x_{in}(t), \quad x_{out}(t) = \mathbf{C}\mathbf{u}(t)$$

where $\mathbf{u}(t)$ is an N-dim state vector (N = order of filter) and **A**, **B**, **C** are matrices describing the linear filter.

Nonlinear time-delayed feedback is described by:

$$x_{out}(t) = \beta \cos^2\left(x_{in}(t-\tau) + \phi_o\right)$$

where β is the feedback strength and τ is the feedback delay such that:

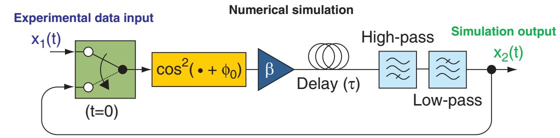
$$x(t) = \frac{\pi V(t)}{2V_\pi}, \quad \beta = \frac{\pi G R P_o}{2V_\pi}$$

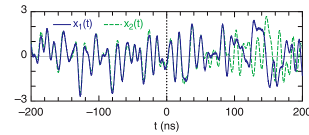
So:

$$\frac{d\mathbf{u}(t)}{dt} = \mathbf{A}\mathbf{u}(t) + \mathbf{B}\beta\cos^2\left(\mathbf{C}\mathbf{u}(t-\tau) + \phi_o\right), \quad x(t) = \mathbf{C}\mathbf{u}(t)$$

Note: In the experiments, we only observe the scalar variable $x(t)$ and not all the components of the vector $\mathbf{u}(t)$.

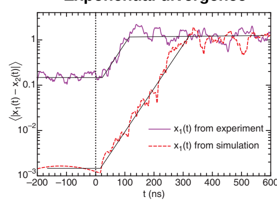
Synchronization & prediction



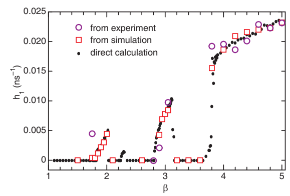


(1) Feed experimental data $x_1(t)$ into numerical simulation in place of feedback signal.
(2) When open-loop synchronization is achieved, release simulation from experimental data and measure the divergence rate

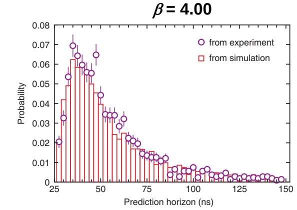
Exponential divergence



Maximal Lyapunov exponents



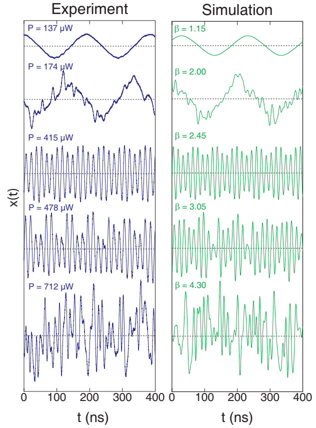
Distribution of prediction horizon times β = 4.00



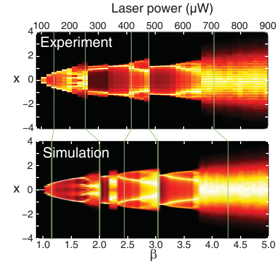
Comparison between experiment & simulation

We study the system by varying the feedback strength β (by adjusting laser power P_o).

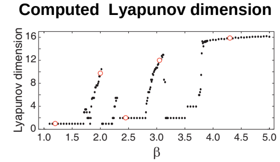
Measured & simulated time series



Measured & simulated bifurcation diagrams

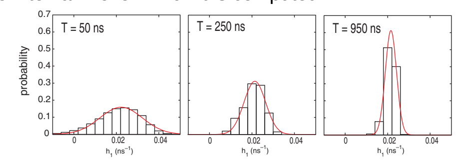


Computed Lyapunov dimension



Distribution of Lyapunov exponents

The distribution of finite-time Lyapunov exponent h_1 will vary depending on the time interval T over which it is computed.



Conclusions

✓ We have an accurate model for this optoelectronic system.
✓ We demonstrate a method for assimilating experimental data into a multi-dimensional, time-delayed model.
✓ We can make predictions for many delay periods when the dynamics are about 15 dimensional
➢ We could use a similar scheme to synchronize and release two experimental systems for a model-independent measurement of maximal Lyapunov exponents.

Ref.: A.B. Cohen, B. Ravoori, T.E. Murphy, R. Roy, arXiv:0809.3777v1 (2008); to appear in PRL