



# The Chaotic Time Reversal Sensor

Biniyam Tesfaye Taddese<sup>2,3</sup>, James Hart<sup>1,2</sup>, Thomas Antonsen Jr.<sup>1,2,3</sup>,  
Edward Ott<sup>1,2,3</sup>, John Rodgers<sup>2</sup>, Steven M. Anlage<sup>1,3,4</sup>  
Department of Physics - University of Maryland, College Park, U.S.A.



center for nanophysics and advanced materials

## Overview

Classically chaotic systems display extreme sensitivity to initial conditions ... which suggests good sensor applications

Sensors often utilize waves (acoustic, electromagnetic, seismic, etc.) for detection

Wave Chaotic systems show extreme sensitivity to perturbations

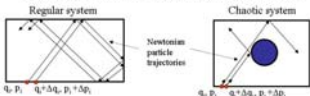
Time Reversal and Spatial Reciprocity are two 'hidden' symmetries of the wave equation that can be exploited to simplify the wave chaotic sensor

We have developed a novel sensor paradigm that combines the extreme sensitivity of wave chaotic systems with the simplicity of time-reversed and spatially reciprocal waves to create:

The **CHAOTIC TIME-REVERSAL SENSOR**

## Wave Chaos?

- Classical chaotic systems have diverging trajectories
- 2-Dimensional "billiard" tables with hard wall boundaries



- Linear wave systems can't be chaotic



- However in the semiclassical limit, you can think about rays

Wave Chaos concerns solutions of wave equations which, in the semiclassical limit, can be described by ray trajectories

## Ray Chaos

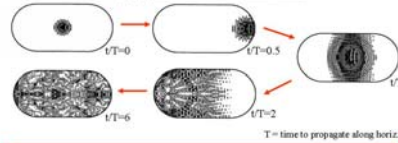
Many enclosed three-dimensional spaces display ray chaos



## Now Consider Wave Propagation in Ray-Chaotic Enclosures

The two-dimensional Stadium Billiard shows Ray Chaos, which is illustrated by the spatial ergodicity of a ray trajectory.

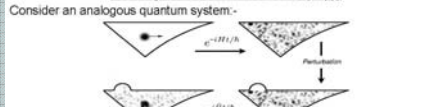
Propagation of a Gaussian Wave Packet



Solve the wave equation (Electromagnetic, Acoustic, Schrödinger equation, etc.) in a ray-chaotic enclosure

Examine the solutions in the semiclassical regime:  $0 < \lambda \ll$  System Size  
Tomovic-Haller PRL 47, 382 (1993)

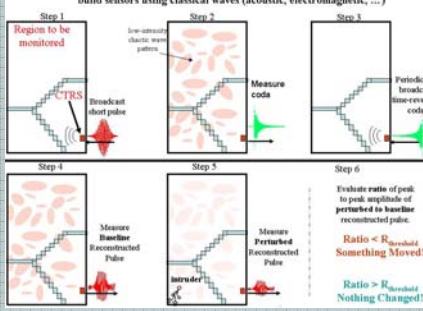
## Quantifying Sensitivity to Perturbation of Wave Chaotic systems using Fidelity or Loschmidt-Echo (M(t))



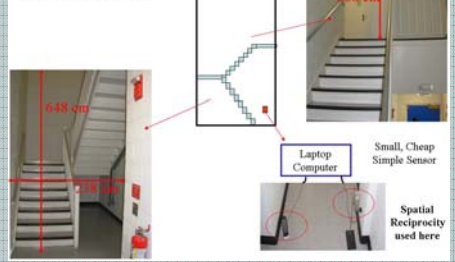
$$M(t) = \left| \langle \Psi_0 | e^{iHt/h} e^{-iHt/h} | \Psi_0 \rangle \right|^2$$

M(t), Loschmidt-Echo, is expected to decay exponentially! (for such perturbations)  
The Loschmidt-Echo is measured using the Scattering Fidelity, which is the Cross Correlation of scattering matrix elements before and after perturbation.  
The ratio of peak to peak amplitude of initial and final state of the illustrated (classical) wave evolution is shown to be indicator of perturbation.

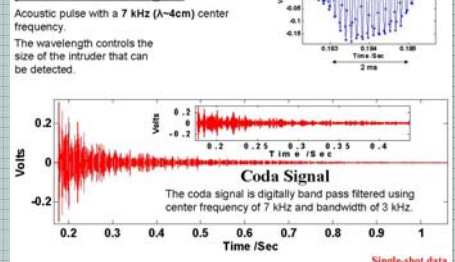
Sensitivity to perturbation of wave chaotic systems can be used to build sensors using classical waves (acoustic, electromagnetic, ...)



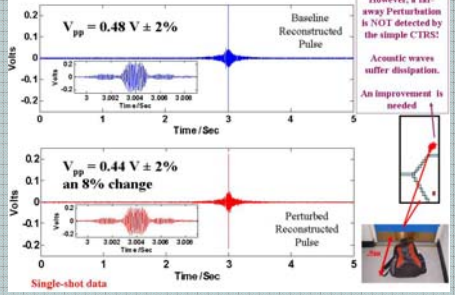
## Acoustic CTRS in a Stairwell



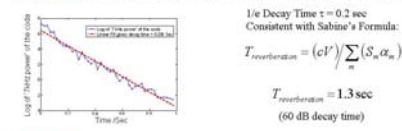
## Acoustic CTRS in Stairwell - The incident pulse and Coda signal



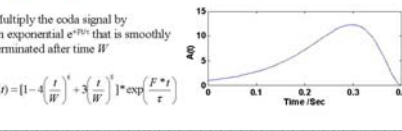
## Acoustic CTRS: Effects of Perturbation



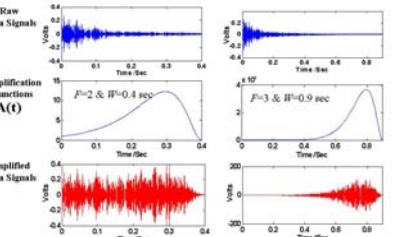
## Problem: Dissipation of Acoustic Waves inside the Stairwell



## Solution: Exponential Amplification to overcome dissipation



## Solution: Exponential Amplification to overcome loss



Both of these amplified coda signals produce excellent detection for non line-of-sight perturbations  
The CTRS is calibrated for a given Enclosure & Type of Perturbation by finding values of parameters which minimize the ratio of V<sub>pp</sub> of Perturbed Reconstructed Pulse to V<sub>pp</sub> of Baseline Reconstructed Pulse.

## Exponential Amplification overcomes loss limitations enabling distant detection

Description of CTRS operation	V <sub>pp</sub> of Baseline Reconstructed Pulse (BRP)	V <sub>pp</sub> of Perturbed Reconstructed Pulse (PRP)	Ratio of V <sub>pp</sub> of PRP to V <sub>pp</sub> of BRP
No Amplification	0.47V (±2%)	0.44V (±2%)	100% → Not Detected!
Amplification with P=2 & P=3	0.54V (±2%)	0.63V (±2%)	100% → Detected!

## Conclusions

- A new sensor paradigm has been developed based on the extreme sensitivity of wave chaotic systems to small perturbations
- Time-Reversal Symmetry and Spatial Reciprocity have been exploited to simplify the sensor
- The sensor has been demonstrated with acoustic waves in a cheap and portable manner
- Exponential amplification can be used to dynamically change the range of sensitivity of the sensor

## Future Work

- Design a network of CTRS sensors to determine the position and size of the perturbation
- Utilize wavelength diversity to sense perturbations of different sizes
- Study use of Fidelity to quantify the perturbation & to classify it under different universality classes
- Realize the CTRS using low power wave source and receiver controlled by an embedded chip

Some Background Publications:  
S. Hemady, et al. Phys. Rev. Lett. 94, 014102 (2005)  
S. M. Anlage, et al. Acta Physica Polonica A 112, 569 (2007)  
http://www.ccr.umd.edu/anlage/AnlageQCChaos.htm  
Many thanks to: E. Bradshaw, R. Prange, S. Fishman, M. Fink, A. Richter, F. Schaefer, S. Hemady