Abstract

For two-dimensional, symmetric double well potentials separated by a tunneling barrier, it has been shown theoretically that quantum mechanical tunneling rates are dependent on the shape of the well in the limit of small quantum wavelengths. Shapes that support chaotic wave functions produce statistically smaller fluctuations in the tunneling rate than classical (integrable) wells. This effect is called “regularization” of tunneling rates and can be analyzed by examining the splitting in energy level between symmetric and antisymmetric wave functions. Utilizing the similarity in the wave natures of a quantum mechanical particle in a symmetric double well and transverse electromagnetic waves in a 2-D cavity, we investigate chaos regularization in coupled microwave cavities both numerically and experimentally. The difference between the antisymmetric and symmetric eigenstate frequencies squared and evanescent electromagnetic wave propagation in a cavity tunnel is shown to be analogous to energy splitting and quantum mechanical tunneling, respectively. This analogy allows investigation of regularization in various microwave cavities and, by extension to a lab-scale model, the behavior of complex quantum mechanical systems.

Methods

Determining Resonant Frequencies

To determine resonant frequencies in the each cavity, we drove weakly coupled ports using a Vector Network Analyzer (VNA). The port has a dual functionality and is able to both send out a wave and measure the reflected response of that waveform inside the cavity.

Distinguishing Antisymmetric and Symmetric Modes

A power source with an alternating phase angle is used to drive a port. Simultaneously, a frequency sweep is conducted on a second port. The response evoked by this pairing allows us to measure both antisymmetric and symmetric resonances.

Comparing Resonant Frequencies

Experimental Cavities

For our barrier is discretized whereas in the electromagnetic extension tunneling is continuous. For this reason, the electromagnetic double well has a high conductive resistivity, limiting tunneling.

Numerical Results

Our goal is to come up with a physical model that we can test experimentally that will demonstrate that this regularization occurs for chaotic wells. Here we use simulation tests as a precursor for our experimental model:

Rectangular Cavity

To create our experimental models, we use a Computer Numerical Control (CNC) milling machine. This creates precise replications of the cavities used in our simulation models onto aluminum plates.

Comparing Resonant Frequencies

We measured energy splitting for a small band of frequencies using the driving phase method (see Figures 9-12). We found that there was a much larger spread for the rectangular cavity compared to the bowtie cavity, as predicted by both theory and our numerical results. This demonstrates that chaos regularization occurs.

Conclusion

Non-Idealities:

- Aluminum has a high conductive resistivity, but still dissipates energy
- The frequencies of driving ports may exclude certain resonant frequencies.
- Our barrier is discretized whereas in the quantum mechanical system the potential barrier is continuous.

Future Work:

- Use a material that has properties more similar to a perfect superconductor
- Find resonance splitting at more locations
- Test other types of integrable and chaotic cavities

Acknowledgments

Anlage Research Group (University of Maryland)


Figure 2. Comparison of tunneling rates found in Lee et al – Integrate well Right – Concave well.

Figure 6. Frequency sweep w/ zero phase shift in the chaotic cavity.

To make our model analogous to a 2-D well potential, we test a 3-D perfectly superconductive microwave cavity with a depth that is shallow relative to the width and length. To ensure we are matching the correct antisymmetric and symmetric waves, we change the boundary conditions at the central barrier. We find that regularization of tunneling rates does occur in the bowtie, or chaotic cavity. Two cavities that have the same width and central barrier height will have the same average energy splitting, as predicted for quantum mechanical models.

Figure 3. Diagram of the central barrier collector for the concave microwave cavity used in the experiment.

Figure 7. Spread of the difference in antisymmetric and symmetric resonant frequency paths measured during the experiment for 4.6 – 8.5 GHz.