Chaos in a Transmission Line Circuit

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With...
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Our Motivation

- As computer clock times become shorter, circuit connections behave more and more like transmission lines.
  
  \[ \text{Length of device} \sim \text{Wavelength of Signal} \]

- Theoretical framework needed to complement the experimental project of Professor Steven Anlage with the help of graduate student Anand Banerjee
Image of a circuit diagram with labels:

- $R_g$
- Transmission Line ($Z_o, T$)
- $V_g(t)$
- $V_{inc}$
- $V_{ref}$
- Diode
- Model with symbols for resistor and nonlinear capacitor
Model for Nonlinear Capacitor

By making the capacitance dependent on the voltage, nonlinear effects are introduced into the circuit.

\[ 5 \leq \frac{C_f}{C_r} \leq 1000 \]
The Delay Differential Equation

Our system is described by a delay differential equation of the form...

\[
\frac{d}{dt}V(t) = C_1 V(t) + C_2 V(t-2T) + C_3 \frac{d}{dt}V(t-2T) + C_4 V_g(t-T)
\]

...where T is the time it takes for a signal to propagate across the length of the transmission line.

• We solved the differential equation using Matlab with Runge-Kutta 4.
• Our program’s solutions were verified by comparing them with phasor solutions in the linear case of the capacitor.
Voltage Wave Forms for Various $V_g$

- $V_g = .5$ V Period 1
- $V_g = 2.25$ V Period 2
- $V_g = 3.5$ V Period 4
- $V_g = 5.25$ V Chaos
Bifurcation Diagram

Vg (Volts)

Strobe Points (Volts)
Bifurcation Diagram
2D Projection of a Chaotic Orbit

\[ V_g = 5 \text{ V} \]
Challenges for the Future

• Ten parameters to explore:
  \[C_f, C_r, g, Z_o, R_g, V_g, \omega, T, V_f, V_{gap}\]

• Experimental verification of numerical results