

Explosive Percolation in Directed Networks

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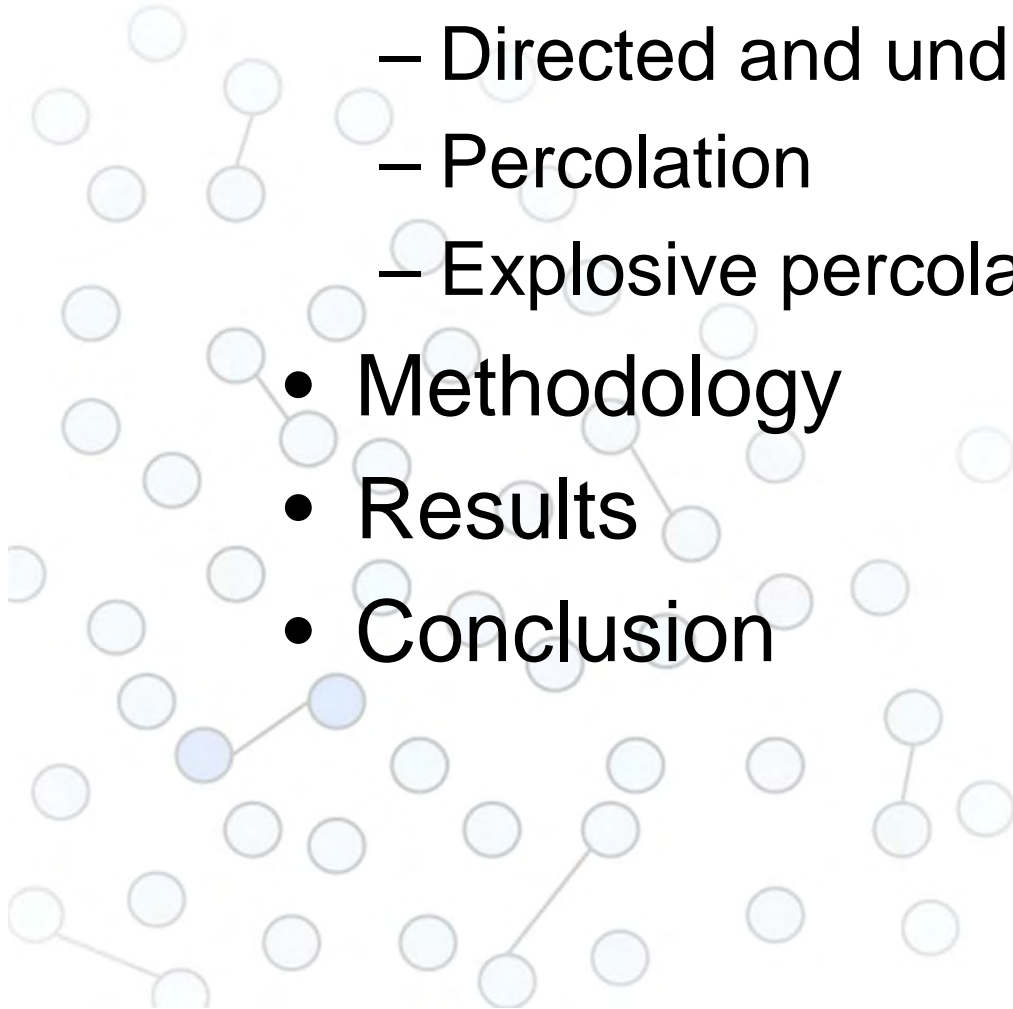
Training and Research Experiences in Nonlinear Dynamics



RUTGERS

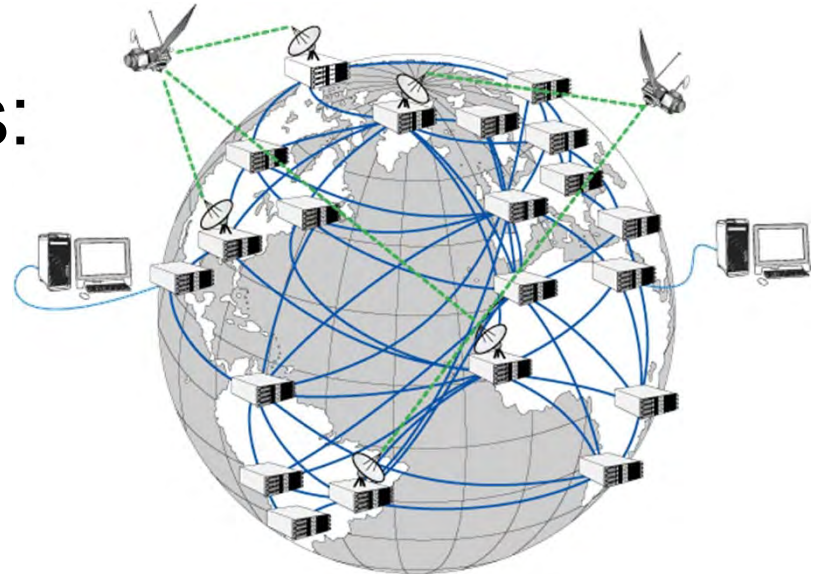
Outline

- Background
 - Directed and undirected networks
 - Percolation
 - Explosive percolation
- Methodology
- Results
- Conclusion



Motivation

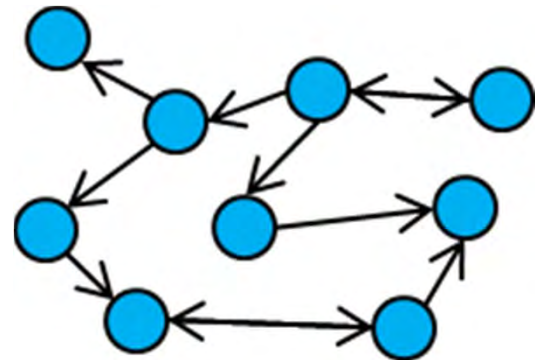
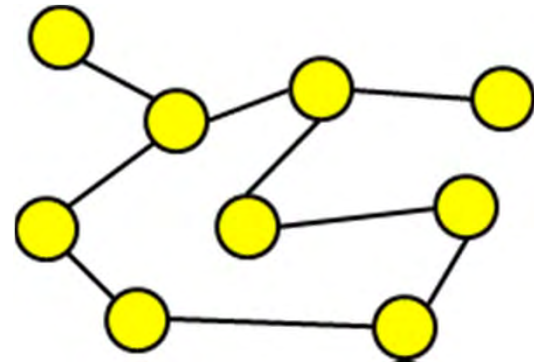
- Real world networks: biological systems, social networks, the Internet, infrastructure, etc.
- Grow by the addition of links
- New class of transitions: “explosive percolation”



Taken from: <http://www.portices.fr/>

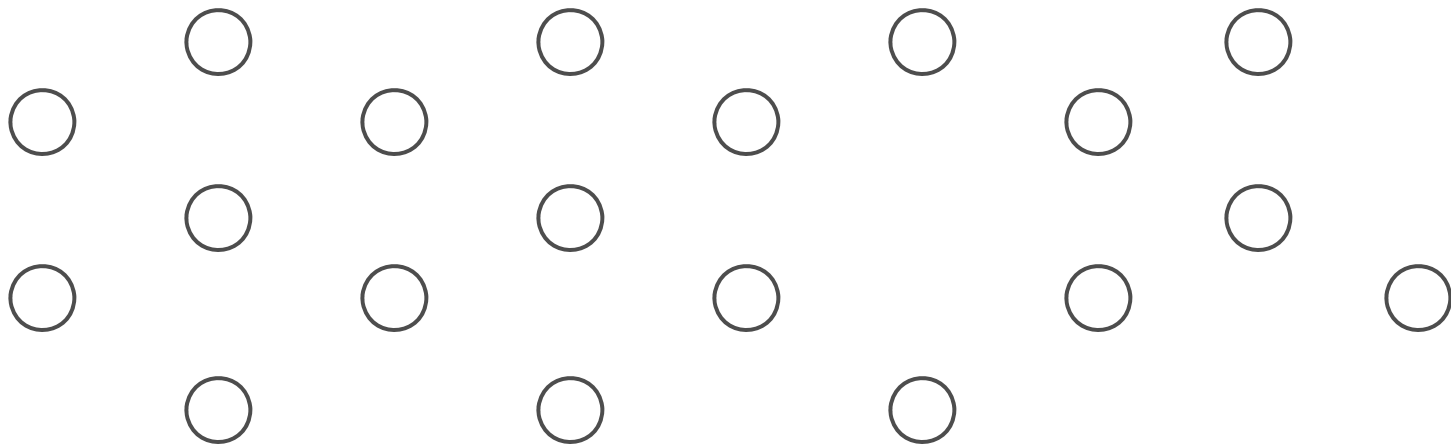
Undirected vs. Directed Networks

- Undirected networks have undirected links
- Directed networks have directed links



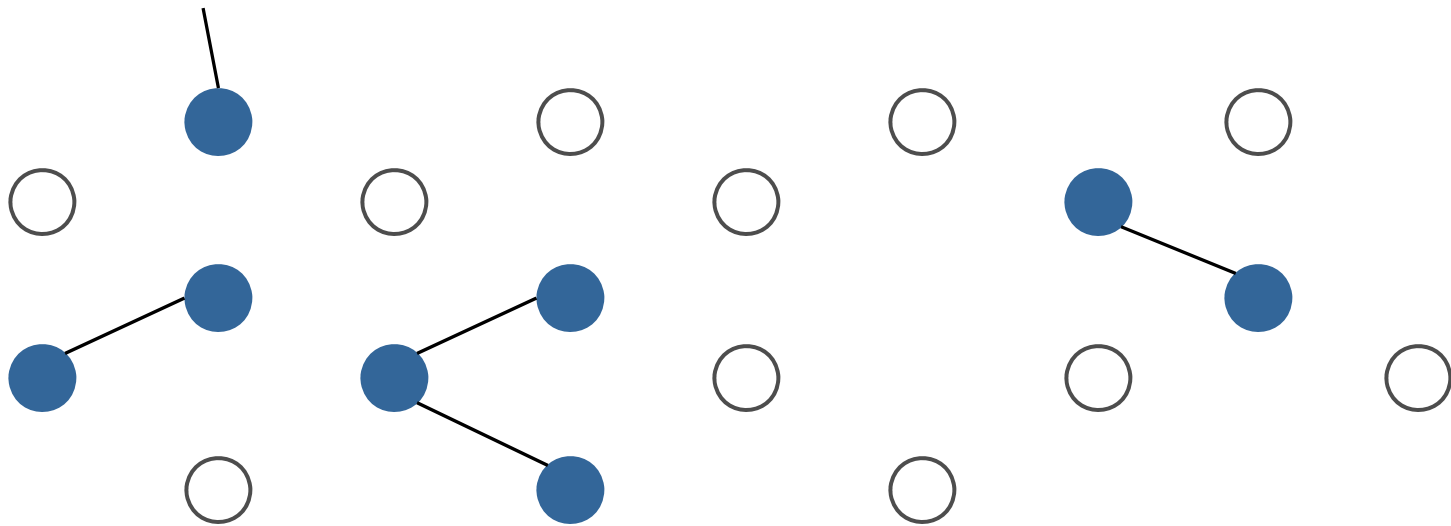
Network Percolation

Formation of *macroscopic* connected component by randomly adding links



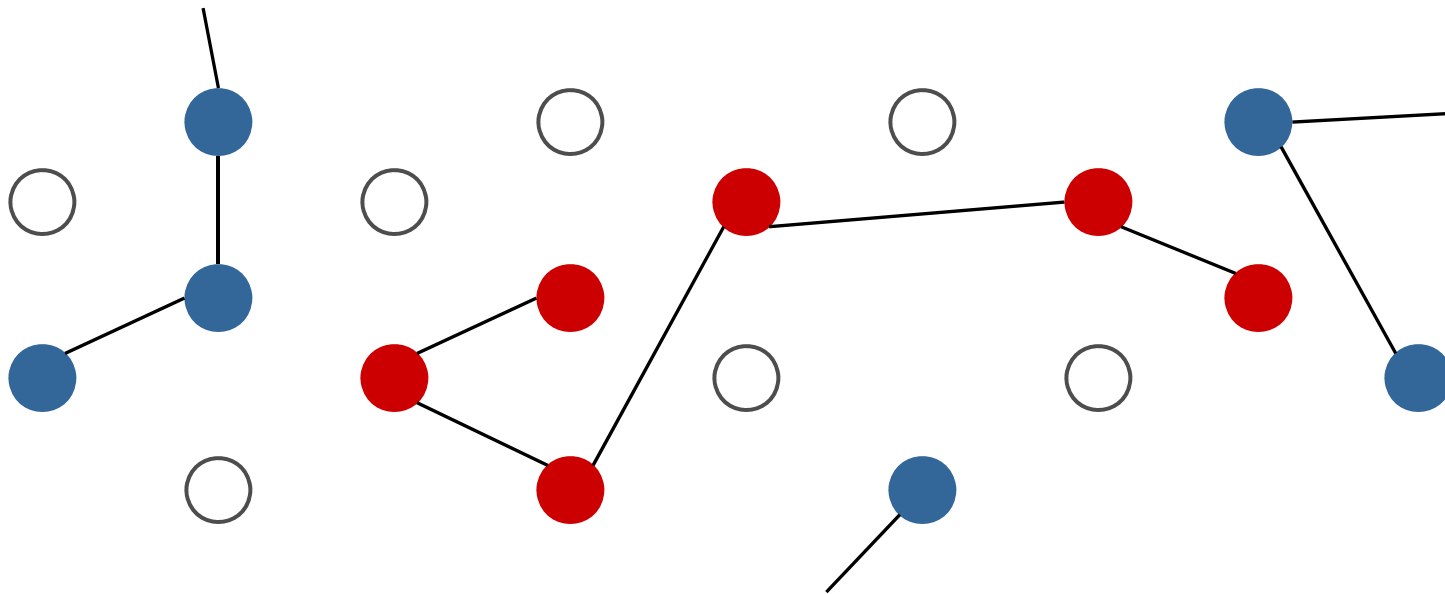
Network Percolation

Formation of *macroscopic* connected component by randomly adding links



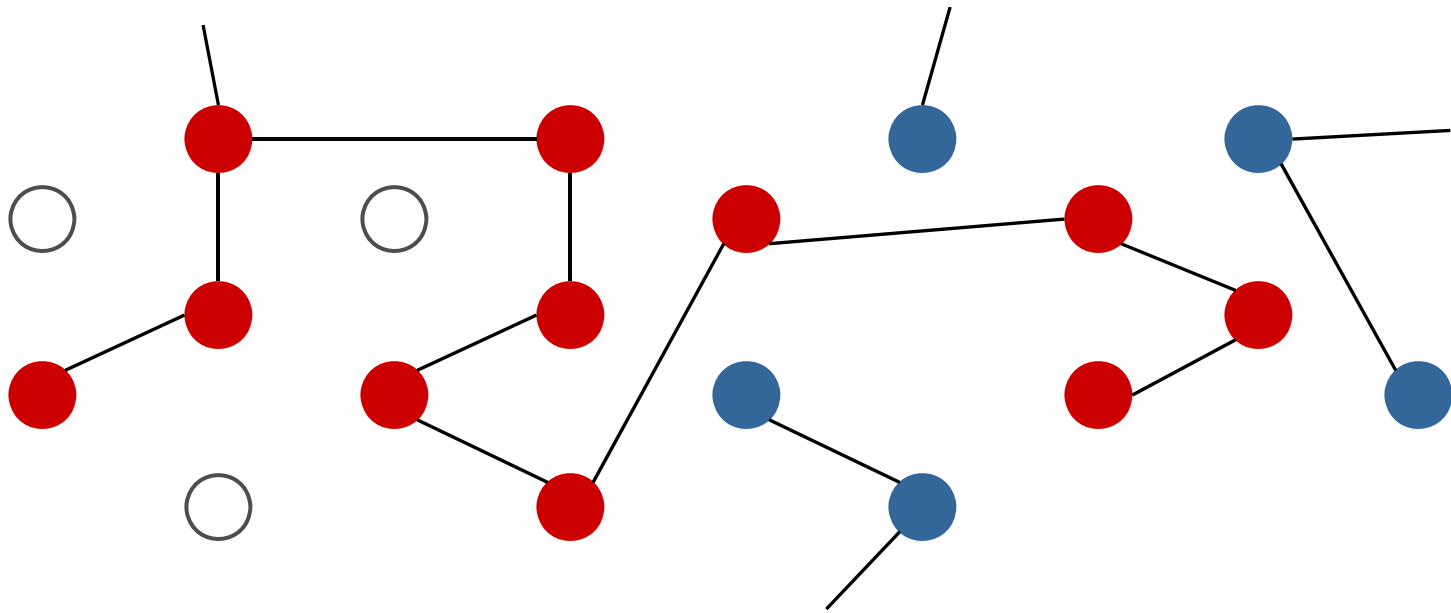
Network Percolation

Formation of *macroscopic* connected component by randomly adding links



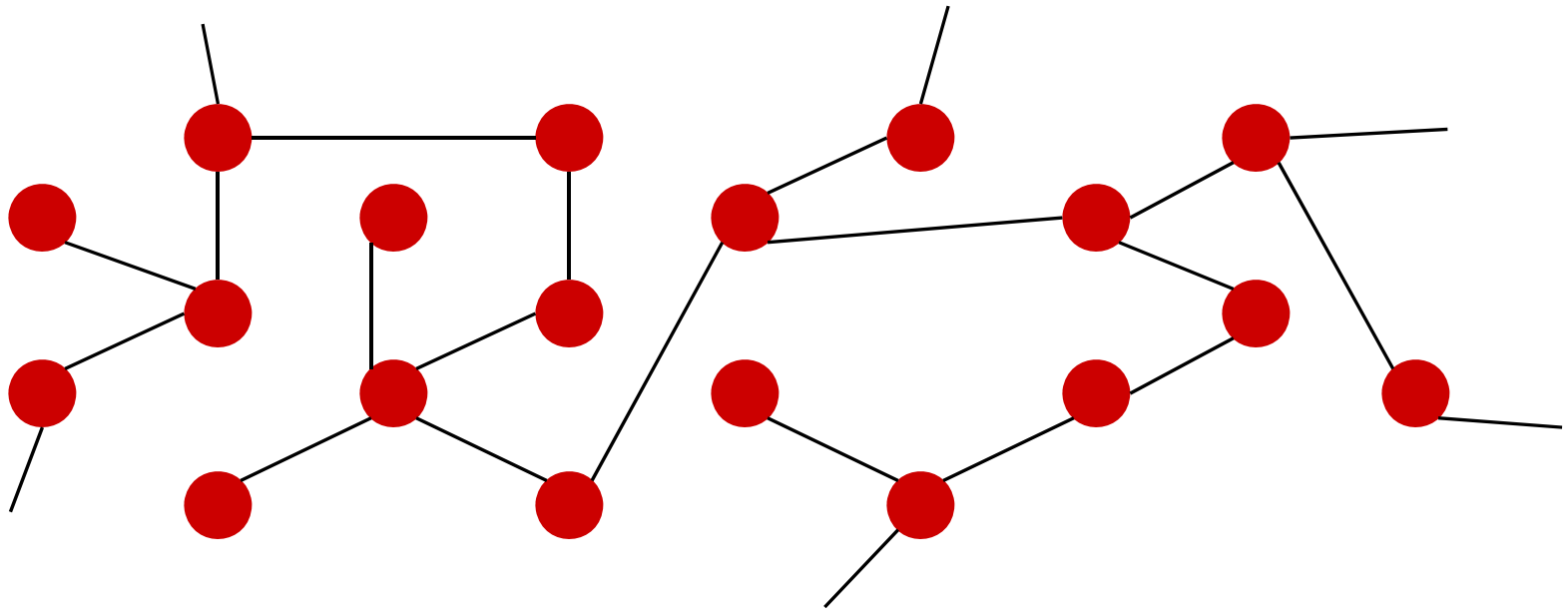
Network Percolation

Formation of *macroscopic* connected component by randomly adding links



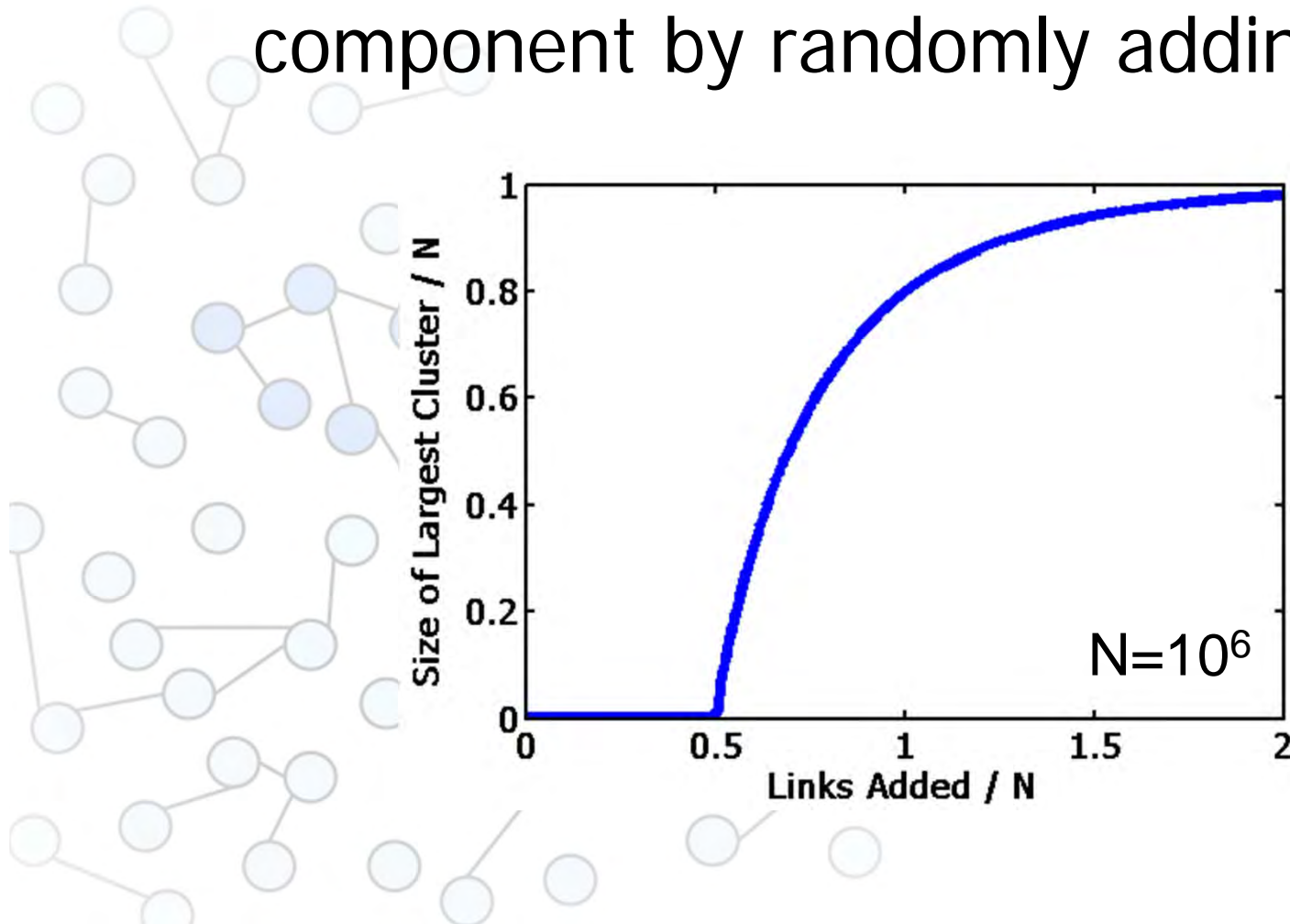
Network Percolation

Formation of *macroscopic* connected component by randomly adding links



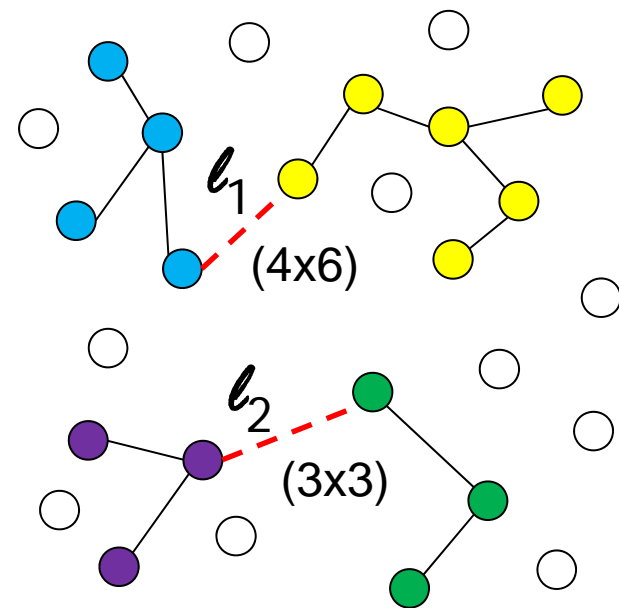
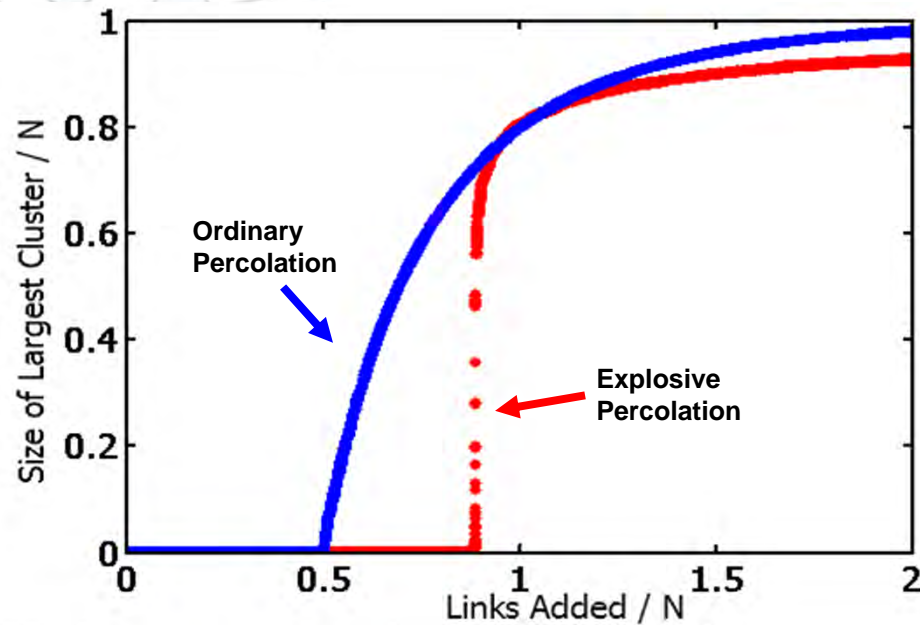
Network Percolation

Formation of *macroscopic* connected component by randomly adding links



Explosive Percolation*

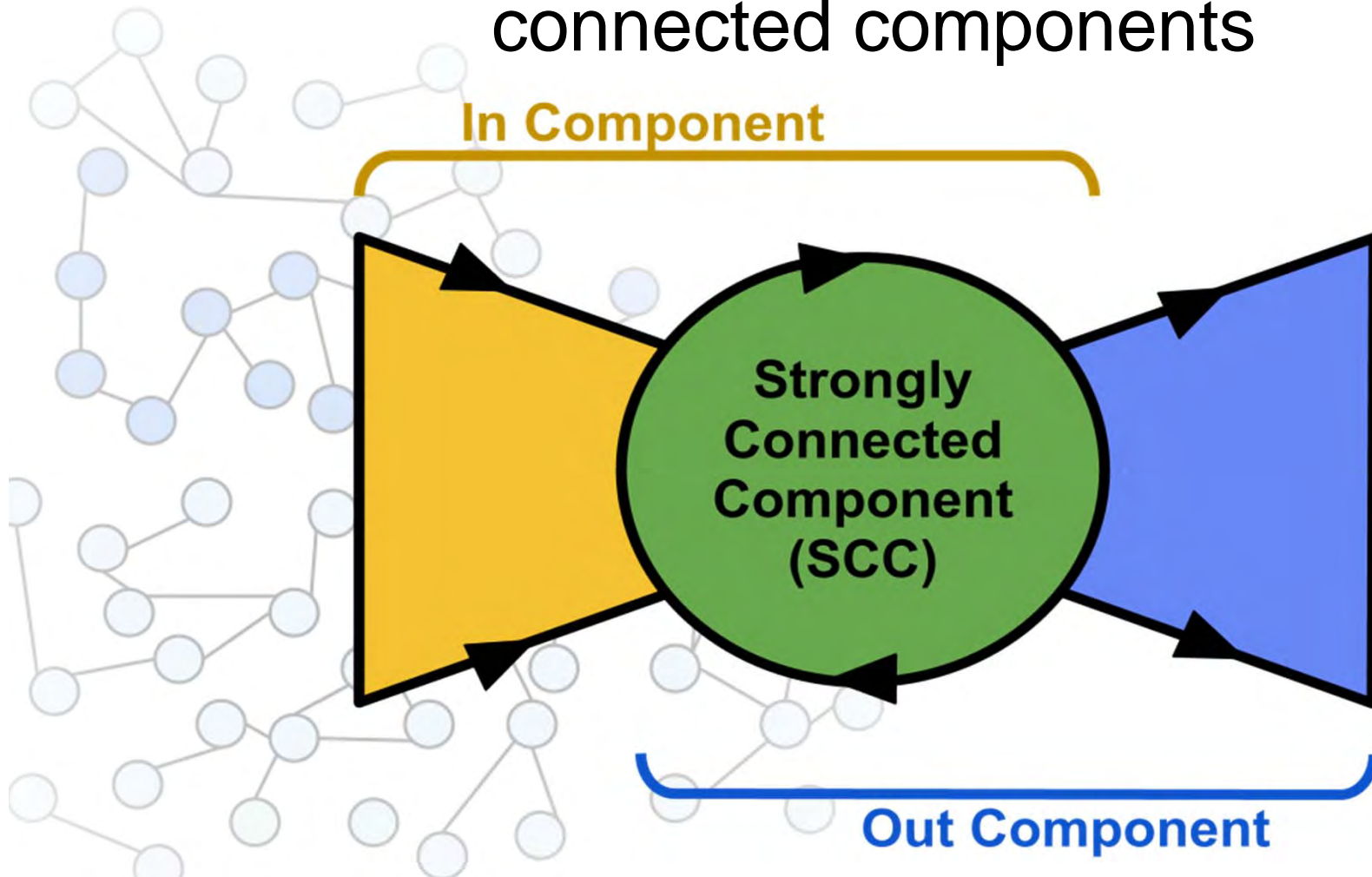
“Competitive” rule for adding links



*D. Achlioptas, R. M. D'Souza, and J. Spencer, Science 323, 1453 (2009).

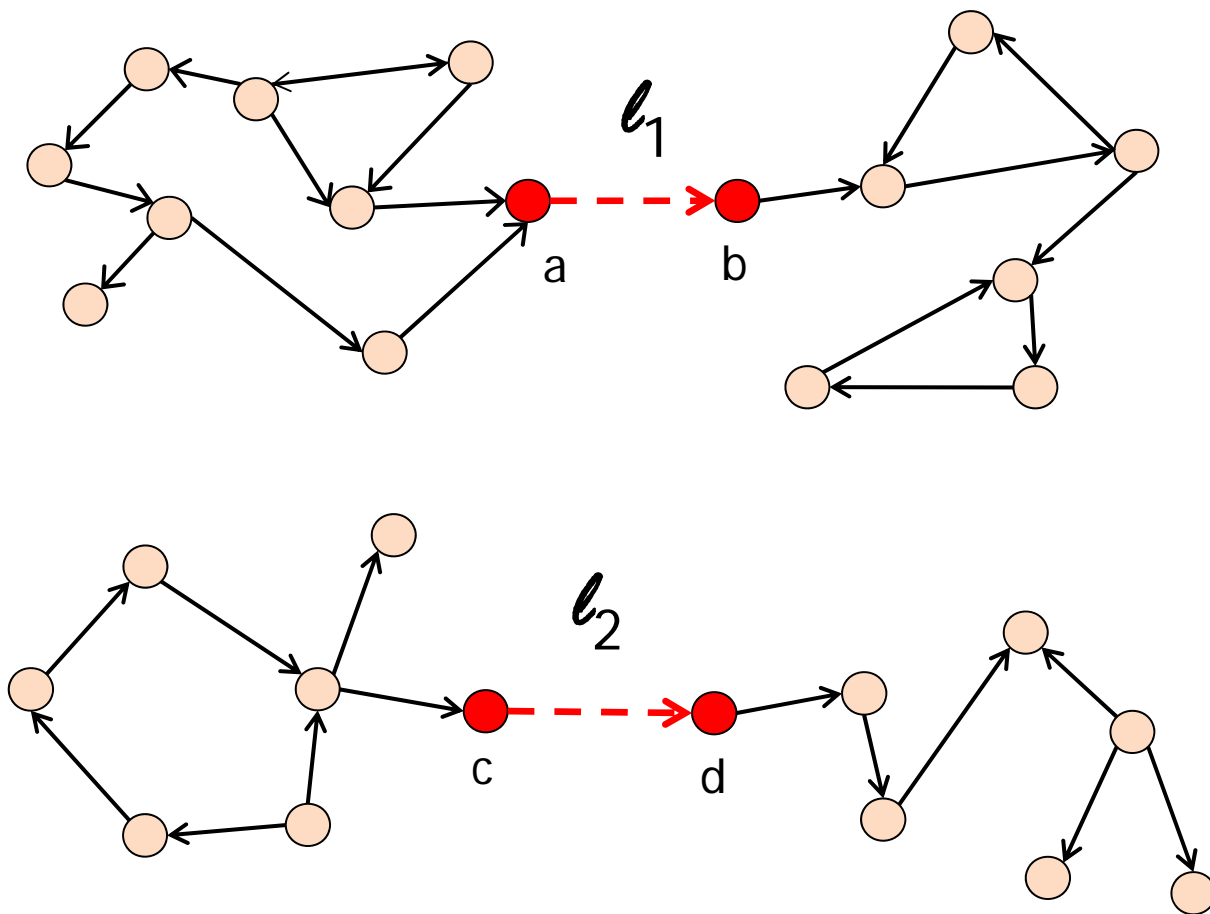
Connected Components

Directed networks have 3 kinds of connected components



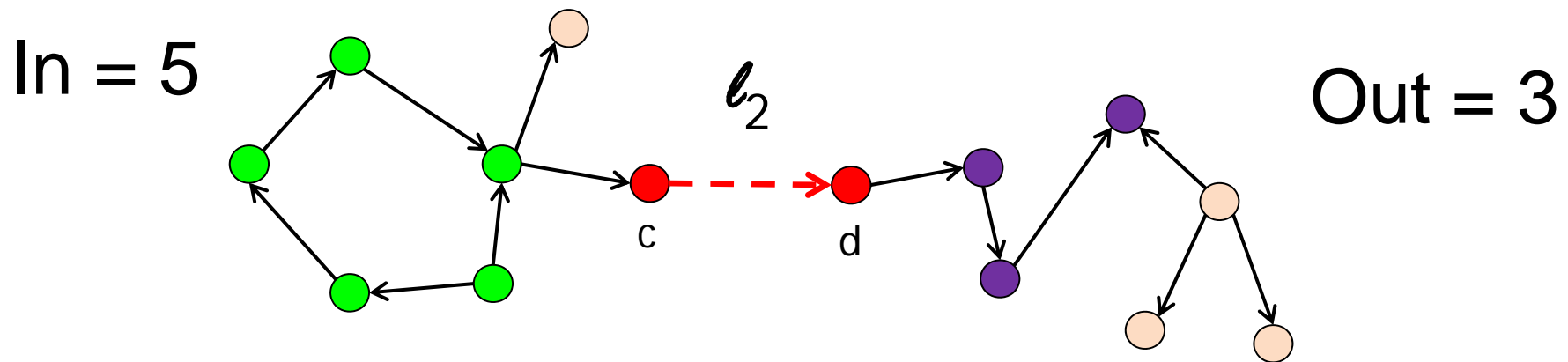
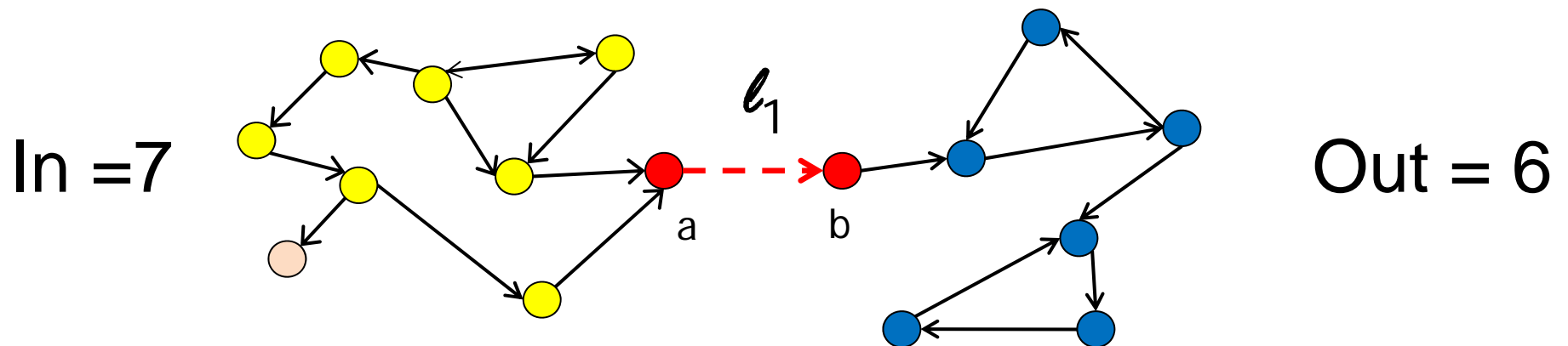
Competitive Network Growth Process

1. Randomly pick 2 candidate links



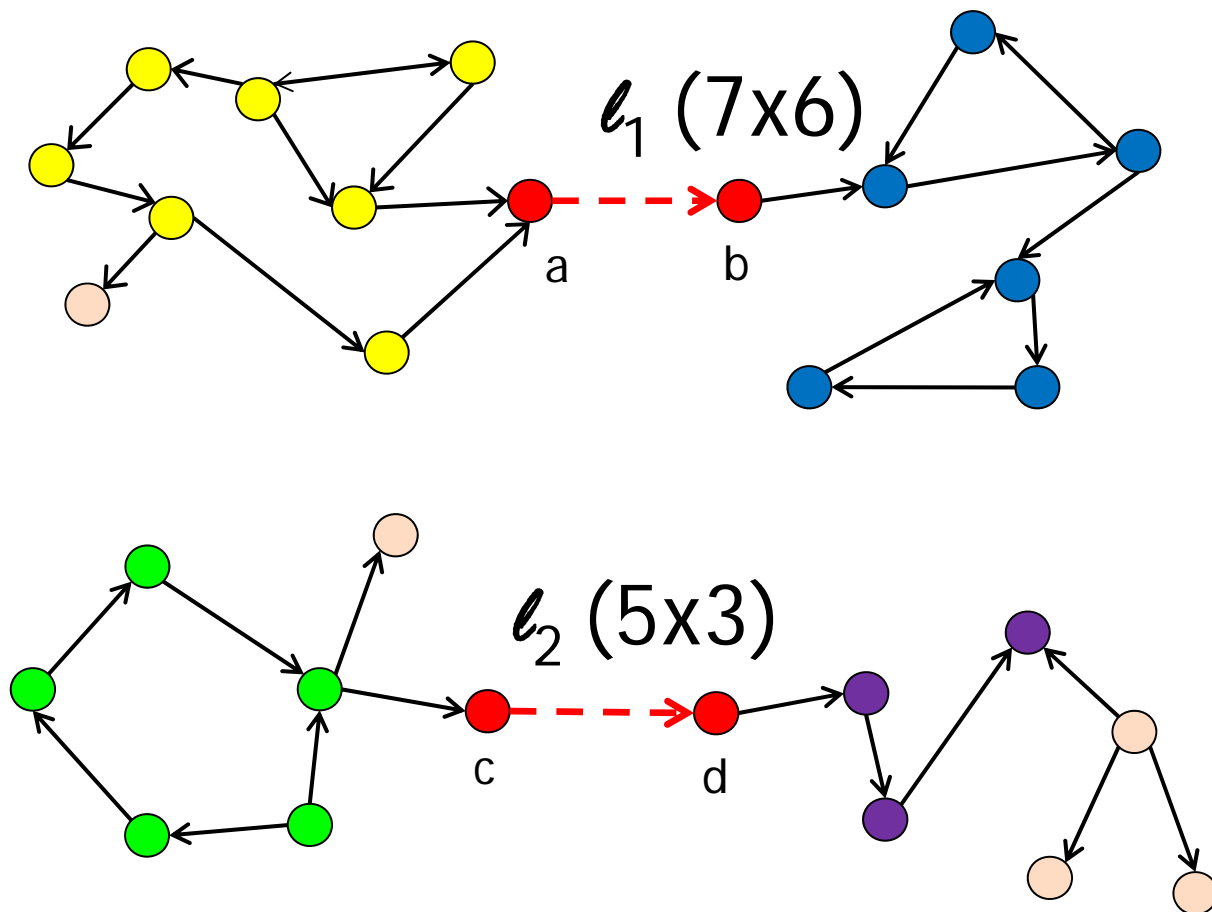
Competitive Network Growth Process

2. Calculate in and out component sizes



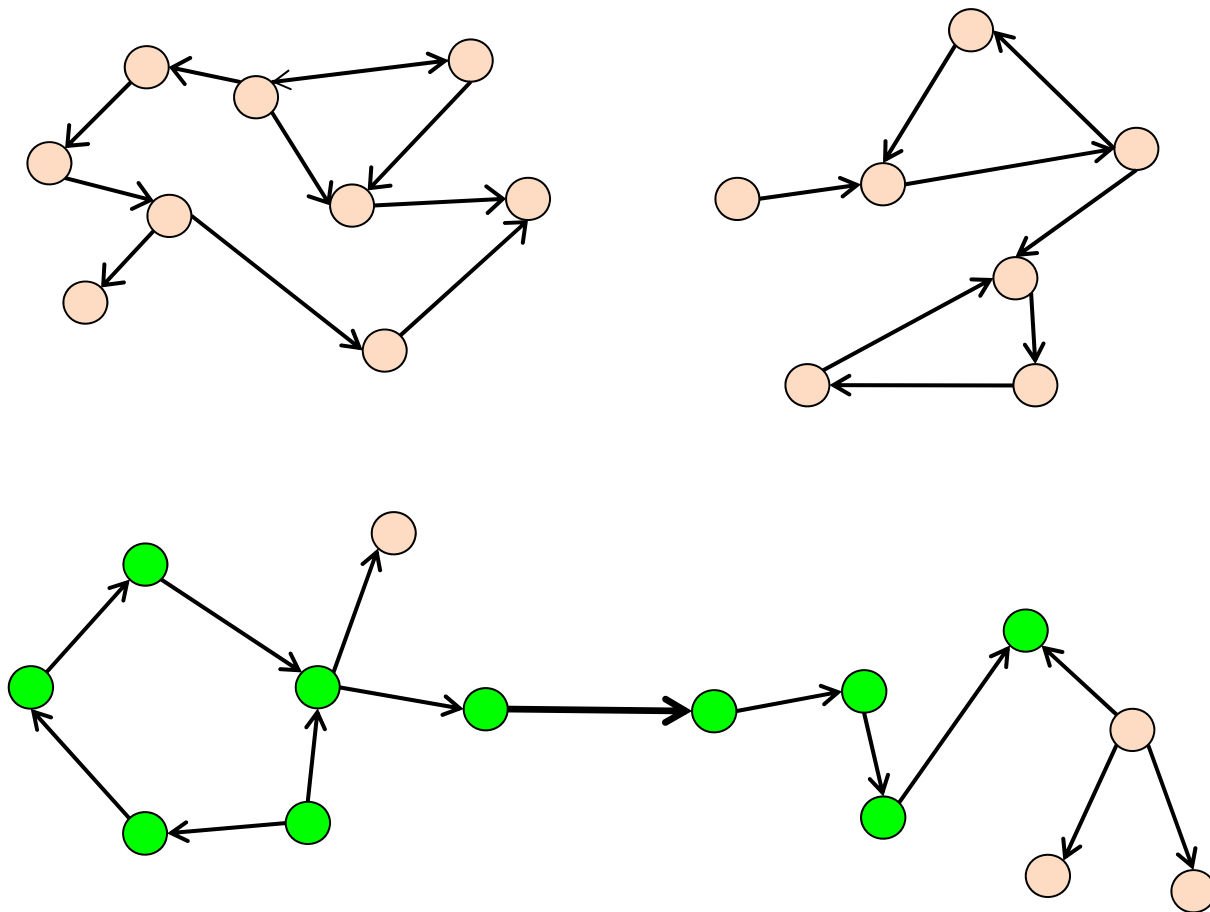
Competitive Network Growth Process

3. Multiply the in and out sizes for each node



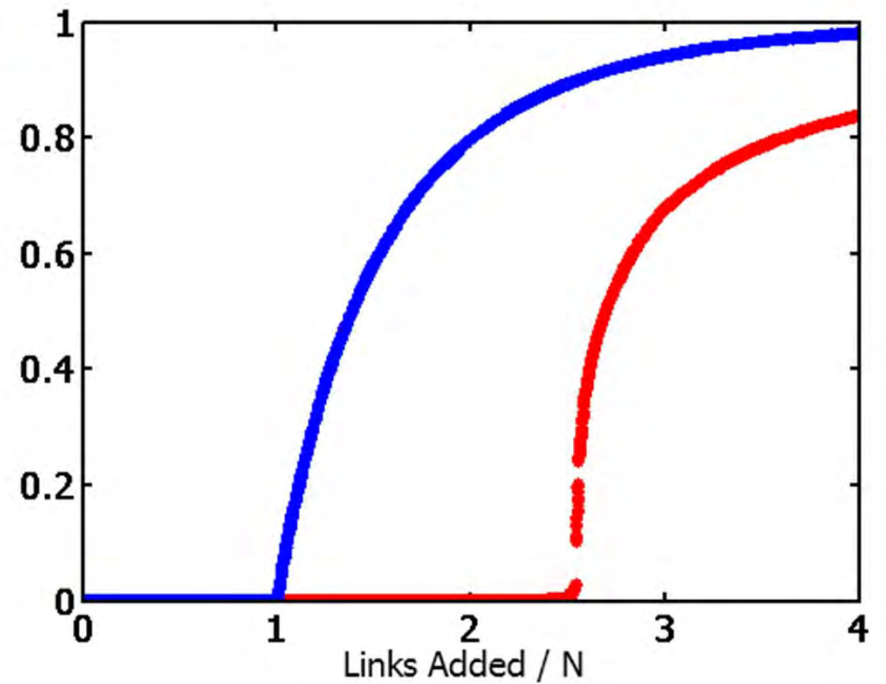
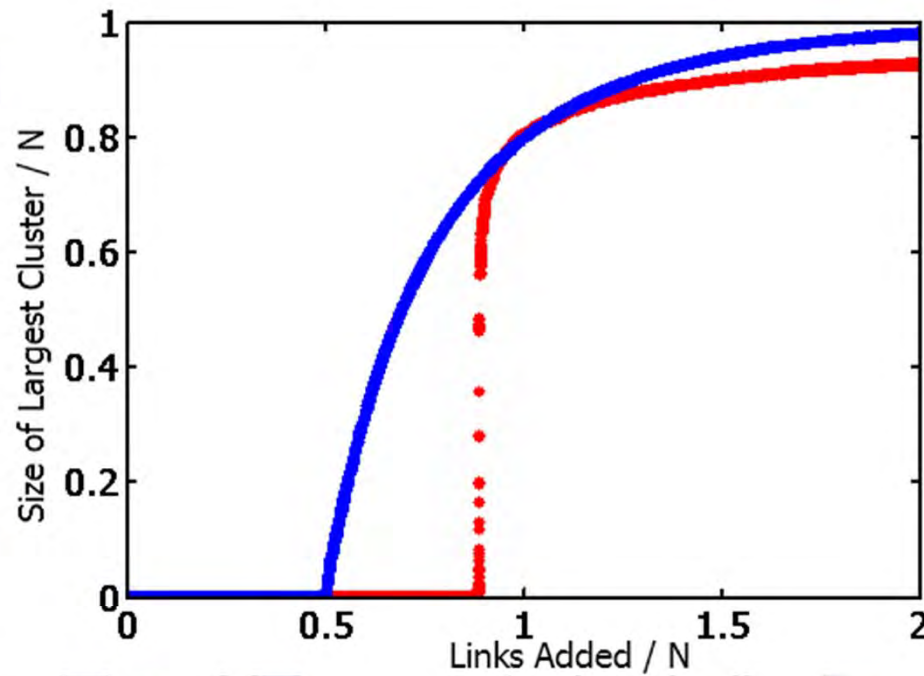
Competitive Network Growth Process

4. Add the link with the smallest product



Results

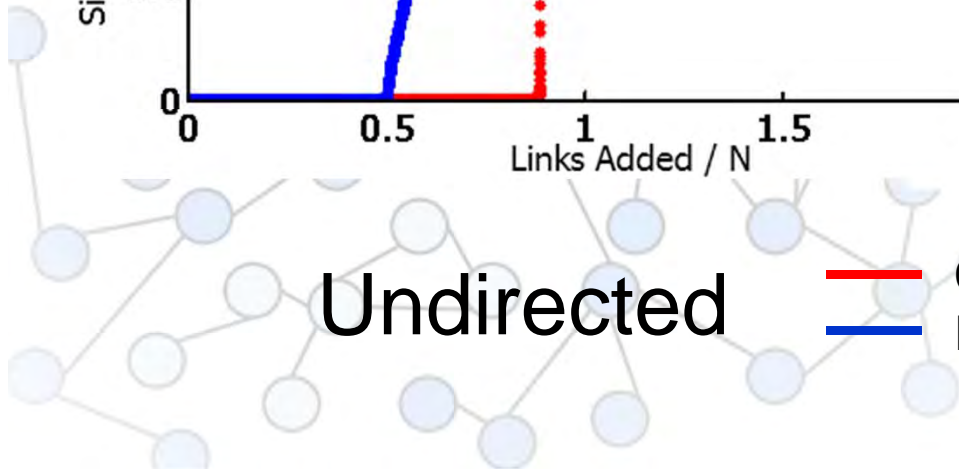
Delayed transition and explosiveness



Undirected

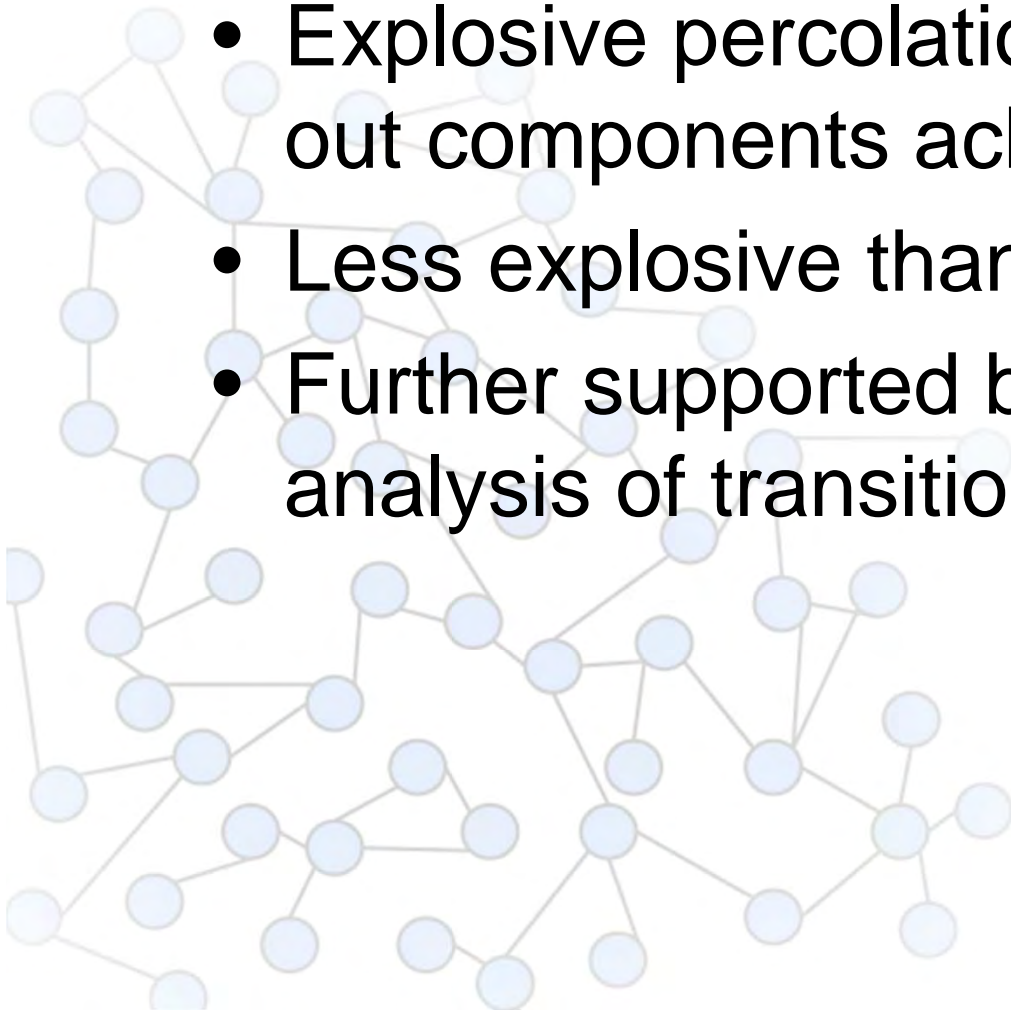
— Competitive
— Random

Directed



Conclusion

- Explosive percolation for giant in and out components achieved
- Less explosive than undirected case
- Further supported by quantitative analysis of transition (see poster)



Acknowledgements

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TREND

NSF

