Potential Flow Model of a Vortex Street Near a Fish-like Body

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Abstract

Vortex streets occur behind bluff objects and traveling fish. Here we construct a potential flow model of a vortex street passing near a fish-like body to simulate velocity and pressure distributions on the fish's surface. This information can be used by an autonomous underwater vehicle to estimate the original flow field.

Flow Model

Complex potential function in ζ plane (flow near circle):

$$W(\zeta,t) = W_{\text{free}}(\zeta) + W_{\text{vortex}}(\zeta,t)$$

Freestream potential:

$$W_{\text{free}}(\zeta) = U_\infty(e^{-i\alpha}(\zeta-\zeta_0) + e^{i\alpha}\frac{a^2}{\zeta-\zeta_0})$$

Vortex potential (single vortex):

$$\frac{i\Gamma_j}{2\pi}\log(\zeta-\zeta_j)$$

'Mirror' vortices added to satisfy boundary conditions:

$$W_{\text{vortex}}(\zeta,t) = \sum_j \frac{i\Gamma_j}{2\pi}[\log(\zeta-\zeta_0) + \log(\zeta-\zeta_j) - \log(\zeta-\zeta_0 - \frac{a^2}{\zeta_j^* - zeta_0})]$$

Joukowsky transformation (maps circle to airfoil):

$$z = \zeta + \frac{c_0^2}{\zeta}$$

Velocity field in z plane (flow near airfoil):

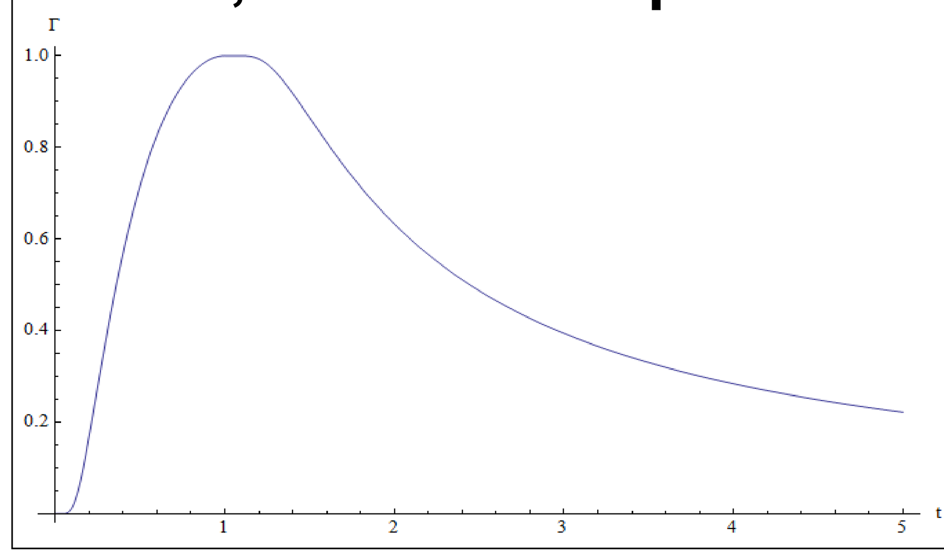
$$f_\zeta^* = \frac{\partial W}{\partial \zeta} \qquad f^*(z,t) = \frac{f_\zeta^*(\zeta(z),t)}{\frac{\partial z}{\partial \zeta}\big|_{\zeta(z)}},\ u = \text{Re}(f^*),\ v = -\text{Im}(f^*)$$

Nomenclature:

U_∞ - Freestream velocity, α - Angle of attack, Γ_j - Circulation strength, ζ_j - Vortex position, a - Circle radius in the ζ plane, c_0 - Joukowsky transformation coefficient, ζ_0 - Circle center in the ζ plane

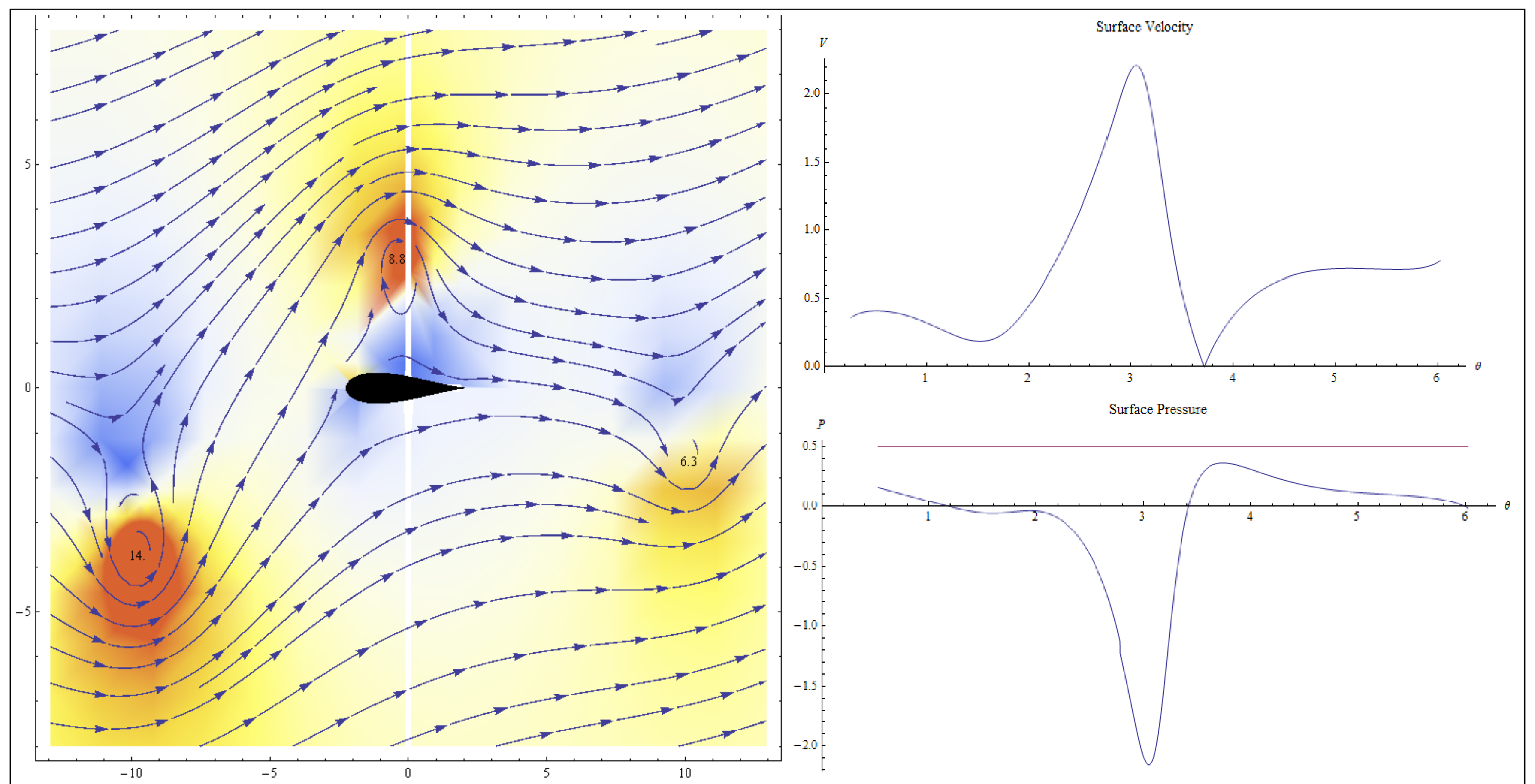
Vortex Decay

Vortex decay due to viscous effects is modeled by defining Γ as a function of time. The decay function is a solution of the Navier-Stokes equations for a single vortex, modified for potential flow.



$$\Gamma(t) = \begin{cases} 1 - e^{-\frac{\beta}{t-\mu}} & t > \mu \\ e^{\frac{(t-\mu)^2}{(t-\mu)^2-\mu^2}} & t > 0 \\ 0 & t \le 0 \end{cases}$$

Flow Visualization



Surface velocity is calculated by taking the norm of the velocity vector at points on the fish's surface. Surface pressure is calculated by Bernoulli's equation for time-dependent potential flows:

$$P(z,t) = P_0 - \frac{1}{2}\rho V(z,t) - \rho\frac{\partial\phi}{\partial t}\Big|_{z,t}$$

(The red line is the pressure at the far field.)

(Simulated) Flow Estimation

Noise added

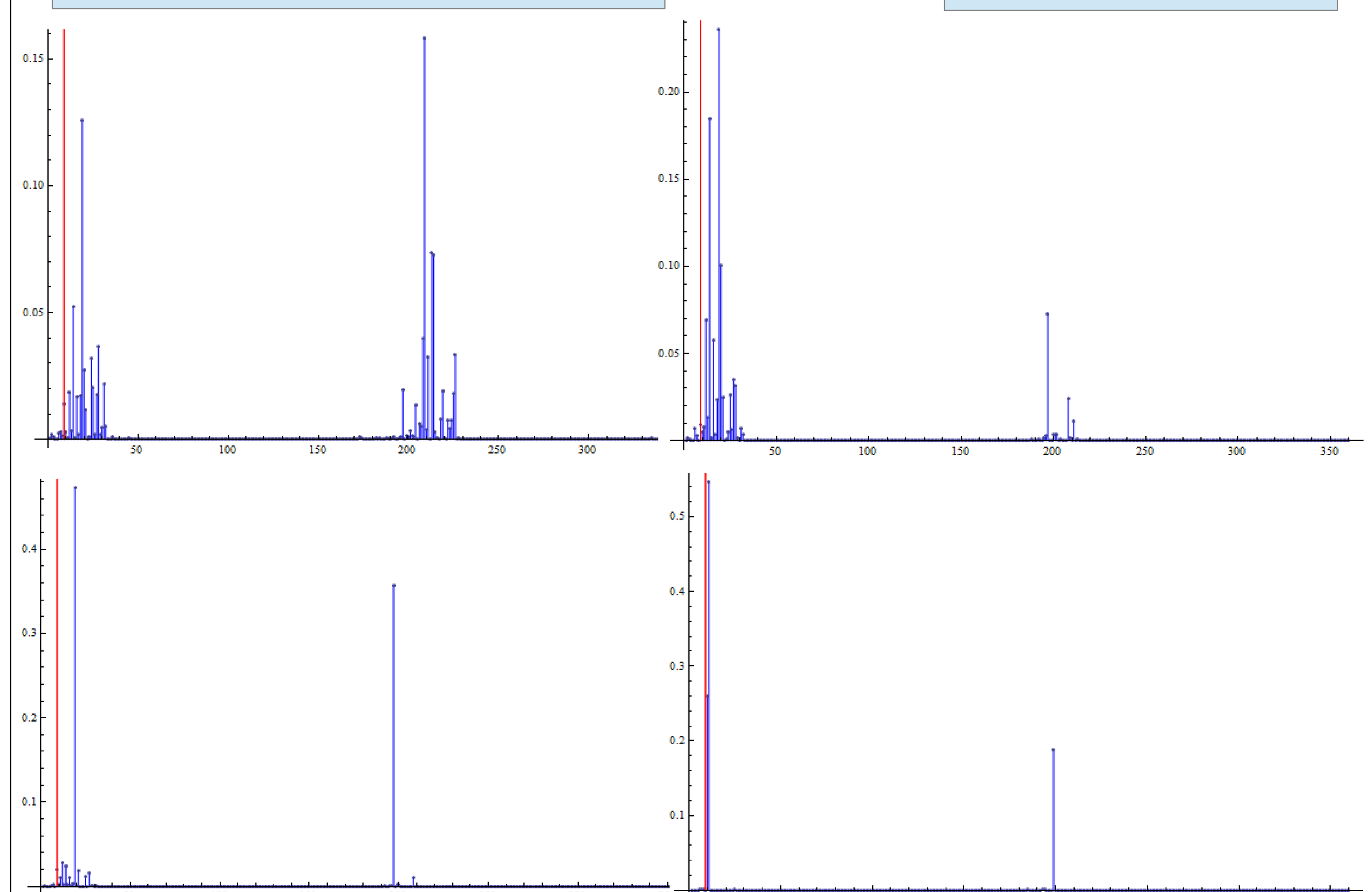
"Measured" flow

Model w/ unknown flow parameter(s)

Model w/ all parameters

Bayesian Filter

Estimated flow parameter(s)



Posterior probabilities for the angle of attack after 1, 2, 4 and 8 time steps with 23 simulated velocity sensors on the surface of the fish with N(0, .25) noise, a freestream velocity of 1 and an 'unknown' angle of attack of 9° (shown as the red line).

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