Potential Flow Model of a Vortex Street Near a Fish-like Body

Joshua Brulé, University of Maryland/TREND 2012
Dr. Derek Paley, CDCL
Flow past a bluff object

• Fluid dynamics is governed by the Navier-Stokes equations:

\[
\rho \left( \frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} \right) = -\nabla p + \nabla \cdot \mathbf{T} + \mathbf{f},
\]

• $1,000,000 prize for proving/disproving existence and smoothness

http://www.grc.nasa.gov/WWW/k-12/airplane/dragsphere.html
Kármán vortex street

Courtesy of Cesareo de la Rosa Siqueira, University of Sao Paulo, Brazil
Potential Flow Modeling

- Flow velocity represented via complex potential function $W(\zeta, t)$
- Real part is x-axis, Imaginary part is y-axis
- Equivalent to *inviscid*, *irrotational* solutions of Navier-Stokes
- Vortex potential:

$$\frac{i \Gamma_j}{2\pi} \log(\zeta - \zeta_j)$$

"A model of the lateral line of fish for vortex sensing."
Ren Z, Mohseni K. 2012
Vortex near a (circular) fish

- (Insert your own spherical cow joke)
- Boundary conditions (i.e. no flow through fish surface) are satisfied by adding 'mirror' vortices inside the circle
- Complex potentials are linear; easy to add in freestream velocity and additional vortices

"A model of the lateral line of fish for vortex sensing."
Ren Z, Mohseni K. 2012
Vortex Near Airfoil

- Joukowsky transformation maps circles ($\zeta$ plane) to airfoils ($z$ plane)

\[ z = \zeta + \frac{c_0^2}{\zeta} \]

- By varying the transform coefficient and the location of the circle (in the $\zeta$ plane) we can produce every fish height/width ratio
Vortex Decay / Position

- An actual vortex decays over time due to viscous effects.
- For potential flow models, we have to add that in 'by hand'.
- Vortex position is prescribed by a path function (in this model, a straight line in the z plane).

\[ \Gamma(t) = \begin{cases} 
1 - e^{-\frac{\beta}{t-\mu}} & t > \mu \\
\frac{(t-\mu)^2}{e(t-\mu)^2-\mu^2} & t > 0 \\
0 & t \leq 0 
\end{cases} \]

Based on Saffman, P.G. Vortex Dynamics. 1992
Potential Vortex Street
(Simulated) Flow field estimation

- "A blind fish can school" (Pitcher, T.J. et. al. 1976)
- Fish can navigate by detecting pressure and velocity distributions on their body's surface
- What about robot-fish?

![Diagram of flow field estimation process]
Estimating angle of attack (prior)

23 virtual velocity sensors, with \( N(0, .15) \) noise, freestream velocity 1, actual AoA = 9 degrees
Estimating angle of attack \((t = 0)\)
Estimating angle of attack (t = 1)
Estimating angle of attack \((t = 2)\)
Estimating angle of attack ($t = 3$)
Estimating angle of attack (t = 4)
Estimating angle of attack ($t = 5$)
Estimating angle of attack (t = 6)
Estimating angle of attack (t = 7)
Estimating angle of attack \((t = 8)\)
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