Stability and Bifurcation Analysis of the Planar Two-Point Vortex System

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Ocean Vortex Pairs

Synthetic Aperture Radar of Ocean Vortex Pair (15 x 13 km), near Sicily in the Mediterranean Sea
(Ivanov and Ginzburg, 2002)

Slocum Glider
(Teledyne Webb Research)
The System

• Two vortices rotate around a common center
• If rotate frame with them, get coherent structures
• (Left: equal circulation strengths in the rotating frame)
Equilibria Solutions
Bifurcation Parameter

- If
  - $\Gamma = \text{circulation strength}$
  - $\Gamma \downarrow 1 \neq 0$
  - $\Gamma \downarrow 1 = \text{positive constant}$

- Then the parameter is $\mu = \frac{\Gamma \downarrow 2}{\Gamma \downarrow 1}$

- Bifurcation points:
  - $\mu = -1$
  - $\mu = 0$
First Bifurcation: $\mu = -1$
Second Bifurcation: $\mu = 0$

- $\mu = -0.08$
- $\mu = 0$
- $\mu = 0.08$
Conclusion

• Found equilibria
• Bifurcations when:
  • Equal and opposite circulation
  • One vortex stops spinning
• Now understand system better
Thank you
Equations: Equilibria

• When $y=0$, solve for $x$:

\[
(\Gamma_{\downarrow 1} + \Gamma_{\downarrow 2}) \gamma_2 x \gamma_3 + (\Gamma_{\downarrow 2} \gamma_2 - \Gamma_{\downarrow 1} \gamma_2)(D)x \gamma_2 - [(\Gamma_{\downarrow 1} + \Gamma_{\downarrow 2}) \gamma_2 + \Gamma_{\downarrow 1} \Gamma_{\downarrow 2}](D \gamma_2) x - (\Gamma_{\downarrow 2} \gamma_2 - \Gamma_{\downarrow 1} \gamma_2) D \gamma_3 = 0
\]

• Otherwise:

\[
x = -D/2 \left( \Gamma_{\downarrow 2} - \Gamma_{\downarrow 1} \right) / \left( \Gamma_{\downarrow 1} + \Gamma_{\downarrow 2} \right)
\]

\[
y = \pm \sqrt{3} / 2 \ D
\]
Equations: Bifurcation

- \( \mu = -1 \)
  \[
  \omega = \Gamma \frac{1}{2\pi D^2} \left(1 + \mu \right)
  \]

- \( \mu = 0 \)
  \[
  \lambda = \pm \sqrt{-12\mu}
  \]