Frequencies Can Be Coupled Oscillators

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What are optoelectronic feedback loops?

- Laser ➔ Modulator ➔ Photodiode ➔ Time Delay ➔ Modulator
- Capable of high-dimensional chaos
How can frequencies be coupled?

• The dynamical behavior of the feedback loop is driven by the nonlinearity of a Mach-Zender modulator:

\[ P_{\text{out}} = P_{\text{in}} \cos(x + \varphi_{\text{in}})^2 \]

• [optical output] = [optical input] * [variable attenuation].
• Using trigonometric identities and a series expansion with $\phi_o = \pi/4$, the most significant terms become:

$P_{\text{out}} = P_{\text{in}}/2 [1 - 2x + 4/3 x^3]$. 

• Using a Fourier transform for $x$:

$$ x(t) = \sum_{j} A_j e^{i \omega_j t} + A_j^* e^{-i \omega_j t} , $$

$$ P_{\text{out}} = P_{\text{in}} / 2 \left[ 1 - 2 \left( \sum_{j} A_j e^{i \omega_j t} + A_j^* e^{-i \omega_j t} \right) + 4/3 \left( \sum_{j} A_j e^{i \omega_j t} + A_j^* e^{-i \omega_j t} \right)^3 \right] . $$
• Each term in the frequency summation of $x$ is coupled to one another by the cubic nonlinearity:

$$P_{\text{out}} = \frac{P_{\text{in}}}{2} \left[ \frac{4}{3} \left( A_{\downarrow j} A_{\downarrow j+1} A_{\downarrow j+2} \right) e^{i(\omega_{\downarrow j} + \omega_{\downarrow j+1} + \omega_{\downarrow j+2}) t} + \ldots \right].$$

• Altering one particular frequency will effect each other frequency, thus the frequencies appear globally coupled.
Experimental Setup

- Select 3 frequencies using comb filter
Cont.

- Experiment Schematic and Implementation
- Filter and Time Delay Implemented on field programmable gate array (FPGA)
Data/Results

- Uncoupled System
Cont.

- Coupled System
Conclusions & Further Research:

• Frequencies appear to display coupled behavior.

• It is not clear if synchrony will occur or under what conditions.

• Further research questions include:
  1. Do similar frequencies tend to have stronger coupling?
  2. Will this coupling effect remain or even grow when more frequencies are allowed?

• The system could potentially resemble a Kuramoto-like scheme of globally coupled oscillators.
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