Avalanches and Entropy Generation in Sand Timers

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Background

• Forest fires, neuronal activity, biological evolution
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• Self-organized criticality
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• Self-organized criticality
  • Power law distributions
  • $P(x) = ax^{-k} \rightarrow \log(P(x)) = \log(a) - k\log(x)$
  • Scale invariance
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  - Scale invariance
- Avalanche definition
- Hourglass
Questions

• How does the way we observe this system affect whether/how we see avalanches?
• How does this affect information content in data?
• Can we relate these?
Experimental Setup

- Detector
- Objective
- Lens
- LED
- Image plane
- Object
- Aperture used as field stop
Aperture Diameter (compared after x9 magnification):
Small aperture – 1.17 mm ≈ 1 grain diameters
Medium aperture – 2.59 mm ≈ 2 grain diameters
Large aperture – 3.58 mm ≈ 3 grain diameters

Orifice sizes:
5 minute timer – 0.66 mm ≈ 4 grain diameters
2 minute timer – 0.81 mm ≈ 5 grain diameters
Time Series

Small aperture, large orifice

<table>
<thead>
<tr>
<th>Normalized Variance ($\sigma/V_{mean}$)</th>
<th>Small aperture</th>
<th>Large aperture</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 min – large orifice</td>
<td>0.11836</td>
<td>0.10736</td>
</tr>
<tr>
<td>5 min – small orifice</td>
<td>0.08734</td>
<td>0.08334</td>
</tr>
</tbody>
</table>
Time Series

Small aperture, large orifice
Size and Duration Distributions

Large aperture, large orifice
Entropy

• Quantification of randomness of system
• Entropy rate calculated by Cohen-Procaccia algorithm – function of temporal and spatial resolution.
• A reliable, computationally inexpensive calculation of entropy rate.
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• D: embedding dimension, N: number of reference points
Discretization of analog signals
Entropy

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- D: embedding dimension, N: number of reference points
- Entropy rate: \( h_D = H_D - H_{D-1} \)
Entropy Generation Rates

Observation Area
5 Minute Timer (0.66 mm diameter) $h_3$

Orifice Size
Large Aperture $h_3$
Conclusions

• Unlikely that we saw avalanches
  • Large enough minimum
  • Power law

• Observation area, orifice size did not affect distributions

• Higher variance in larger orifices, smaller observation areas

• Correspondingly higher entropy generation rates for these data sets
Future Work

• Different orifice to particle size ratios
• Move vertical observation position along hourglass neck