Interactions Between Drain Vortices in a Rotating System

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Vortices

• Swirls in fluid flows
• Ubiquitous in real fluids (Lautrup, 2011)
• Crucial to our understanding of turbulence (Frisch, 1995)
• Vorticity ($\omega$) is defined by:

$$\omega = \nabla \times \mathbf{v}$$
Rotating Fluids

Navier-Stokes Equations:

\[
\frac{D\mathbf{v}}{Dt} = -\frac{\nabla p}{\rho} - \nu \nabla^2 \mathbf{v}
\]

\[\nabla \cdot \mathbf{v} = 0\]

\(\mathbf{v}\) : velocity field
\(p\) : Pressure field
\(\rho\) : Density
\(\nu\) : viscosity
Rotating Fluids

Navier-Stokes Equations:

\[
\frac{D\mathbf{v}}{Dt} = -\frac{\nabla p}{\rho} - \nu \nabla^2 \mathbf{v} - 2\boldsymbol{\Omega} \times \mathbf{v}
\]

\[\nabla \cdot \mathbf{v} = 0\]

\[\boldsymbol{\Omega} : \text{Rotation rate}\]

Coriolis acceleration
Rotating Fluids

Navier-Stokes Equations in geostrophic flow:

\[
\frac{\partial \mathbf{v}}{\partial t} \approx -\frac{\nabla p}{\rho} - 2\Omega \times \mathbf{v}
\]

\[\nabla \cdot \mathbf{v} = 0\]
Consequences

Taylor-Proudman Theorem:
\[ \frac{\partial v}{\partial z} \approx 0 \]

Geostrophic wind:
\[ v = \frac{\Omega \times \nabla p}{2\Omega^2} \]
Consequences

Taylor-Proudman Theorem:

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Flow invariant with height
Consequences

Taylor-Proudman Theorem:

\[ \frac{\partial v}{\partial z} \approx 0 \]

Geostrophic wind:

\[ v = \frac{\Omega \times \nabla p}{2\Omega^2} \]

Flow invariant with height

Flow perpendicular to pressure gradient
Experiment Setup

- 30 cm diameter tank
- 7.5 mm drain holes, 10 cm apart
- Drained water pumped back into the tank through perforated hose around the edge
- Full setup rotates
- Corotating video setup
Line-like vortices

• Long, thin vortices

\[ \nu = \frac{C}{r} \hat{\phi} \]

• Satisfy geostrophic requirements

• Expected orbit frequency:

\[ f \sim \frac{C}{g^2} \]

Vortex Triangle
(Krishnamurthy, Aref et. al. 2003)
Data

Plot of vortex orbit frequency \( (f) \) v. separation distance \( (g) \)

- Equation: \( f = A g^b \)
  - \( b = -1.4 \pm 0.2 \)
  - \( R^2 = 0.899 \)
  - \( \chi^2 = 0.030 \)

- Equation: \( f = a_1 + A g^{-2} \)
  - \( A = 11.1 \pm 0.75 \)
  - \( R^2 = 0.895 \)
  - \( \chi^2 = 0.030 \)
Data

$f$ as a function of $\Omega$

difference may suggest hysteresis

$f$ and $g$ as a functions of $\Omega$

apparently related
Conclusions

- These vortices seem very similar to vortex filaments.
- Geostrophic vortex generator works, and can be used to study filament-like vortex interactions.
- Applications to turbulence.
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References


